

Residual Stiffness Analysis of Flexural Concrete Elements with Composite Reinforcement Systems

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Abstract

Various materials and reinforcement technologies have been created, but this versatility causes a severe engineering problem – there is no versatile technique suitable for the development of efficient reinforcement system. In reinforced concrete systems, residual stiffness of the element can estimate the efficiency of the reinforcement. The stiffness is closely related to the structural integrity of cracked sections resisting the deformation increase of the member. This study introduces a simplified approach for the flexural stiffness analysis. It employs a new testing layout designed with the purpose to form multiple cracks in a small laboratory specimen. The proposed analytical model is based on the following assumptions: smeared crack concept; linear strain distribution within the section depth; elastic behaviour of reinforcement and compressive concrete; a rectangular distribution of stresses in the tensile concrete. The latter assumption enables obtaining a closed-form solution of the residual stiffness problem in terms of the equivalent tensile stresses in the concrete. The achieved solution requires neither iterative calculations nor a description of the loading history. The proposed methodology is also acceptable for estimating residual stiffness of elements with various combinations of reinforcement. The application of the proposed technique is illustrated experimentally. Several composite reinforcement schemes, including internal steel and glass fibre reinforced polymer (GFRP) bars, carbon fibre reinforced polymer (CFRP) sheets and near-surface mounted (NSM) strips in different combinations, are considered. The analysis of the test results reveals a substantiate efficiency of the external CFRP reinforcement systems concerning the internal reinforcement as the reference. The application of the CFRP sheets substantially increases the effectiveness of the tensile concrete. In essence, the decay of the equivalent stresses acting to the tensile concrete does not exceed 50% of the tensile strength of the concrete in the presence of the external reinforcement. However, the formation of a critical shear crack caused the failure of all specimens with composite reinforcement. That is a consequence of low resistance of the fibre reinforced polymer materials to a shear load.

1 Introduction

There is a general steady increase of application of innovative construction materials and concrete, as the case of using fibre reinforced polymers (FRP) as reinforcement. Various materials and reinforcement technologies have been developed. However, there is no uniform methodology to compare the mechanical characteristics of different reinforcing systems. Residual stiffness of the element can estimate the efficiency of reinforcement systems.

The purpose of this study is to offer a rational way of composing efficient composite reinforcement systems. Residual stiffness of flexural members is the focus of the research. It is closely related to the structural integrity of cracked sections of the concrete resisting the deformations of the element.

Tension-stiffening models can represent the stiffness in an averaged manner. Numerous studies investigated the tension-stiffening issue. However, only several works addressed flexural effects. Fundamental studies by Kaklauskas & Ghaboussi [1] and Torres et al. [2] could be mentioned in this context. Elaborate numerical procedures are an intrinsic attribute of the “exact” approaches [3]. Iterative nature of the algorithms often complicates applicability of the exact techniques: the calculation errors are accumulated following the load history [3]. The development of more reliable algorithms employed the reinforcement-related tension-stiffening concept was the consequence of the further improvements [4-6]. Such models, however, are not useful for the analysis of the elements reinforced with a combination of different types of internal bars, near-surface mounted strips and external sheets.

This study offers a new simplified approach for the flexural stiffness analysis introducing the equivalent stresses to measure the residual stiffness of the element. The proposed methodology is acceptable for estimating the residual stiffness of elements with different combinations of reinforcement. The developed analytical model requires neither iterative calculations nor a description of the loading history. The application of the proposed technique is illustrated experimentally representing a closed-form solution of the flexural stiffness problem in terms of the equivalent tensile stresses in the concrete.

2 Description of the beam specimens

The testing layout was designed with the purpose to form multiple cracks in a small laboratory specimen. That enables to reduce scatter of the result due to the crack formation peculiarities. The geometry of the beams was chosen using the iterative trial-and-error procedure. The satisfaction of the simplified modelling assumption (rectangular distribution of stresses in the tensile concrete) was used as the design criterion. In other words, the exact tension-stiffening diagram [3] had to have the shape as close as possible to rectangular.

Figure 1 shows the cross-sections of the specimens; Table 1 describes the parameters of the beams. In this table, h , b , and d are the height, width, and effective depth of the cross-section; A_r and E_r are the area and elasticity modulus of the reinforcement, respectively. The table also defines the compressive strength of the $\varnothing 150 \times 300$ mm concrete cylinder at 28 days ($f_{c,28}$) and at age (t) of testing (f_{cm}).

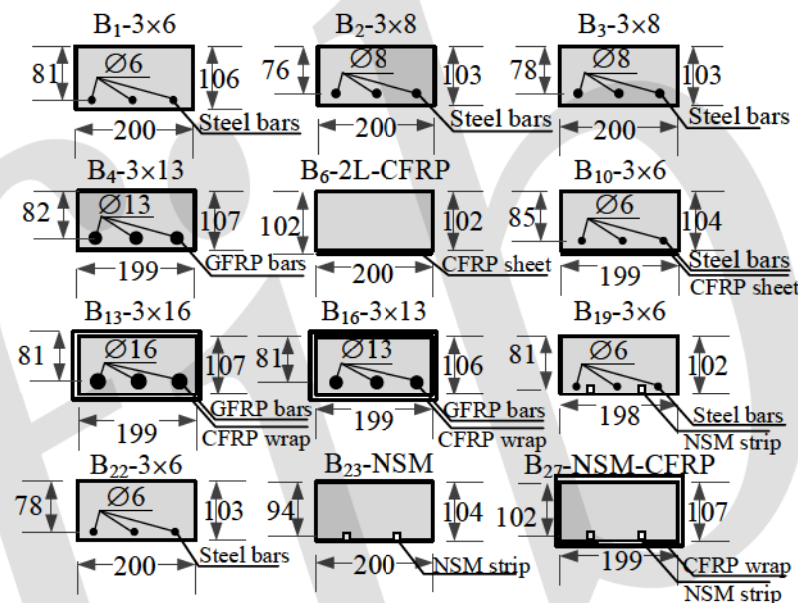


Fig. 1 Cross-sections of the beam specimens (dimensions are in mm).

Table 1 Main parameters of the beam specimens.

Specimen	h , mm	b , mm	d_1 , mm	d_2 , mm	A_{r1} , mm ²	A_{r2} , mm ²	E_{r1} , GPa	E_{r2} , GPa	$f_{c,28}$, MPa	t , days	f_{cm} , MPa	δ_1 , kPa	δ_2 , kPa
B1-3×6	106	200	81	—	83.2	—	202.1	—	53.2	43	57.2	2.36	—
B2-3×8	103	200	76	—	149.8	—	206.7	—	53.2	42	57.1	1.46	—
B3-3×8	103	200	78	—	149.8	—	205.7	—	60.4	38	69.5	2.62	—
B4-3×13-GFRP	107	199	82	—	380.1	—	46.0	—	55.9	39	68.6	2.67	3.32
B6-2L-CFRP	102	199	—	103	—	23.2	—	230.0	61.8	48	69.7	4.82	7.45
B10-3×6-2L-CFRP	104	199	79	105	83.2	23.2	202.1	230.0	58.3	49	61.9	3.52	4.64
B13-3×16-GFRP (shear)	107	199	81	—	593.7	—	46.0	—	51.7	45	61.8	1.87	2.11
B16-3×13-GFRP (shear)	106	199	80	—	380.1	—	46.0	—	60.4	46	69.5	2.52	3.23
B19-3×6-NSM	102	198	78	92	83.2	28.0	202.1	170.0	63.1	57	70.3	2.31	—
B22-3×6	103	200	78	—	89.2	—	202.1	—	58.3	57	61.9	2.33	—
B23-NSM	104	200	94	—	28.0	—	170.0	—	61.0	62	68.1	2.95	4.05
B27-NSM-CFRP (shear)	107	199	95	107	28.0	23.2	170.0	230.0	61.0	63	68.1	2.73	2.98

As it can be observed in Fig. 1, most of the tested specimens had a combined reinforcement. Therefore, Table 1 specifies two reinforcement types. Except for B19-3×6-NSM, the subscript “1” of the parameters d , A_r , and E_r corresponds to the internal reinforcement (either bars or NSM strips), whereas the index “2” describes the external

sheets. Considering the specimen B₁₉₋₃×6-NSM, the subscripts “1” and “2” correspond to the internal bars and NSM strips, respectively. Physical parameters of the steel reinforcement were determined experimentally (through the tensile tests of three samples), whereas the values from the mill-certificates are provided for the non-metallic reinforcements.

The beams were cast using steel forms; they were unmoulded in 2-3 days after the casting. All specimens reinforced with internal bars were stored in water to reduce the shrinkage effect. The NSM system and CFRP sheets were attached to the dry specimens before the tests. The specimens were dried for approximately 24 hours before gluing the sheets.

3 Beam tests and discussion of the results

The 1000 mm long specimen is tested under a four-point-bending scheme with 600 mm pure bending zone and two 150 mm shear spans, a . That fits the Eurocode 2 [7] condition of elements not requiring design of shear reinforcement, i.e. $a \leq 2d$, where d is the effective depth of cross-section (Table 1).

The specimens were produced in several batches using the same concrete compositions. The composition of the concrete (for one cubic meter) is following: 356 kg of cement CEM I 42.5 R, 163 l of water, 177 kg of limestone powder, 890 kg of 0/4 mm sand, and 801 kg of 4/16 mm crushed aggregates; 1.97% (by the cement weight) of the superplasticiser *Mapei Dynamon XTend* and 3.5 kg of the admixture *SCP 1000 Optimizer*. The beams were reinforced with either 13 mm and 16 mm GFRP bars or 8 mm and 6 mm steel bars (Fig. 1). Several samples had NSM CFRP strips and external CFRP sheets attached to the tension surface of the specimen. The previous study [8] indicated the insufficient resistance of FRP composites to shear loads. In this study, three beams had CFRP wrapping in the shear zones to avoid premature failure of the specimens. These specimens are designated as “(shear)” in Table 1. Two CFRP loops (each of 100 mm width) were placed close to each other in the shear zone, as shown in Fig. 2b. The same unidirectional *MapeWrap C UNI-AX* CFRP sheets were used as external and shear reinforcement. The equivalent thickness of the dry material was 0.166 mm.

As shown in Fig. 2, the surface deformations were assessed using linear variable displacement transducers (LVDT) L_{10} - L_{15} attached in two continuous lines to the side surface of the specimen. The vertical displacements in pure bending zone were measured by nine LVDT (L_1 - L_9 , Fig. 2). The data logger ALMEMO 5690-2 collected the output results of all LVDT and the load cell. Figures 3 and 4 show the surface strains and vertical displacement assessment results.

Deformations and crack pattern of the opposite side surface of the specimens were fixed with the help of digital image correlation system (DIC). Two cameras IMAGER E-LITE 5M were used for this purpose. The cameras were placed on a tripod at the 3.0 m distance from the specimen. The gap between the cameras was equal to 0.4 m. The cameras, incorporating a charge-coupled device (CCD) detector, have a resolution of 2456×2085 pixel at the 12.2 fps rate. Such arrangement of the apparatuses enables a crosscheck of the test outcomes: the collected measurements enable to construct three each other independent moment-curvature diagrams, which can be used for the analysis.

Figure 5 demonstrates the evolution of the cracks identified by the DIC system highlighting the cracking sequence. The cracking patterns are related to certain bending moments indicated at the schemes. Figure 6 shows the final crack pattern of the beams.

The formation of a critical shear crack caused the failure of all specimens with composite reinforcement. That is a consequence of low resistance of the fibre reinforced polymer materials to a shear load. (The beam specimens had no shear reinforcement.) An increase in the area of FRP reinforcement has no significant effect on the shear resistance – the doubled number of CFRP sheets increase the load-bearing capacity only by 20%. The external wrapping acting in the shear zone (the specimens B₁₃, B₁₆, and B₂₇) also had no substantial effect on the shear resistance of the beam specimens. Although the load-bearing capacity of the strengthened specimens was increased by almost 30%, the formation of a critical shear crack caused the failure of the strengthened specimens. Another essential result was observed in the specimen B₁₉ reinforced with a combination of the 6 mm steel bars and NSM CFRP strips (Fig. 1). Notwithstanding the effect of the steel bars, the shear failure was the consequence of the bending test. That makes a requirement of the minimum shear resistance mandatory for the design of structural elements with composite reinforcement systems.

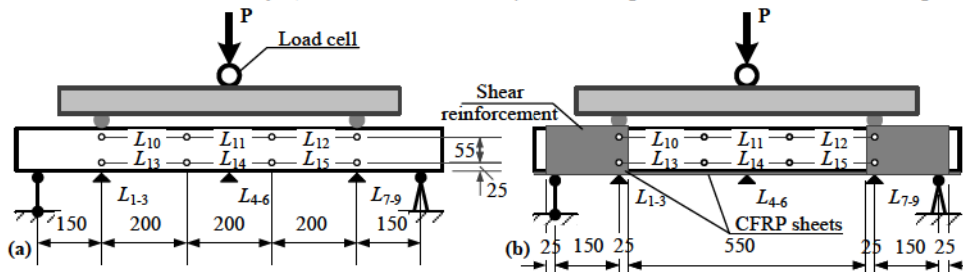


Fig. 2 Loading scheme of typical (a) and strengthened in shear (b) beams (dimensions are in mm).

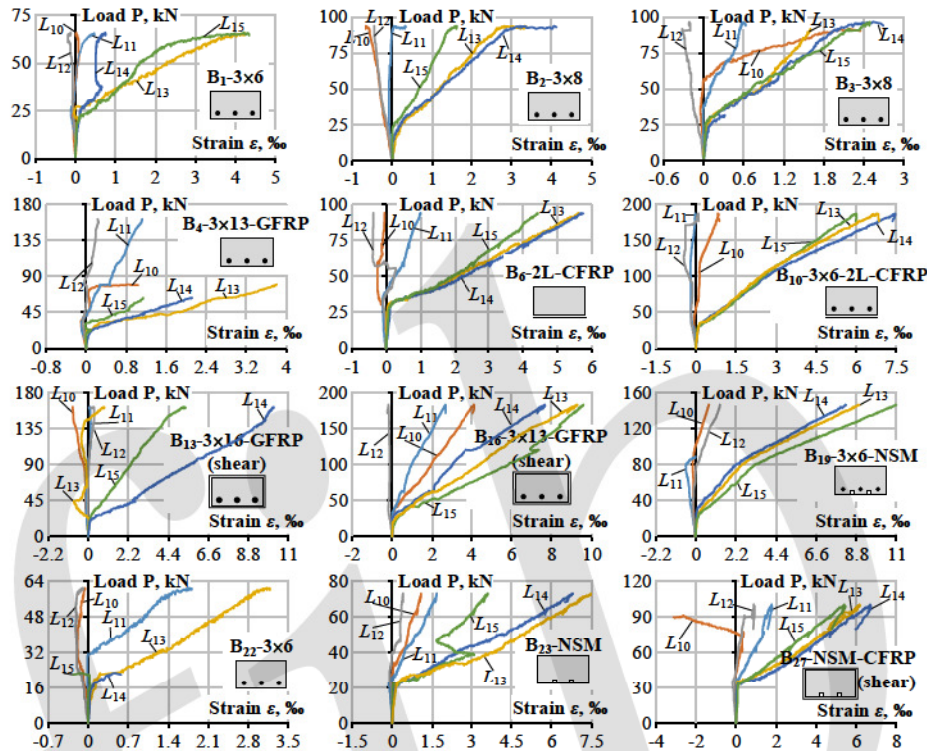


Fig. 3 Surface deformations of the beams.

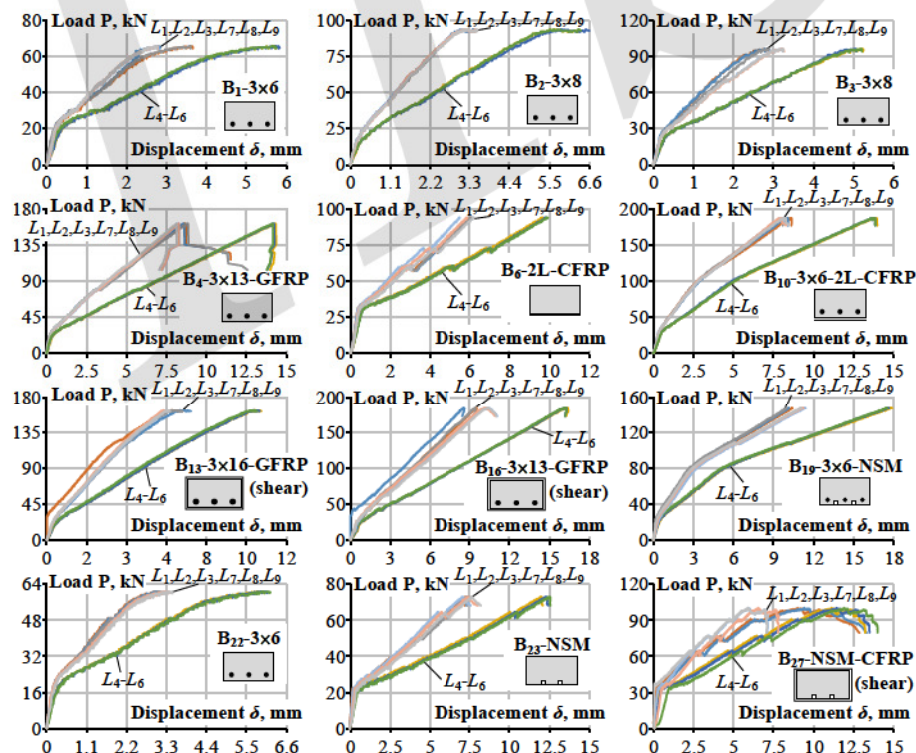


Fig. 4 Vertical displacements of the beams.

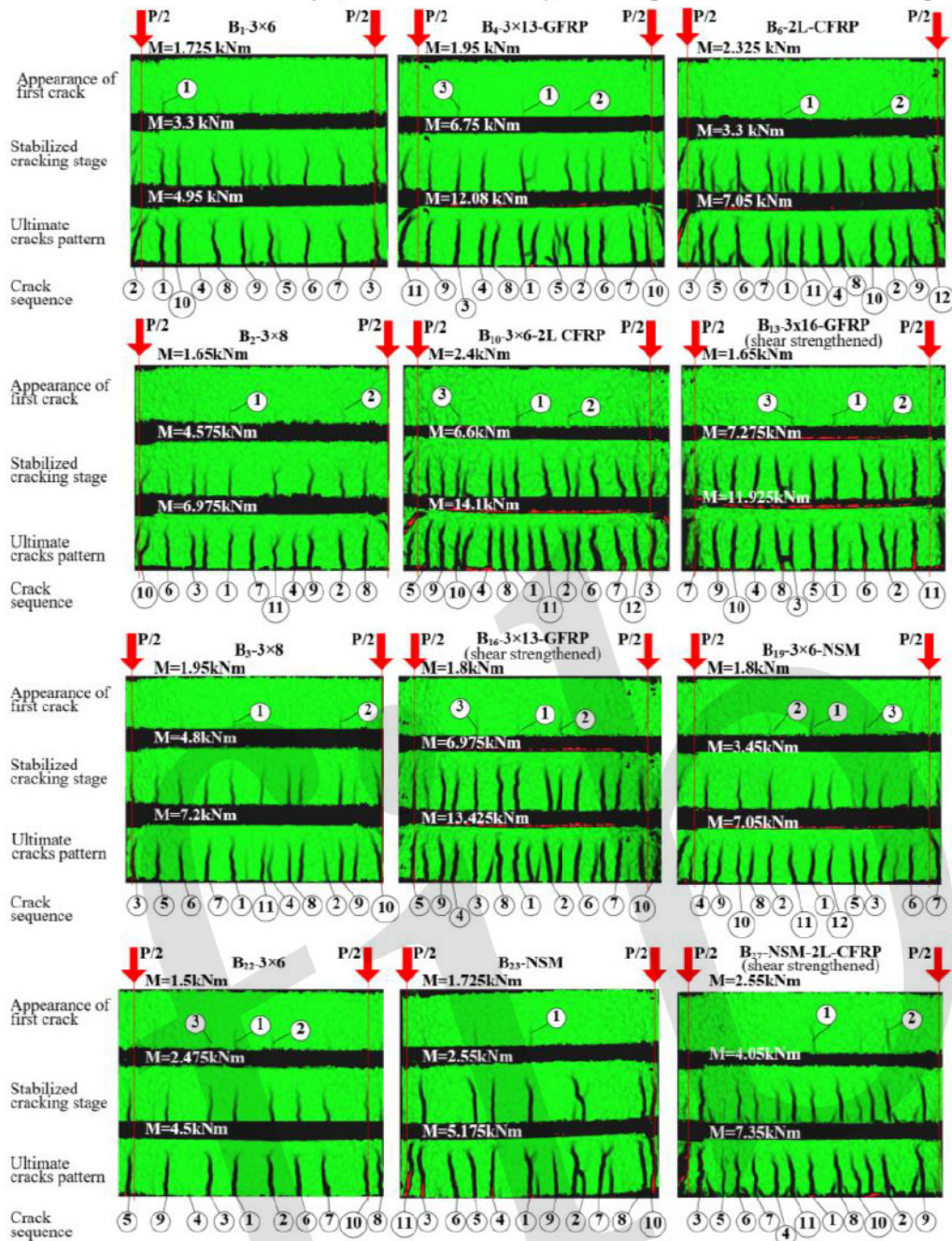


Fig. 5 Cracking schemes identified by the DIC system.

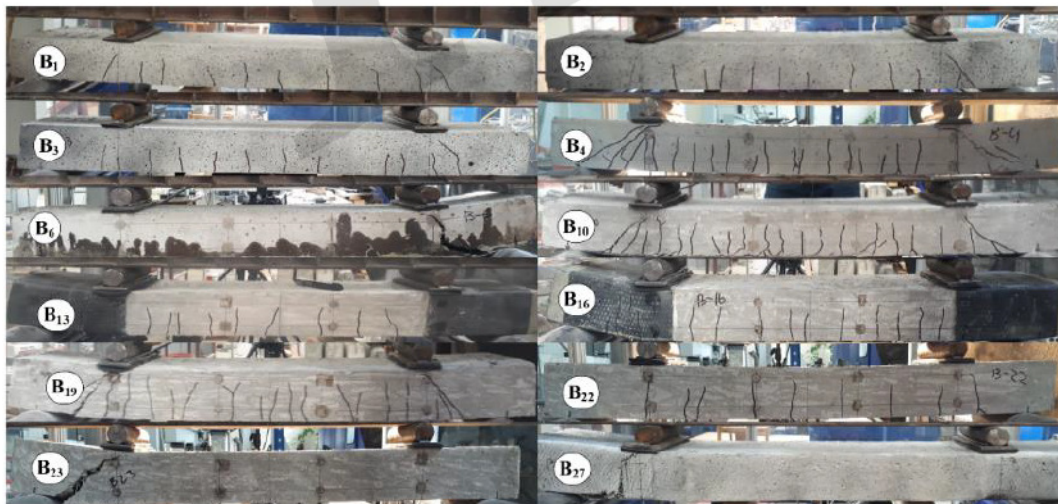


Fig. 6 Final crack pattern of the beams.

4 The residual stiffness analysis

The stiffness analysis is based on the moment-curvature response of the pure bending zone. The monitoring scheme (Fig. 2) enables the curvature estimation in different ways: from vertical displacements of LVDT L_1 - L_9 and from surface deformations identified using the LVDT L_{10} - L_{15} or DIC system. The averaged curvature can be obtained from the vertical displacement readings assuming a circular arc shape of the deflection curve along the pure bending zone [9] as follows

$$\kappa = \frac{2 \cdot \delta}{(l_b/2)^2 + \delta^2} = \frac{8 \cdot \delta}{l_b^2 + 4 \cdot \delta^2}, \quad \delta = \frac{\delta_4 + \delta_5 + \delta_6}{3} - \frac{\delta_1 + \delta_2 + \delta_3 + \delta_7 + \delta_8 + \delta_9}{6}, \quad (1)$$

where δ is the deflection over the pure bending zone; δ_i is the displacement obtained by LVDT L_i ($i = 1 \dots 9$, Fig. 2); l_b is the length of the bending zone (equal to 600 mm).

The surface strains were employed for the assessment of the curvature as follows

$$\kappa = \frac{(\varepsilon_{13} + \varepsilon_{14} + \varepsilon_{15}) - (\varepsilon_{10} + \varepsilon_{11} + \varepsilon_{12})}{3 \cdot h_e}, \quad (2)$$

where ε_i is the strain estimated using the reading by the LVDT L_i ($i = 10 \dots 15$, Fig. 2); h_e is the distance between the gauging lines (equal to 55 mm). A similar procedure was applied to estimate the curvature using the DIC system that is capable of determining relative displacements between any points recognised at the monitoring surface [10].

The analysis of the alternative curvature values enables of identifying the measurement errors. For instance, an illumination alteration or movement of the camera can corrupt the DIC results; the crack appearance at the LVDT support can distort the measurement results; a cyclic load can cause the erroneous outcomes of the LVDT devices. The presence of the overlapping measurements increases the reliability of the monitoring results that can be verified by comparing the alternative outcomes.

Figure 7a shows the moment-curvature diagrams constructed using the surface deformations captured by the DIC system. Figure 7b gives the corresponding equivalent stiffness values. The latter diagrams were derived using the proposed analytical model. That implies the following assumptions: smeared crack approach; linear strain distribution within the section depth; elastic behaviour of reinforcement and compressive concrete; rectangular distribution of stresses in the tensile concrete. The latter assumption enables an explicit solution of the residual stiffness problem expressed in terms of the equivalent stresses, σ_t^* , acting to the tensile concrete.

Figure 8 describes the assumed model that employs the transformed section method, combining different types of reinforcement (Fig. 8a) in one modified component (Fig. 8b). The following equations determine the effective depth d_r , area A_r , and elasticity modulus E_r of the transformed reinforcement:

$$d_r = \frac{\sum_{i=1}^3 E_i A_i d_i}{\sum_{i=1}^3 E_i A_i}, \quad A_r = \frac{1}{E_r} \sum_{i=1}^3 E_i A_i, \quad E_r = E_1, \quad (3)$$

where E_i is the elasticity modulus of the i -th reinforcement component; Fig. 8a defines other notations.

The modelling is based on the equilibrium equations of internal forces and bending moments to the centroid of the equivalent tensile stress diagram (Fig. 8d):

$$\mathbf{F}_t^* + \mathbf{F}_r - \mathbf{F}_c = 0, \quad \mathbf{F}_r \left(d_r - \frac{h + y_c}{2} \right) + \mathbf{F}_c \left(\frac{h + y_c}{2} - \frac{y_c}{3} \right) - \mathbf{M}_{ext} = 0, \quad (4)$$

where $\mathbf{F}_t^*$ is the *equivalent* resultant force of the tensile stresses in the concrete; $\mathbf{F}_r$ and $\mathbf{F}_c$ are the internal forces acting to the tensile reinforcement and the compressive concrete; y_c is the depth of the compressive zone. In the above equations, the internal forces can be expressed through the respective stresses:

$$\sigma_t^* (h - y_c) b + \sigma_r A_r - \sigma_c \frac{y_c b}{2} = 0, \quad \sigma_r A_r \left(d_r - \frac{h + y_c}{2} \right) + \sigma_c \frac{y_c b}{2} \left(\frac{3h + y_c}{6} \right) - \mathbf{M}_{ext} = 0. \quad (5)$$

For the given curvature, κ , the stresses in the tensile reinforcement and the most compressed concrete layer (Figs. 8c and 8d) can be expressed using the conditions of strain compatibility:

$$\sigma_r = \kappa (d_r - y_c) E_r, \quad \sigma_c = \kappa y_c E_c. \quad (6)$$

Taking into account expressions (6), equation (5) can be modified as

$$\kappa (d_r - y_c) E_r A_r \left(d_r - \frac{h + y_c}{2} \right) + \kappa y_c E_c \frac{y_c b}{2} \left(\frac{3h + y_c}{6} \right) - \mathbf{M}_{ext} = 0 \quad (7)$$

and expressed as a third order polynomial

$$C_3 y_c^3 + C_2 y_c^2 + C_1 y_c + C_0 = 0 \quad (8)$$

with coefficients

$$C_3 = \frac{\kappa E_c b}{12}, \quad C_2 = \frac{\kappa}{4}(2E_s A_s + E_c b h), \quad C_1 = \frac{\kappa E_s A_s}{4}(h - 3d), \quad (9)$$

$$C_0 = \frac{\kappa d^2 E_s A_s}{2} \left(2 - \frac{h}{d}\right) - M_{ext}.$$

The polynomial (8) has three roots. From the condition $0 < y_c \leq h$, the position of the neutral axis is defined as

$$y_c = \frac{1}{3C_3} \left\{ 2\bar{C} \cos \left(\frac{1}{3} \cos^{-1} \left\{ -\frac{27C_3^2 C_0 - 9C_3 C_2 C_1 + 2C_2^3}{2\bar{C}^3} \right\} \right) - C_2 \right\}, \quad \bar{C} = \sqrt{C_2^2 - 3C_3 C_1}. \quad (10)$$

The equivalent average stress in the tensile concrete is determined using equation (5); Fig. 8c defines the corresponding strain. The following formulas express those parameters:

$$\sigma_t^* = \kappa \frac{y_c^2 b E_c - 2(d - y_c) E_s A_s}{2(h - y_c) b}, \quad \varepsilon_t^* = \kappa \frac{h - y_c}{2}. \quad (11)$$

Figure 9 illustrates the residual stiffness tendencies observed in the specimens with composite reinforcement. The analysis of the residual stiffness (expressed in term of the equivalent stresses σ_t^*) reveals the substantial efficiency of the external CFRP reinforcement system concerning the internal reinforcing schemes as the reference. The area under the equivalent stress diagram can determine the deformation energy related to the relative contribution of the cracked concrete. Table 1 gives the energy values δ_1 and δ_2 corresponding to the deformation ε_t^* equal to 2‰ and 3‰. The maximal energy values are highlighted in the table. The efficiency decreases with increasing the reinforcement ratio. That is a well-known fact [1], but the effect of the external sheets should be the object of further research.

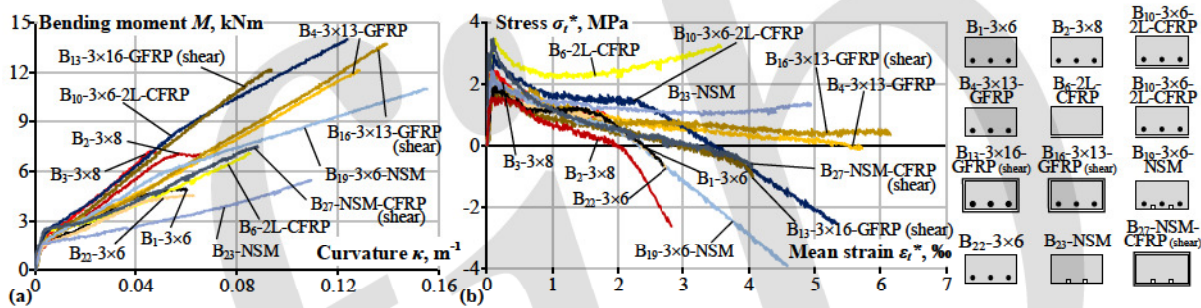


Fig. 7 The residual stiffness analysis: (a) moment-curvature diagrams of the beams; (b) the corresponding equivalent residual stresses in the tensile concrete σ_t^* , see equation (11).

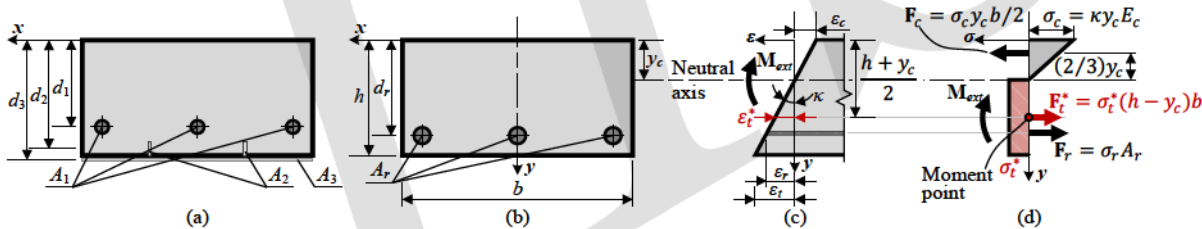


Fig. 8 Analytical model of the element subjected to bending moment M_{ext} : (a) RC section; (b) transformed section; (c) strain profile; (d) stresses and internal forces acting to the section.

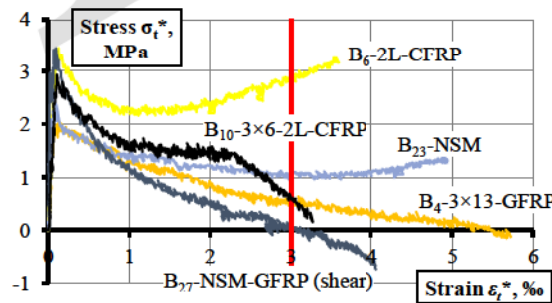


Fig. 9 Equivalent stress-strain diagrams of the selected specimens.

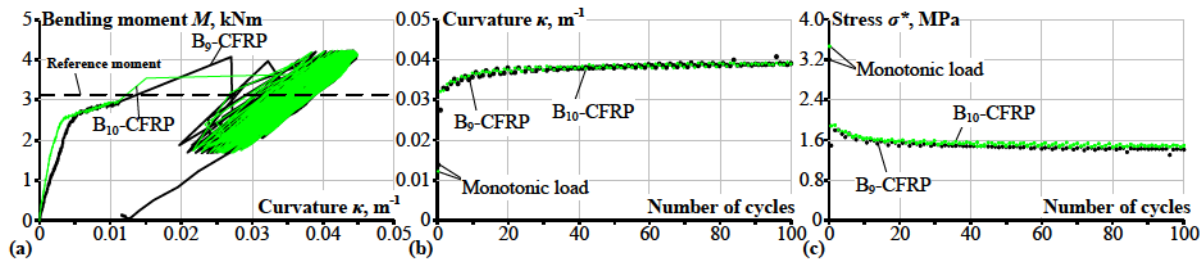


Fig. 10 Analysis of beam specimens subjected to cyclic load [8]: (a) moment-curvature diagram; (b) reference curvatures; (c) the resultant equivalent stresses σ_e^* .

The ability to analyse the residual stiffness of elements subjected to cyclic loading is an essential feature of the proposed analytical model. Figure 10a demonstrates the moment-curvature diagrams of the beam specimens subjected to cyclic load. The residual stiffness analysis is related to an arbitrarily set reference bending moment. The dashed line in Fig. 10a indicates the reference load. The curvature evolution is estimated by using one curvature value at each loading cycle that corresponds to the ascending of diagrams shown in Fig. 10a. Figure 10b reveals an increase in the curvature with load cycles. Figure 10c demonstrates the application results of the proposed analytical model clarifying the stiffness decay tendency.

5 Conclusions

- The proposed methodology is suitable for estimating residual stiffness of elements with various combinations of reinforcement: any kinds of internal bars, external sheets, and near-surface mounted strips can be used in different combinations.
- The external CFRP sheets demonstrated the most efficient deformation resistance among the considered reinforcement systems. This effect is to be the object of further research.

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