Vilniaus universitetas
Fizikos fakultetas
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YUKAWA COUPLINGS FOR GRIMUS NEUFELD MODEL

Bachelor studies final work

Computer physics and modeling

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Allowed to defend	2018-05-24
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Vilnius 2018

Vilniaus universitetas
Fizikos fakultetas
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JUKAVA KOEFICIENTAI GRIMUS NEUFELD MODELYJE

Pagrindinių studijų baigiamasis darbas

Kompiuterinė fizika ir modialiavimas

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Vilnius 2018

Contents

1 Introduction

For the better part of last century it was thought that neutrinos are massless. The resolve of solar neutrino problem proved that neutrinos must have mass. Further experiments showed that that mass must be very small compared to other particles. The theory in Standard Model doesn't explain these masses. It is still a mystery to this day.

Grimus and Neufeld [1] introduced a model how neutrinos could get masses. Later Grimus and Lavoura [2] improved theoretical understanding and gave mathematical expression for mass calculations. In the model one neutrino gains mass through seesaw mechanism and second one from one loop correction.

The main goals in this bachelors thesis were to familiarize with Grimus Neufeld model, rederive and program the analytical expression for neutrino masses in the model and find expressions for Yukawa couplings in the same model. I continued from my bachelor practice work [3] about Standard Model and seesaw mechanism.

In first four chapters I explain the fundamentals of Standard Model relevant for the work. In fifth and sixth chapters I introduce Grimus Neufeld model. Finally I explain what calculations took place in seventh chapter. Lastly I show my results and discuss them in conclusion and summary.

2 Group Theory

Group theory is the most important theoretical mechanism in particle physics. Only because of it, it is possible to account for different forces: electromagnetism, weak and strong forces in our model. It dictates how they are depicted in the Lagrangian - the main theoretical structure that is used in creation and prediction of any model.

2.1 Groups

Group is abstract set of some elements: discrete or continuous and some operation defined on then. For example all real numbers(except 0) and multiplication form a group. Formal definition of a group is:

*If we have set of objects G and some interaction between those objects *, the group candidate (G, *) is a Group only, and only if it has these 4 following properties:*

1. *Closure:* $g_1 \in G \wedge g_2 \in G : g_1 * g_2 \in G$
2. *Associativity:* $g_1 \in G \wedge g_2 \in \wedge g_3 \in G : (g_1 * g_2) * g_3 = g_1 * (g_2 * g_3)$
3. *Identity:* $g \in G \wedge \exists e \in G : e * g = g * e = g$
4. *Inverse:* $\forall g \in G : \exists g^{-1} \in G : g * g^{-1} = g^{-1} * g = e$

Mentioned example of all real numbers minus 0 and multiplication have all of these properties. Simply groups are just some elements that through an operation interact with each other.

Generally groups don't have Commutative property. If they do they are called Abellian groups, if don't non Abellian.

2.1.1 Representations

Groups don't have unique way to writing them. There are infinite ways to write elements of any group. And if group's elements are expressed in matrices they can have any dimension. Certain logical system or a rule of writing elements in a group is called representation. If g are elements of the group (G,*): $g \in G$ then representation of that group D(G) must obey the relation:

$$D(g_1) * D(g_2) = D(g_1 * g_2) \tag{1}$$

Groups have infinite amount of representations. But their general, underlying structure always stays the same. For example group can be initially defined to have certain logic to it how it acts on modulus space. And changing its representation wont change that. The structure is what, ultimately, defines the group.

In particle physics representations are square matrices of some dimension and numbers. Representations can have different dimensions, though for every group there exists representation that cannot be reduced to smaller dimension - Irreducible representation. It is the simplest and neatest way to express the group.

2.2 Lie Groups

Groups with continuous elements are called Lie Groups. Each element is parameterized by one or multiple continuous variables φ_n: $g(\varphi_1, .., \varphi_n)$ and thus have infinite elements.

Lie groups in particle physics are mainly of four categories(with their notation in parentheses): orthogonal ($O(N)$), special orthogonal($SO(N)$), unitary($U(N)$) and special unitary($SU(N)$). These are N dimensional matrices that either are real or complex, multiplied with self transpose or hermitian conjugate reduce to unit matrix and either are special(symbol S) or not. Special means that matrix's determinant(or modulus) is equal 1: $|U(N)| = 1$.

It is worth noting that these matrices govern the structure of the group. But representations of them can have any dimension. The representation in which the elements have same dimension and the matrix that group represents is called Fundamental.

The representation of the group is governed by group's generators. Generator of a group is what spawns the representation and is defined as T_a

$$T_a = -i \left.\frac{\partial D_n}{\partial \varphi_a}\right|_{\varphi_a=0} \tag{2}$$

where D_n is representation with dimension n. For any group there are infinite amount of generators, just as representations. Representation of a group with m generators and m parametrization variables φ is generated as follows

$$D_n(\varphi_a) = e^{i\varphi_a T_a} \tag{3}$$

where in the exponent summation over a is assumed. It is written in Einstein's summation notation.

Generators of the group follow very important commutation relation:

$$[T_a, T_b] = i f_{abc} T_c \tag{4}$$

where f_{abc} is called structure constant of the group. If structure constant known then everything about the group is known and it can be written in any representation.

2.2.1 $SU(2)$

This group is one if not the most important group in Standard Model. $SU(2)$ is group of transformation:

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \tag{5}$$

where a, b, c and d are complex numbers. Now using unitary and special matrix properties:

$$A^{-1} = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = A^{\dagger} = \begin{pmatrix} a^* & c^* \\ b^* & d^* \end{pmatrix} \tag{6}$$

so we get

$$A = \begin{pmatrix} a & b \\ -b^* & a^* \end{pmatrix} \tag{7}$$

where determinant must be equal 1. In this there are two free complex or 4 free real numbers, since to describe complex number you need two real. From determinant requirement one can drop one of those 4 and there are left 3 free numbers. So $SU(2)$ group has 3 parameters.

Out of three generators it is chosen that the third one is always diagonal, by convention. The representation of this group is described by the highest eigenvalue j the third generator can get. So they range from 0 to infinity with $\frac{1}{2}$ increments: 0, $\frac{1}{2}$, 1, $1\frac{1}{2}$, 2, ... The dimension itself of the representation is $(2j+1) \times (2j+1)$.

The $j = \frac{1}{2}$ representation is especially important. The generators in this representations are:

$$T_1 = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, T_2 = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, T_3 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{8}$$

which you can recognize as Pauli matrices multiplied by one half. The structure constant tensor for SU(2) is Levi-Civita tensor ϵ_{abc}.

2.3 Lorentz Group

Lorentz group is one of the most important groups in Standard Model. It governs how mathematical objects we use to describe particles transform when we move from one observational frame to the next. It is the basis for the Standard Model.

2.3.1 Lorentz Transformations

In special relativity time and space are treated equally. This means that two observers moving relative to each other view the same event in different positions and times. The Galilean transformations no longer work.

Spacetime four vector is defined as vector of four components:

$$x^\mu = \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} \quad \mu \in \{0,1,2,3\} \tag{9}$$

where t is time, x, y and z are coordinates in observers inertial frame. c is speed of light which is chosen to be equal 1, along with plank constant h = 1 in so called God units. If there are two observers, the original one and another one moving with speed v in direction of x axis. Then if there is an event in spacetime the original observer will register at spacetime vector x in his frame, while moving observer at spacetime vector x' in his reference frame. Turns out the product $x^2 + y^2 + z^2 - t^2$ is always independent of reference frame. Mathematically it can be written as:

$$t^2 - x^2 - y^2 - z^2 = x_\mu x^\mu = x^\mu x_\mu \tag{10}$$

Where $x_\mu = \eta_{\mu\nu} x^\nu$ is covariant spacetime four vector and $\eta_{\mu\nu}$ is Minkowski metric with signature $+---$.

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{11}$$

Minkowski metric is unique to space we live in - Minkowski space: with 3 spatial dimensions and one time dimension.

The four momenta p^μ is a momenta vector in Minkowski space:

$$p^\mu = \begin{pmatrix} \frac{E}{c} \\ p_x \\ p_y \\ p_z \end{pmatrix} \quad \mu \in \{0,1,2,3\} \tag{12}$$

With very important relation:

$$p^2 = p_\mu p^\mu = \frac{E^2}{c^2} - \overrightarrow{p}^2 = m^2 c^2 \tag{13}$$

It is called on mass shell relation.

There are two types of Lorentz transformations rotations of coordinates with each other and rotations of coordinate with time, former being called Euler transformations and latter Boosts. Six in total.

If there are 2 observer frames, one stationary s and one moving in x direction with speed $v_x$compared the stationary one. Then one boost B and Euler transformation E that rotates coordinates around y axis would be:

$$B_x = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad R_y = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & 0 & \sin\theta \\ 0 & 0 & 1 & 0 \\ 0 & -\sin\theta & 0 & \cos\theta \end{pmatrix} \tag{14}$$

Where $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ and $\beta = \frac{v}{c}$. These transformations act on spacetime four vectors: $x'^{\mu} = (B_x)^{\mu}_{\nu} x^{\nu}$, $x''^{\mu} = (R_y)^{\mu}_{\nu} x^{\nu}$. They are elements of Lorentz group that are parameterized by four elements of spacetime four vector.

2.3.2 Structure and Representations

When one calculates the generators for Lorentz group using Lorentz transformations one can find (with using change of basis) that 6 Lorentz generators form two separate groups with 3 generators each that are closed under relation (4). With the structure constant of ϵ_{abc} for both which means that those two subgroups are copies of $SU(2)$!

The representations of Lorentz group are given by two values of j: (j, j') for each $SU(2)$ copy. Copies are totally separate the total dimension of transformations is going to be $[(2j+1)(2j'+1)] \times [(2j+1)(2j'+1)]$. Most important representations have to be with values $\left(\frac{1}{2}, 0\right)$,$\left(0, \frac{1}{2}\right)$ and $\left(\frac{1}{2}, \frac{1}{2}\right)$.

Representation of (0,0) is a scalar, and scalars don't transform anything.

The $\left(\frac{1}{2}, 0\right)and\left(0, \frac{1}{2}\right)$ representations are both in 2×2 dimensions and act on 2 dimensional modulus space vectors ω which are called spinors. The $\left(\frac{1}{2}, 0\right)$ representation is called Left-Handed spinor representation and $\left(0, \frac{1}{2}\right)$ Right-Handed spinor representation. Because they describe transformations of spinors that are part of Left-Handed and Right-Handed coordinate systems respectively. Spinors that transform to them are called Left-Handed and Right-Handed accordingly. Them both together are called Weyl spinors. Weyl spinors are basically spinors with only one handedness or chirality.

Representation that uses both parts of Lorentz group$\left(\frac{1}{2}, \frac{1}{2}\right)$is called vector representation. Because it has dimension of 4 and transforms spacetime vectors (9).

2.3.3 Left and Right handed spinors

Weyl spinors can be interchanged with itself if they represent the same particle. For that is used quantity $i\sigma^2$, where $\sigma^2$is the second Pauli matrix and generaly it has the expression of

$$i\sigma^2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{15}$$

Using this we can change between handedness of Weyl spinors(which are of two components):

$$\omega_L = i\sigma^2 \omega_R^* \tag{16}$$

Similarly:

$$\omega_R = i\sigma^2 \omega_L^* = i\sigma^2 \left(i\sigma^2 \omega_R^*\right)^* = i\sigma^2(-i\sigma^2)\omega_R = \omega_R \tag{17}$$

where the identity $i\sigma^2(-i\sigma^2) = 1$ is used.

3 Fermions

Neutrinos are ferminions and thus it is needed to understand what governs fermion dynamics and what mathematical structure Standard Model employs for $\frac{1}{2}$spin particles. Also it is important to understand difference between different kinds of fermion fields, mainly Dirac and Majorana fermions. Since seesaw mechanism employs use of only the latter one.

3.1 Clifford Algebra

Clifford algebra in n dimensions is set of n matrices γ^μ where $\mu \in \{0, 1, ..., n-1\}$such that anti commutation relation

$$\{\gamma^\mu, \gamma^\nu\} = -\eta^{\mu\nu} I_{4\times 4} \tag{18}$$

is satisfied. Where η Minkowski metric (11). In context of this work the dimension n is chosen to be equal 4. Turns out there are many solutions to Clifford Algebra and they all are connected with similarity transformation. The most straightforward one being so called Weyl or Chiral representation:

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix} \tag{19}$$

where $\sigma^\mu \in \{1, \sigma^1, \sigma^2, \sigma^3\}$, $\overline{\sigma}^\mu \in \{1, -\sigma^1, -\sigma^2, -\sigma^3\}$ and σ are Pauli matrices. Solutions to Clifford algebra, so called gamma matrices are connected to Lorentz group in the way that they can be used to create Lorentz generators in any representation. Also the representation of gamma matrices define the exact form of Dirac and Majorana spinors in 4 dimensions.

3.2 Dirac equation

All fermion fields must obey Dirac equation:

$$(i\gamma^\mu \partial_\mu - m)\,\psi = 0 \tag{20}$$

This, again, is expressed in God units where c = h = 1. The term $\partial_\mu \in \left(\frac{\partial}{\partial t}, \vec{\nabla}\right)$ is four gradient(gradient in the 4dimensional spacetime). Since gamma matrices are in dimension of 4, they also must act on 4 component spinor ψ. This spinor is called Dirac spinor and generally in itself has both Left-Handed and Right-Handed parts or two Weyl spinors with different chiralities. In Chiral representation of gamma matrices Dirac spinor has the form:

$$\psi = \begin{pmatrix} \omega_L \\ \omega_R \end{pmatrix} = \begin{pmatrix} \omega_{L1} \\ \omega_{L2} \\ \omega_{R1} \\ \omega_{R2} \end{pmatrix} \tag{21}$$

Where ω is corresponding chirality(handednes) Weyl spinor with 2 elements. Or more fitting to Standard Model:

$$\psi = \begin{pmatrix} \omega_L \\ i\sigma^2 \omega_L'^* \end{pmatrix} \tag{22}$$

Where ω_L and ω_L' are not necessarily the same field.

The expresion $\overline{\psi}$ is called Dirac adjoint and is defined as:

$$\overline{\psi} = \psi^\dagger \gamma^0 \tag{23}$$

3.2.1 Chirality

While in Chiral representations of gamma matrices it is always easy to get right and left handed parts, they are just second and first 2 component Weyl spinors in the dirac spinor. How ever there is useful tool to get them in any representation. Gamma 5 matrix is defined as:

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 \tag{24}$$

With it it is possible to make chirality operators:

$$L = \frac{1}{2}\left(1 - \gamma^5\right) \tag{25}$$

$$R = \frac{1}{2}(1 + \gamma^5) \tag{26}$$

Typical Dirac spinor will have both left and right chiral parts $\psi = \psi_L + \psi_R$

Then using (25) and (26) it is possible to get those parts individually:

$$\psi_L = L\psi, \quad \psi_R = R\psi \tag{27}$$

3.3 Majorana Fermions

While Dirac fields can be complex Majorana can only be real. General condition for reality of spinor field is:

$$\psi = C\overline{\psi}^T \tag{28}$$

Where C is called charge conjugation matrix and has form in chiral representation:

$$C = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix} = \begin{pmatrix} -i\sigma^2 & 0 \\ 0 & i\sigma^2 \end{pmatrix} \tag{29}$$

Majorana fields, defined with this gain some interesting properties. First of all they become real(always). Second, since C is charge conjugation operator it means that Majorana particles are anti particles of themselves. Lastly the Lorentz covariant conjugate $\psi^{(c)} = C\overline{\psi}^T$ switches the chirality of the spinor:

$$\begin{pmatrix} -i\sigma^2 & 0 \\ 0 & i\sigma^2 \end{pmatrix} \left(\begin{pmatrix} \omega_L \\ 0 \end{pmatrix}^\dagger \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \right)^T = \begin{pmatrix} -i\sigma^2 & 0 \\ 0 & i\sigma^2 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \omega_L^* \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ i\sigma^2\omega_L^* \end{pmatrix} \tag{30}$$

It makes the left field right. Majorana particle is made from two different chiral fields that are, at the core, the same:

$$\psi_M = \begin{pmatrix} \omega_L \\ i\sigma^2\omega_L^* \end{pmatrix} \tag{31}$$

It also interesting that charge conjugation operator C seems to be 4 dimensional equivalent of (15) for Dirac spinors.

3.4 Lagrangian for Fermions

Lagrangian expression for fermions is given by Dirac Lagrangian:

$$\mathcal{L}_{\mathrm{D}} = \overline{\psi}\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi \tag{32}$$

Using Euler-Lagrange equation:

$$\partial_{\mu}\left(\frac{\partial\mathcal{L}_{\mathrm{D}}}{\partial\left(\partial_{\mu}\psi\right)}\right) = \frac{\partial\mathcal{L}_{\mathrm{D}}}{\partial\psi} \tag{33}$$

It is possible to get out the Dirac equation (20). First term in the parenthesis is kinetic term for the field, which represents how field dynamics will occur. Second term is mass term, and for seesaw mechanism it is very important. Using both Dirac and Majorana formulation one can extract Dirac and Majorana mass terms from this Lagrangian.

3.4.1 Majorana Mass Term

Majorana particles have mass terms similar to the one in the Dirac Lagrangian. If we have Right-Handed real Weyl field ω_R the Majorana spinor of it is going to be:

$$\psi = \begin{pmatrix} i\sigma^2\omega_R^* \\ \omega_R \end{pmatrix} \tag{34}$$

Then in the Lagrangian the mass term is going to be:

$$m\overline{\psi}\psi = m\begin{pmatrix} \omega_R^{\dagger}(-i\sigma^2) & \omega_R^T \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} i\sigma^2\omega_R^* \\ \omega_R \end{pmatrix} = m\left[\omega_R^{\dagger}(-i\sigma^2)\omega_R + \omega_R^T i\sigma^2\omega_R\right] \tag{35}$$

Now if choose to represent each Weyl field ω_R with a 4 dimensional Dirac spinor instead of 2 dimensional and see that charge conjugation matrix is real (29) and using identities: $CC^{\dagger} = 1$ and $C^T = -C$ we can rewrite above equation neatly as:

$$m\overline{\psi}\psi = m\left[\omega_R^T C^{-1}\omega_R + H.c.\right] \tag{36}$$

H.c. means Hermitian conjugate. This is the Majorana mass term. It will be useful while pursuing the seesaw mechanism.

4 The Standard Model

The theory of Standard Model is the best physical system we, as a species, have to try to explain laws and origin of nature. It is extensive collection of mathematical structures, methods and physical unification attempts that best describe our universe. It is needed to understand how it works, and how it is established in order to understand how to extend it with methods like the seesaw.

4.1 Gauging the Symmetry

In order to introduce main forces to the Standard Model one needs to do something called gauging the symmetry. First of all it is good to note that Dirac Lagrangian (31) isn't changed under multiplication by any complex number or just $e^{i\alpha}$ - a phase. It is said that Lagrangian is invariant under global U(1) symmetry. It is invariant because if we transform our field:

$$\psi \to e^{i\alpha}\psi \tag{37}$$

the Lagrangian wont change since the extra phase factors of U(1) will just cancel out. And it is global because the phase angle α is constant in the dependence on spacetime coordinates, so it is the same in every point in spacetime.How ever if αwould depend on spacetime coordinates it would become local and change Lagrangian (19) since it would introduce extra derivative in the first term before cancellation of phase factor:

$$\mathcal{L}_{\mathrm{D}} = \overline{\psi}\left(i\gamma^{\mu}\partial_{\mu} - m - \gamma^{\mu}\partial_{\mu}\alpha\left(x^{\mu}\right)\right)\psi \tag{38}$$

The Gauging of Symmetry is a method to introduce well known fundamental forces to our theory. At first we break the symmetries manifesting in Lagrangian, in (37) case it was U(1). Now in order to get some theory of fundamental force out of it we need to fix it, so Lagrangian becomes invariant again, even under that certain symmetry(our case U(1)). That is done with introduction of two things: first there is a field A_{μ}, called Gauge field. In this case it is defined to transform under mentioned U(1) as:

$$A_{\mu} \to A_{\mu} - \frac{1}{q}\partial_{\mu}\alpha\left(x\right) \tag{39}$$

Then we change our derivative to covariant derivative D_{μ} that introduces the new field to the theory and Lagrangian:

$$D_{\mu} = \partial_{\mu} + iqA_{\mu} \tag{40}$$

By definition particles charge associated with a force is defined as a multiplier in corresponding forces gauge field's term. So in this case it is q. Introducing this to the Lagrangian gets rid of the unwanted terms, thus giving Lagrangian its invariance back. In this case the fundamental force is electromagnetism and charge q is electric charge. The introduced gauge field is electromagnetic 4 potential vector defined as $A^{\mu} = (\rho, \vec{A})$. The kinetic term for newly added field is defined as:

$$\mathcal{L}_{Kin.A} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \tag{41}$$

where $F_{\mu\nu}$ is electromagnetic field strength tensor: $F^{\mu\nu} = \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}$. Other fundamental forces(except gravity) are added with other certain groups: weak force with SU(2), strong with SU(3) and as discussed above electromagnetism with U(1).

4.2 Representations of Standard Model

The Standard Model has defined certain representations for particles that it contains. The main group of standard model, which gauge all the forces, as mentioned above is $SU(3)_C \times SU(2)_L \times U(1)_Y$. The subscript C stands for color, as $SU(3)$ creates (with gauging) color force or strong nuclear force. L stands for left. It means that it acts on left handed Weyl spinors that underlying entire Standard Model. And lastly Y stands for hypercharge, and interacts with regular charge in electro-weak force theory - the unification of weak and electromagnetic forces.

Particles in Standard Model can transform under only certain set of representations of groups mentioned above. Those representations are described by triplet of numbers in which first number is dimension of SU(3) representation, second dimension of SU(2) and last is hypercharge of U(1).

For leptons there are only two important representations and they are $\left(1, 2 - \frac{1}{2}\right)$ and $(1, 1, +1)$. Both describe particles that don't interact with strong nuclear force, since they "transform" under dimension 1 representation of SU(3) so they don't transform at all. In former representation 2 means that it transforms as a doublet with Left-Handed SU(2), while latter doesnt transform with it and represents Right-Handed particle. In standard model the only Higgs field has representations $\left(1, 2, -\frac{1}{2}\right)$. This means it has two symmetries: left handed SU(2) and U(1).

4.3 Particles of Standard Model

Particles in SM are introduced as left handed doublets and right handed singlets. Leptons and quarks are introduced in generations - doublets that transform under $\mathrm{SU}(2)_L$ and contain two particles whom are Left-Handed Weyl spinors. There are 3 generations in total for leptons and quarks.

The lepton generations are:

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \tag{42}$$

Which contain charged particles: electron, muon and tau coupled to their respecting neutrinos: electron neutrino, muon neutrino and tau neutrino.

These doublets correspond to $\left(1, 2 - \frac{1}{2}\right)$ representation. Anti particles can be added as right handed singlets under $(1, 1, 1)$ transformations.

Standard Model has only 1 Higgs doublet ϕ:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix} \tag{43}$$

where $\phi^+$is charged Higgs field and ϕ^o is neutral one.

4.4 The Higgs Mechanism

Higgs mechanism is probably the main theory in standard model. Due to it's effects all the massive gauge fields gain mass and in turn our reality works the way it does: weak nuclear force gauge bosons get mass and become short range and photon doesn't so electromagnetic force remains theoretically infinite in range.

Lagrangian for scalar particles is Klein-Gordon Lagrangian[4]:

$$\mathcal{L}_{KG} = \partial^\mu \phi^\dagger \partial_\mu \phi - m^2 \phi^\dagger \phi = -\partial^\mu \phi^\dagger \partial_\mu \phi - V(\phi^\dagger, \phi) \tag{44}$$

Second term on the right can be thought as a potential term V. The scalar field ϕ has mass because of it. When there is no particle i.e. $|\phi| = 0$ the lagrangian vanishes and we have perfect vacuum. However in nature it is not always the case. At small energies the Higgs gains non-zero vacuum value and its potential term becomes:

$$V = \gamma \left(\phi^\dagger \phi - \frac{1}{2} v^2 \right)^2 \tag{45}$$

where γ and Φ are real constants. In turn Higgs gets something called mexican hat potential, and non zero vacuum value. Now it has infinite vacuum states where $|\phi| = v$ which is nonphysical. However Higgs still has it's two symmetries: $SU(2)_L$ and $U(1)$. In Higgs mechanism they are both used to "rotate" Higgs to only have only one vacuum state.

It's done by redefining Higgs to have real vacuum expectation value. Vacuum expectation value(VEV) of Higgs is denoted $\langle 0 |\phi| 0 \rangle$. We use SU(2) symmetry to accumulate all of the VEV to neutral component of doublet and U(1) to making it real:

$$\begin{aligned} \langle 0 |\phi^+| 0 \rangle &= 0 \\ \langle 0 |\phi^o| 0 \rangle &= v \end{aligned} \tag{46}$$

Then redefining the Higgs doublet to be:

$$\phi = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}} \left(v + h(x) + iG^o \right) \end{pmatrix} \tag{47}$$

Where $G^+$and $G^o$are charged and neutral Goldstone bosons. This use of simmetry is called gauge fixing because when we define Higgs this way it doesnt have SU(2) and U(1) symmetry anymore.

This new Higgs doublet combined with local symmetry breaking gives masses to all massive gauge bosons - they get new quadratic terms with v and other parameters.

4.5 Yukawa coupling

Interaction between scalar and spinor fields in the Lagrangian is described by Yukawa coupling term:

$$\mathcal{L}_{Yuk} = y\phi\overline{\psi}\psi \tag{48}$$

Where ϕ is scalar field and ψ is our fermion field and y is coupling constant between those two. Higgs mechanism is introduced through scalar field ϕ and it gives mass to coupled fermion field.

The coupling to a doublet and right handed particle $\overline{p}$ looks like this:

$$\mathcal{L}_{Yuk} = -y_c \epsilon^{ij} \phi_i \overline{p}_c L_j - y_n \widetilde{\phi}^{\dagger i} \overline{p}_n L_i + H.c. \tag{49}$$

Where y_c and $y_n$are coupling constants to charged and neutral parts of lepton doublet, L is lepton double itself and H.c. means hermitian conjugate of previous terms. It is needed for completion of the theory.

Continuing with the breaking of the symmetry the scalar field is chosen to be (46) - the unitary gauge. This means that in second term only second element of doublet survives, and in second term only first, and choosing the doublet to be electron one it is then:

$$\mathcal{L}_{Yuk} = -\frac{y_c}{\sqrt{2}}(v+h)\overline{p}_c e - \frac{y_n}{\sqrt{2}}(v+h)\overline{p}_n \nu_e + H.c. \tag{50}$$

From this now we can get the mass, continuing with second term only:

$$\mathcal{L}_{Yuk} = -\frac{y_n}{\sqrt{2}}(v+h)\overline{p}\nu_e - \frac{y_n}{\sqrt{2}}(v+h)\nu_e^\dagger\overline{p}^\dagger = -\frac{y_n}{\sqrt{2}}(v+h)\left[\overline{p}\nu_e + \nu_e^\dagger\overline{p}^\dagger\right] \tag{51}$$

Then if Dirac spinor ζ is introduced: $\zeta = \begin{pmatrix} \nu_e \\ \overline{p}^\dagger \end{pmatrix}$ the expression above can be rewritten as:

$$\mathcal{L}_{Yuk} = -\frac{y_n v}{\sqrt{2}}\overline{\zeta}\zeta \tag{52}$$

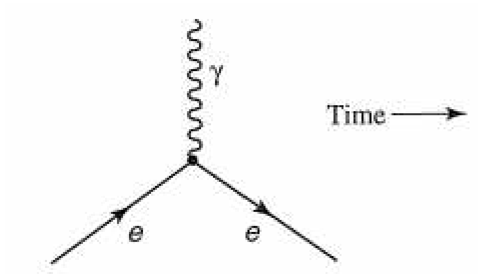
The first term gives mass to the ζ and in turn to the introduced particle $\overline{p}$. This is Dirac mass term, which term it is presented is exactly the same in Dirac Lagrangian (31)

5 Feynman diagrams

Feynman diagrams are method to quantize physical processes that include elementary particles. They are very important in calculating physical quantities needed for experiment evaluation. This section is taken mainly from [5]. The propagator section is from [6, 7].

5.1 Vertex of interaction

One main element of any Feynman diagram is a vertex of interaction. It is a vertex that connects 3 lines to a single point - interaction. The vertex, or any Feynman diagram for that matter has implied time evolution:

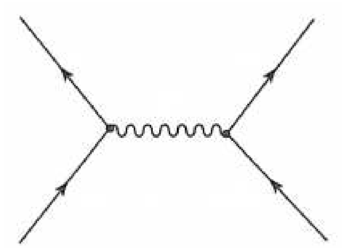


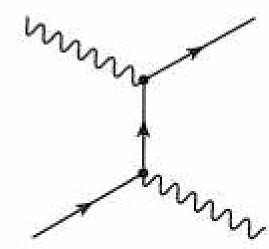
Markers e symbolize electron, γ photon. The arrows symbolize passage of time. This vertex means that there was an electron, he interacted with a photon and then there was electron again. Interaction with a photon can mean anything - that electron absorbed it, emitted it, scattered it or was scattered by it. This is basic vertex for electromagnetic force.

The arrow of time can be inverted, thus marking the anti particle.

5.2 Complex diagrams

Individual vertexes can be combined to form more complex ones that that represent more real processes. For example two of the electromagnetic vertexes can be combined to form Feynman diagrams:

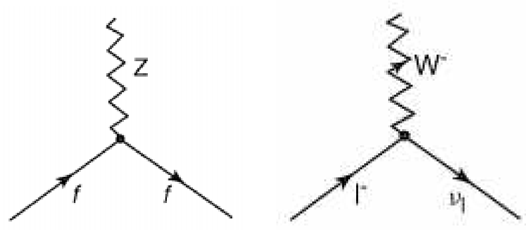




First one symbolizes electron - positron pair annihilation and pair creation. Second one is Compton scattering. In these diagrams, the lines in between - photon in first one and electron in second one are called virtual particles. Because it is impossible to detect them in these particle interactions thus they are theorized to be there.

5.3 Weak nuclear force

The Standard Model describes only three out of four elementary forces: strong and weak and electromagnetic forces. Neutrinos can only interact with weak because they don't have charge and are fermions they don't interact with the strong force. While electromagnetic force has only one boson - photon, weak force has 3: Z and $W^{\pm}$ who are neutral, positive and negative respectively. Corresponding vertices for Z and $W^{-}$are:

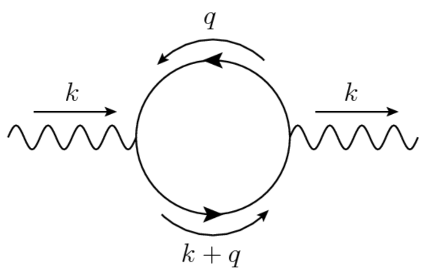


Where f is any fermion, l is lepton and ν_l is neutrino corresponding to that lepton family. In the second diagram W^-boson takes charge from lepton and transforms it to corresponding neutrino. As conservation of charge demands.

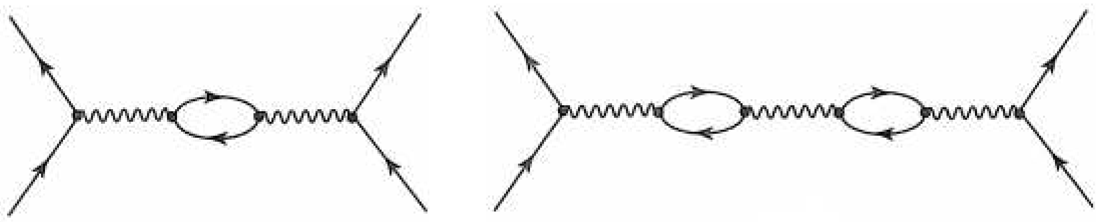
5.4 Loops

Every Feynman diagram is characterized by what particles goes into it and what goes out. That means what happens inside, the virtual interactions don't uniquely define the process. This opens door to having multiple diagrams for the same process. The permutations on how to put different vertices inside the diagram increase number of possible diagrams.

One permutation is called loop. It is combination of two vertices that connect with the same input and output parts to each other[8]:



Here momenta of all particles are labeled: k of real particles and q of virtual ones. This picture represents a vacuum polarization - when photon spontaneously creates electron and positron pair that annihilates with each other shortly afterwards recreating a photon. In principle any traveling particle can have any number of these loop parts. This means that there exists infinite amount of possible diagrams for any process. For example electron positron anhilation/recreation process can have one, two or more loops:



A Feynman diagram without loops is called tree level diagram.

5.5 Loop corrections

Feynman diagrams are not just a method to visualize particle processes but also a tool to convert theory to measurable values. Each diagram can be used to calculate process's scattering cross section or decay rate for comparison to the experiment. To do this one uses Feynman rules that specifically describe what needs to be done to extract those values. They involve writing coupling constant for every vertex in the diagram, writing propagator for every internal and outgoing line and then integrating over all virtual particle momenta space to get amplitude for a process defined with the diagram.

Since there are infinite number of diagrams what is the correct way to calculate anything? Answer is that amplitudes from all the diagrams should be added together. The key is that each new diagram with more loops contribute more coupling multipliers which are in most cases less than 1: $g < 1$. This means that bigger and more complex diagrams with more loops add less of a difference - the series converge.

Then it is up to the user to calculate everything to precision of his/hers liking.

Loops symbolize particles self energy - energy particle has due to interaction with it self. For example traveling neutrino could spontaneously release Z boson and shortly after that reabsorb it thus interacting with itself.

5.5.1 The propagator

The propagator R of a line in Feynman diagram is a statistical weight to go from one vertex to another. Its defined as:

$$R = \frac{i}{p^2 - m^2} \tag{53}$$

Where p and m is momentum and mass of that particle line. In a line there can be any amount of loops. Loop self energy $\Sigma(p^2)$ is defined such that quantity $-i\Sigma(p^2)$ is a correctional multiplier for a propagator for each loop present in the line.

The mass of a particle is defined as the position of the pole of the propagator R. Since (13) is on mass shell: $m^2 = p^2$. But since self energies affects the propagator it also affects the mass of the particle! It is one of the main tools that Grimus Neufeld Lavoura use in their model.

6 The Grimus-Neufeld Model

The Grimus-Neufeld model (GN) is the main subject of interest in this thesis. Original model was suggested by Grimus and Neufeld in [1] and later developed for more practical calculations by Grimus and Lavoura in [2]. This chapter is mainly based on those two papers.

In GN model they add n_R amount of left handed neutrinos and n_h amount of Higgs doublets to the Standard Model. They then explore a possible way that currently known 3 neutrinos of the Standard Model gain mass through seesaw mechanism, through the interaction with different Higgses in loop corrections.

For purposes of this thesis and calculations I assume that $n_R = 1$ and $n_h = 2$ also. One additional right handed neutrino and one additional Higgs doublet. The explored model is that seesaw gives mass to heaviest neutrino while first order loop corrections - loop correction of just one loop gives little bit of mass to second neutrino while first one always stays massless.

6.1 Seesaw mechanism

The seesaw mechanism concerns the mass matrix and it's eigen values. For example if we have mass matrix M populated by two elements A and B where $B \gg A$:

$$M = \begin{pmatrix} 0 & A \\ A & B \end{pmatrix} \tag{54}$$

The eigen values :

$$\begin{aligned} \lambda_1 &= \tfrac{B+B\sqrt{1+4\frac{A^2}{B^2}}}{2} \simeq B \\ \lambda_2 &= \tfrac{B-B\sqrt{1+4\frac{A^2}{B^2}}}{2} \simeq -\tfrac{A^2}{B} \end{aligned} \tag{55}$$

where in Taylor series around A = 0 were used to expand second mass term were used. Now one eigenvalue stays the big value B but other one becomes $\frac{A^2}{B}$. Which means that when B becomes bigger second value goes smaller. This technique is called seesaw mechanism.

GN use it to introduce one small mass to one left handed neutrino. That is achieved by adding heavy right handed neutrino to Standard Model. The big mass of right handed neutrino corresponds to big value B in this example.

6.2 Two Higgs doublets

The GN model variant that I research uses two Higgs doublets: $\phi_1$and ϕ_2. Currently only one Higgs is known and been experimentally verified. But to harness both Higgses one needs to deal with Higgs mechanism and breaking of Higgs's vacuum. This subchapter is largely based on [9].

The two Higgses should both have portion of VEV that is currently known to be v = 246GeV. The initial distribution should be parametrized by vector $\hat{v} \in \mathbb{R}^2, |\hat{v}| = 1$ such that

$$\langle 0 \,|\phi_k|\, 0\rangle = \langle \phi_k \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ \hat{v}_k \end{pmatrix} \quad k \in \{1, 2\} \tag{56}$$

Where a is component index. Then one can choose vector $\hat{w} \in \mathbb{R}^2, |\hat{w}| = 1$ which is orthogonal to $\hat{v}$:$\hat{v}\hat{w} = 0$. Then using these vectors one can define new Higgses $H_1$and H_2 as follows:

$$H_1 = \hat{v}_k \phi_k, \;\; H_2 = \hat{w}_k \phi_k \tag{57}$$

The new Higgses then will have VEVs:

$$\begin{aligned} \langle H_1 \rangle &= \tfrac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \\ \langle H_2 \rangle &= \begin{pmatrix} 0 \\ 0 \end{pmatrix} \end{aligned} \tag{58}$$

When vacuum gains non zero VEV then only first Higgs doublet will break. Furthermore these Higgses can be parametrized as follows:

$$H_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}} (v + h + iG^0) \end{pmatrix} \tag{59}$$

$$H_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}} (H + iA) \end{pmatrix} \tag{60}$$

Where all new variables are real, but only h, H and A gain masses when symmetry breaks. This gives us four massive Higgs fields: h, H, A and H^+. G^+ and G^o are goldstone bosons which never gain mass when higgs breaks.

Generally the 3 fields can mix together while interacting with other SM particles. How ever for purposes of this thesis only h and H can mix:

$$\begin{pmatrix} h' \\ H' \\ A' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} h \\ H \\ A \end{pmatrix} \tag{61}$$

Where θ is a mixing angle and h' is real higgs that has been discovered and verified in SM.

6.3 GN Lagrangian and terms

GN write up general form of multi Higgds doublet model:

$$\mathcal{L}_{GN} = -\sum_{k=1}^{n_h} \left(H_k^\dagger \bar{\ell}_R \Gamma_k + \widetilde{H}_k^\dagger \bar{\nu}_R \Delta_k \right) D_L - \frac{1}{2} \bar{\nu}_R C M_R \bar{\nu}_R^T + H.c. \tag{62}$$

Where H.c means hermitian conjugate of everything that came before, $\bar{\ell}_R$ is right handed lepton, $\bar{\nu}_R$ right handed heavy neutrino mentioned before. D_L is vector of SM lepton doublets - its a 3 component vector where each element is one doublet from generations defined in (40). Γ_k and Δ_k are dimensional vectors of Yukawa couplings to right handed lepton $\bar{\ell}_R$ and right handed heavy neutrino $\bar{\nu}_R$. C is charge conjugation matrix and M_R is Majorana mass term discussed in section 3.4.

First term in (58) is obviously Yukawa interaction term and second one is Majorana mass term. Masses gained from Yukawa part can be expressed in so called Dirac mass term M_D:

$$M_D = \frac{1}{\sqrt{2}} \sum_k v_k \Delta_k = \frac{v}{\sqrt{2}} \Delta_1 \tag{63}$$

Where v_k is VEV for k'th Higgs, but since we put all of the VEV to first Higgs in previous subchapter: $v_1 = v, v_2 = 0$.

The Majorana mass term defines mass that only right handed neutrino $\bar{\nu}_R$ gains by interacting with itself. Dirac mass describes mass that neutrinos get when SM left handed neutrinos interact with right handed one.

Now these mass terms can be written together in general neutrino mass matrix:

$$M_{D+R} = \begin{pmatrix} 0 & M_D^T \\ M_D & M_R \end{pmatrix} \tag{64}$$

This is seesaw matrix has dimension $(3+1) \times (3+1)$ because it was chosen to add only 1 right handed neutrino.

The seesaw mechanism gives a mass to one SM's neutrino at tree level, and GM explicitly wrote it:

$$M_v^{tree} = -M_D^T M_R^{-1} M_D \tag{65}$$

The two Higgses in GM model mixes with eachother, so Yukawa couplings Δ_k must mix too:

$$\Delta_b = B_{bk}\Delta_k, \;\; b \in \{h', H', A'\} \tag{66}$$

Where B is mixing matrix from (61). The way it is written the first column of B corresponds to Δ_1 and next two columns Δ_2. Little b stands for mixed state h', H' or A'.

6.4 GN one loop correction

Loop correction in GN model gives mass to second out of three neutrinos, leaving only one mass-less. Grimus and Lavoura explicitly write the correction:

$$\delta M_L = \sum_b \frac{m_b^2}{32\pi^2} \Delta_b^T \left(\frac{1}{M_R} \ln \frac{M_R^2}{m_b^2} \right) \Delta_b + \frac{3m_z^2}{32\pi^2} M_D^T \left(\frac{1}{M_R} \ln \frac{M_R^2}{m_z^2} \right) M_D \tag{67}$$

where m_b is mass of b field, m_z is mass of neutral Z boson of weak interaction. It is worth noting that relation (67) can be written like this only when one heavy right handed neutrino is added and Majorana mass term M_R is just a number.

The complete left handed, SM, neutrino mass matrix then is:

$$M_\nu = M_\nu^{\text{tree}} + \delta M_L \tag{68}$$

It incorporates both: seesaw and loop correction contributions to the neutrino masses.

The matrix represents the masses of three left handed Standard Model neutrinos. Singular are the true masses of them.

7 Calculations

In this chapter I describe how I carried out the calculations needed to find the Yukawa couplings. Then I show results obtained. The calculations were done using Mathematica software package. Specifically version 9.

7.1 The approach

The approach was to calculate neutrino mass matrix (68), to find the eigenvalues and compare them to known squared mass differences Δm^2_{31} and Δm^2_{21}.

At first the set and free parameters were identified. Set parameters are known values to our model that reduce the variable number by initializing them to numerical values. Those were [10]:

1. Bigger mass difference between neutrinos $\Delta m^2_{31} = (2.55 \pm 0.04) * 10^{-12}\,\mathrm{GeV}$
2. Smaller mass difference between neutrinos $\Delta m^2_{21} = (7.56 \pm 0.19) * 10^{-14}\,\mathrm{GeV}$
3. Neutral Z boson mass $m_z = 91.19\,\mathrm{GeV}$
4. Standard Model Higgs mass $m_{h'} = 125.09\,\mathrm{GeV}$
5. Standard Model Higgs Vacuum expectation value(VEV) $v = 246\,\mathrm{GeV}$

These parameters are put into calculations at final step.

Free parameters were those that were input arguments into geting a and $|b|$. They were:

1. Higgs mixing angle θ
2. Majorana mass of one right handed neutrino m_r
3. The modulus of Dirac mass vector c
4. The mass of second neutral Higgs $m_{H'}$
5. The mass of third neutral Higgs $m_{A'}$

The first step was to implement calculation of neutrino mass (68) matrix in which Yukawa couplings Δ_k are free parameters. The eigenvalues are then equated to known neutrino masses. The relation connects all of the parameters together. How ever solving it for Yukawaw couplings was proven to be very hard to nearly impossible due to the size and complexity of equation itself. It involves 3×3 matrix witch is the biggest hurdle to overcome with finite computer resources.

In order to reduce the relation to more manageable form I reduced 3×3 matrix expression from (68) to 2×2 subblock. I achieved that by parametrizing Yukawa couplings Δ_k.

7.2 Mass matrix reduction

To reduce mass matrix (68) I parametrize Yukawa couplings Δ_k in three dimensional complex space with base V_i. The base is row vectors of a PMNS matrix.

The PMNS chapter is mainly from [11, 5]

7.2.1 PMNS matrix

Neutrinos that are detected in neutrino detectors aren't the ones we are interested in. This is because we can only detect neutrinos that interact with their corresponding charged leptons i.e. electron neutrino only interacts with electron and so on. This is how flavour is defined.

However flavourful neutrinos don't have defined mass. Only eigenstates of Hamiltonian or mass eiganstates have it. Thus mass neutrinos mix to form flavour ones:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \tag{69}$$

where numbered neutrinos are mass eigenstates and the mixing matrix is PMNS matrix, named after Bruno Pontekorvo, Ziro Maki, Masami Nakagawa and Shoichi Sakata.

It is unitary matrix that is parametrized by 4 angles: $\theta_{12}, \theta_{13}, \theta_{23}$ and δ. Where first three are mixing angles of neutrinos and fourth one is CP(Charge-Parity) violating phase. The matrix itself is described as:

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \tag{70}$$

Where $s_{ij}$and c_{ij} are sin and cos of angle θ_{ij} correspondingly.

One can calculate the matrix since all the angles are known [10] to some degree($\pm 1\sigma$): $\theta_{13} = 8.44^\circ \pm 0.18^\circ$, $\theta_{12} = 34.5^\circ \pm 1.1^\circ$, $\theta_{23} = 41.0^\circ \pm 1.1^\circ$ and $\delta = 252 \pm 56$. But it is unnecessary since, for my purposes, the explicit form is not needed.

PMNS matrix is Unitary. One can divide it in to orthonormal, complex, row vectors V_i, $i \in \{1, 2, 3\}$:

$$U_{PMNS} = \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} \tag{71}$$

And $V_i^* V_j^\dagger = \delta_{ij}$.

7.2.2 Yukawa coupling parametrization

The neutrino mass matrix of 68 is explicitly defined by Yukawa coupling vectors Δ. Dirac mass term is just Δ_1 and Δ_b is just superposition of normal deltas. Thus it is convenient to express them through vectors V 71. Then we parametrize Yukawa couplings Δ_1 and Δ_2 accordingly:

$$\begin{aligned} \Delta_1 &= cV_3^* \\ \Delta_2 &= aV_2^* + bV_3^* \end{aligned} \tag{72}$$

where a and c are real constants and b is complex one. Constant c is defined as:

$$c = \frac{\sqrt{2}}{v} |M_D| \tag{73}$$

It is just the modulus of the Dirac mass vector.

As mentioned above the orthonormal vectors V_i are known. And one needs only to find parameters c, a and b to completely know Yukawa couplings. In work described in this thesis the parameter c is set to be free, a and b are the ones to be determined.

7.2.3 Parametrized terms

The terms of interest in 68 are $M_D, \Delta_{h'}, \Delta_{H'}, \Delta_{A'}$. Following the parametrization in (68), and (59) with (62) we have:

$$M_D = \frac{vm_d}{\sqrt{2}} V_3^* \tag{74}$$
$$\Delta_{h'} = a \sin\theta V_2^* + (m_d \cos\theta + b\sin\theta) V_3^* \tag{75}$$
$$\Delta_{H'} = a \cos\theta V_2^* + (-m_d \sin\theta + b\cos\theta) V_3^* \tag{76}$$
$$\Delta_{A'} = a V_2^* + b V_3^* \tag{77}$$

Now it is clear that every term in 68 will have in the form of one of the following matrices: $V_2^\dagger V_2^*, V_2^\dagger V_3^*, V_3^T V_2$ and $V_3^T V_3$(since couplings are row vectors).
Redefining PMNS matrix:

$$U_{PMNS} = \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = V$$

Partially diagonalizing the mass matrix we are going to put it between V and $\hat{V}$:

$$\widetilde{M_\nu} = V^* M_\nu V^\dagger \tag{78}$$

Where $\widetilde{M_\nu}$is reduced neutrino mass matrix. Finally since $V_i^* V_j^\dagger = \delta_{ij}$ only elements of (68) that will survive will be $(2,2), (2,3), (3,2)$and $(3,3)$. And they wont have any leftover factors of any V_i vector.
After the reduction:

$$\widetilde{M_\nu} = \frac{1}{32\pi^2 m_r} \begin{pmatrix} 0 & 0 & 0 \\ 0 & a^2 \left(L_A + c_\theta^2 L_H + L_h s_\theta^2\right) & A \\ 0 & A & B \end{pmatrix} \tag{79}$$

Where abbreviations are:

$$A = a\left(|b|\, e^{i\mathrm{ph}} L_A + |b|\, e^{i\mathrm{ph}} c_\theta^2 L_H + |b|\, e^{i\mathrm{ph}} L_h s_\theta^2 + c c_\theta s_\theta \left(L_h - L_H\right)\right) \tag{80}$$
$$B = |b|^2 e^{2i\mathrm{ph}} L_A + c_\theta^2 \left(c^2 L_h + |b|^2 e^{2i\mathrm{ph}} L_H\right) + |b|^2 e^{2i\mathrm{ph}} L_h s_\theta^2 + 2\,|b|\, c e^{i\mathrm{ph}} c_\theta s_\theta \left(L_h - L_H\right) + c^2 L_H s_\theta^2 + 3c^2 L_z - 32\pi^2 c^2 \tag{81}$$
$$L_h = m_{h'}^2 \ln \frac{m_r^2}{m_{h'}^2},\ L_H = m_{H'}^2 \ln \frac{m_r^2}{m_{H'}^2},\ L_A = m_{A'}^2 \ln \frac{m_r^2}{m_{A'}^2},\ L_z = m_z^2 \ln \frac{m_r^2}{m_z^2} \tag{82}$$
$$c_\theta = \cos\theta,\ s_\theta = \sin\theta \tag{83}$$

The mass matrix is seen to only have non zero values in 2×2 block. As expected. It shows that the approach is correct.
The remaining non zero 2×2 block in $\widetilde{M_\nu}$ hereafter is going to be noted by $M_{\nu block}$.

7.3 The results

The reduced 2×2 block allows to continue the calculations. The eigenvalues of the block were equated to known neutrino masses $\Delta m_{21}^2$and Δm_{31}^2. And then solved for parameter a^2:

$$a^2 = -1024\pi^4 m_r^2 \sqrt{\Delta m_{21}^2 \Delta m_{31}^2} \big(L_A\big(-L_h \cos^2(\theta) - \sin^2(\theta) L_H - 3L_z + 32\pi^2\big) - L_h\big(L_H + \sin^2(\theta)\big(3L_z - 32\pi^2\big)\big) + \cos^2(\theta) L_H \big(32\pi^2 - 3L_z\big)\big)^{-1} \tag{84}$$

With the first answer computed parameter b is next. To simplify calculations further b was substituted to exponential form:$b = |b|\, e^{iph}$where ph is phase of complex parameter.

Generally block doesn't have real eigenvalues because it is not hermitian. To make it hermitian I multiplied it with hermitian conjugate of itself:

$$M^2_{\nu block} = M^\dagger_{\nu block} M_{\nu block} \tag{85}$$

This trick also makes eigenvalues to be squared. The trace of a matrix is always equal to sum of eigenvalues λ_i of it:

$$Tr(M) = \sum_i M_{ii} = \sum_i \lambda_i \tag{86}$$

The trace of $M_{\nu block}$ was equated to the sum of neutrino mass squared. The relation was rearranged for$|b|$and result is fourth order polynomial:

$$u_4 |b|^4 + u_3 |b|^3 + u_2 |b|^2 + u_1 |b| + u_0 = 0 \tag{87}$$

The coefficients u_l depend on all parameters including phase of b. They are quite massive in volume and complexity. Their full and simplified expressions can be found in Appendix A.

The polynomial only has real roots when it crosses below 0 in b space. Minimal testing has been done with test values of free parameters. Two test sets for parameter values have been chosen:

Table 1: Free parameter values for test set one

Name	Value
m_r	10^6GeV
c	$0.8\sqrt{m_r \sqrt{\Delta m_{31}^2}}$
$m_{H'}$	978.672GeV
$m_{A'}$	962.67GeV
$\sin\theta$	0.9998

Table 2: Free parameter values for test set two

Name	Value
m_r	10^6GeV
c	$0.8\sqrt{m_r \sqrt{\Delta m_{31}^2}}$
$m_{H'}$	699.503GeV
$m_{A'}$	450.389GeV
$\sin\theta$	-0.76415

The resulting polynomial and corresponding graphs are below. For physical purposes such that model would be consistent with Standard Model the relevant range for $|b|$ is $(0, 1)$.

None of the explored test values give polynomial that moves below zero. This means only points with complex solutions were found.

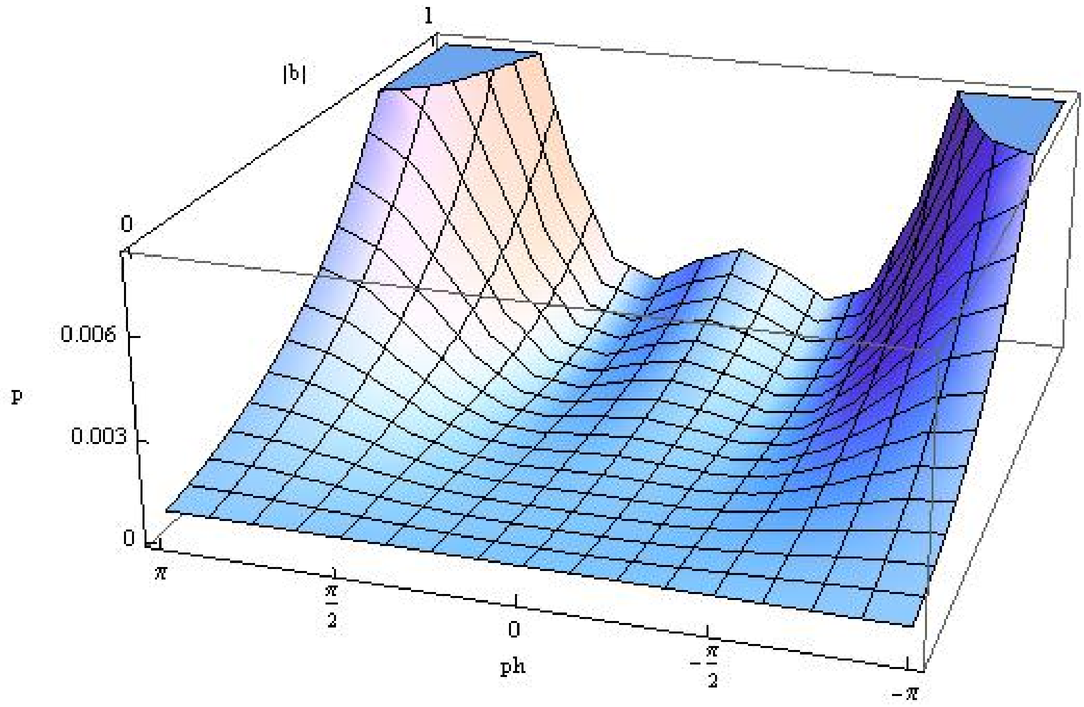


Figure 1: 3D plot of polynomial (87) in b space of parameters: modulus of b : $|b|$and phase of b ph. With free parameter values are from Table 1. The complete expression of polynomial p is $p = 0.00339928|\mathrm{b}|^4 + \left(-0.00476065|\mathrm{b}|^3 - 0.00234845|\mathrm{b}|\right)\cos(\mathrm{ph}) + 0.00335376|\mathrm{b}|^2\cos(2\mathrm{ph}) + 0.00166681|\mathrm{b}|^2 + 0.00082148$ Its shown that polynomial doesn't have real roots

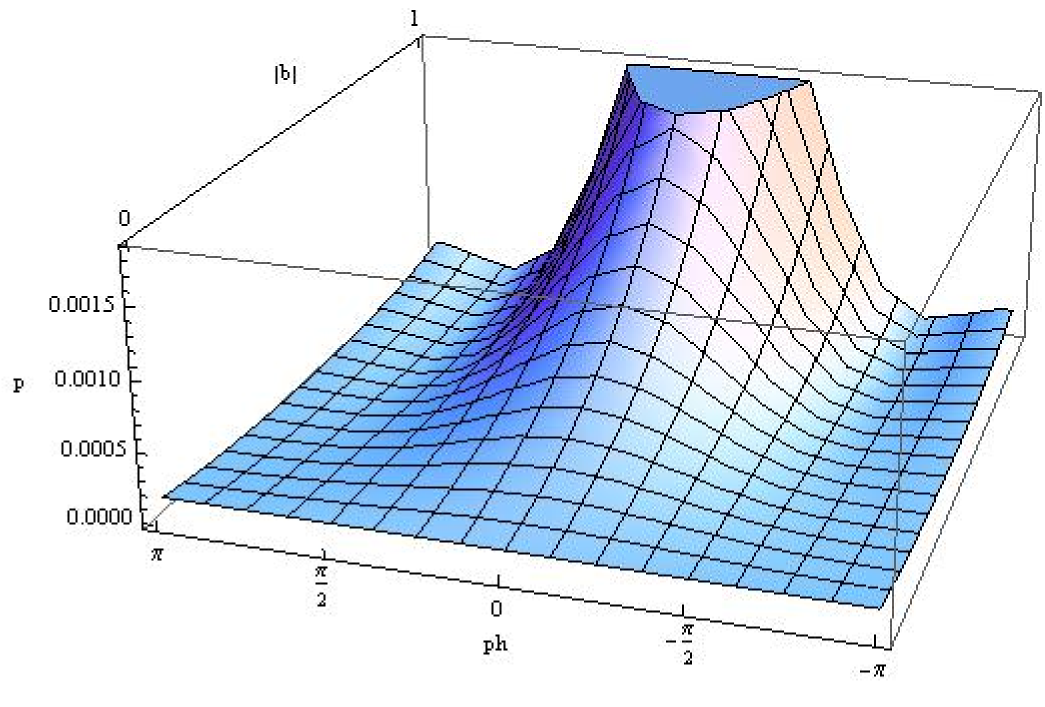


Figure 2: 3D plot of polynomial (87) in b space of parameters: modulus of b : $|b|$and phase of b ph. With free parameter values are from the Table 2 . The complete expression of polynomial p is $p = 0.00054485|\mathrm{b}|^4 + \left(0.00100705|\mathrm{b}|^3 + 0.000560953|\mathrm{b}|\right)\cos(\mathrm{ph}) + 0.000606994|\mathrm{b}|^2\cos(2\mathrm{ph}) + 0.000465331|\mathrm{b}|^2 + 0.000163325$ Its shown that polynomial doesn't have real roots

8 Conclusions

The main result of GN (68) was implemented in Mathematica software. The first attempt wasn't operationally useful since it overloaded computer resources available in order to analytically solve relations involving 3×3 matrix. Solution was to reduce the matrix to 2×2 block. New relations involving the block now could be solved for.

The program is capable of analytically finding expressions for Yukawa coupling parameters a and $|b|$. The Mathematica software package allows for further analytical or numerical manipulation and visualization of results.

The solution for parameter a is exact. But full solution for parameter $|b|$requires one to solve fourth order polynomial (87). Since there are many free parameters in the model we were capable to only investigate a few points. Two complex solutions were found. In order to find real solutions one needs to research parameter space further.

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Paulius Juodsnukis

JUKAVA KOEFICIENTAI GRIMUS NEUFELD MODELYJE

Santrauka

Didesnėje praėjusio amžiaus dalyje buvo manoma, kad neutrinai neturi masių. Tyrinėjant saulės vidaus veikimo principus užfiksuota tik trečdalis teorijoje numatytų neutrinų. Atlikus kitokiu principu pagrįstus matavimus buvo nustatyta, kad neutrinai osciliuoja - periodiškai keičia savo tipą. Teoriniame lygmenyje tai yra įmanoma, tik tada, kai jie neturi vienodų masių. Vadinasi kai kurie neutrinai privalo turėti mases.

Eksperimentiškai matuojant, buvo nustatyta, kad jos yra labai mažos [12]: $10^{-3} - 10^{-5}$eV eilės. Tuo tarpu gerai žinomos barioninės dalelės, tokios kaip protonai ir neutronai, turi mases 10^9eV eilės. Toks didžiulis - 12-14 eilių skirtumas yra labai įdomus šiuolaikinėje dalelių fizikoje.

Viena iš teorijų, galinčių paaiškinti neutrinų mases yra, Grimus - Neufeld(GN) modelis [1]. GN teorijoje SM yra papildomas antruoju Higgs'o dubletu ir vienu masyviu neutrinu. Didelę masę turintis pridėtinis neutrinas pakeičia masės matricą ir savo ruožtu "atsveria" vieno iš SM neutrino masę į mažesnę pusę. Tai vadinamas sverto principas. Tokios teorijos rėmuose vienas Standartinio Modelio neutrinas įgauną masę per sverto mechanizmą, ir antrasis per vienos kilpos pataisas. Vienas iš neutrinų lieka be masės.

Tyrinėjamame GN modelyje svarbų vaidmenį atlieka Jukava koeficientai. Jukava koeficientai sieja skaliarinių laukų, tokių kaip Higgs'o laukas, ir spinoriu laukų, tokių kaip leptonai, sąveikas. Jukava sąveikos kartu su Higgs'o mechanizmu duoda leptonams jų mases.

Šis bakalauro baigiamasis darbas yra mano kursinio darbo [3], kuriame tyrinėjau Standartinį Modelį ir sverto mechanizmą, tęsinys. Pirmuose keturiuose skyriuose supažindinu su Standartinio Modelio pagrindais. Penktajame ir šeštajame skyriuose, paaiškinu GN modelį, ir septintajame savo atliktus skaičiavimus. Rezultatus apibendrinu išvadose - aštuntame skyriuje.

Pagrindiniai šio darbo tikslai buvo: (1) susipažinti su GN modelio teorija, (2) suprogramuoti teorinę GN modelio masės išraišką [2] ir (3) naudojantis ja išreikšti Jukava koeficientų sprendinius.

Darbo tikslai buvo pasiekti. Užprogramavus GN modelio masės išraišką (68) ir sulyginus su žinomais duomenimis gauta lygtis. Panaudojus šią lygtį,buvo pradėtas skaičiavimas išspręsti Jukava narių reikšmėms, kurio Mathematica programinis paketas nesugebėjo užbaigti per priimtiną laiką. Siekiant sumažinti skaičiavimu kieki, GN modelio masės išraiška buvo dalinai diagonalizuota į antros eilės kvadratinės matricos bloką, su kuriuo paprasčiau dirbti. Tas buvo pasiekta parametrizuojant Jukava koeficientus tiesinėje trimat erdvėje.

Jukava koeficientu išraiškos gautos analitiniu pavidalu. Jos toliau gali būti tyrinėjamos analitiškai arba skaitmeniškai, vizualizuojamos ar kitaip apdorojamos daugybės įvairių programinių paketų pagalba.

Parametrizavimo koeficientas a išreikštas tiksliai. Tačiau koeficientas $|b|$ randamas tiktai kaip ketvirtos eilės polinomo (87) šaknys, kurios nebūtinai yra realios. Buvo patikrinti du parametrų rinkiai ir rasta, kad jie negali atitikti ieškomo realaus parametro $|b|$ nes jie atitinka kompleksines šaknis. Norint įsiaiškinti ar sprendinys turi realių šaknų, reikalingas tolimesnis parametrų erdvės tyrinėjimas.

Appendix A

Exact expressions of the coefficients u_i in polynomial (87). Most of them a very large in size therefore the abbreviations e_i are used in defining them. Additional abbreviations (82) and (83) are used.

$$\begin{aligned}
u_0 =& \frac{L_H^2 s_\theta^4 c^4}{1024 m_r^2 \pi^4} + \frac{c_\theta^4 L_h^2 c^4}{1024 m_r^2 \pi^4} + \frac{9 L_z^2 c^4}{1024 m_r^2 \pi^4} + \frac{c_\theta^2 L_h L_H s_\theta^2 c^4}{512 m_r^2 \pi^4} - \frac{L_H s_\theta^2 c^4}{16 m_r^2 \pi^2} \\
&+ \frac{3 L_H L_z s_\theta^2 c^4}{512 m_r^2 \pi^4} - \frac{c_\theta^2 L_h c^4}{16 m_r^2 \pi^2} + \frac{3 c_\theta^2 L_h L_z c^4}{512 m_r^2 \pi^4} - \frac{3 L_z c^4}{16 m_r^2 \pi^2} + \frac{c^4}{m_r^2} - \Delta m_{21}^2 - \Delta m_{31}^2 \\
&+ \frac{2\sqrt{\Delta m_{21}^2 \Delta m_{31}^2} c_\theta^2 L_h^2 s_\theta^2}{e_1} + \frac{2\sqrt{\Delta m_{21}^2 \Delta m_{31}^2} c_\theta^2 L_H^2 s_\theta^2}{e_1} - \frac{4\sqrt{\Delta m_{21}^2 \Delta m_{31}^2} c_\theta^2 L_h L_H s_\theta^2}{e_1} \\
&+ \frac{1024 \Delta m_{21}^2 \Delta m_{31}^2 m_r^2 \pi^4 L_h^2 s_\theta^4}{e_2} + \frac{1024 \Delta m_{21}^2 \Delta m_{31}^2 m_r^2 \pi^4 L_A^2}{e_2} \\
&+ \frac{1024 \Delta m_{21}^2 \Delta m_{31}^2 m_r^2 \pi^4 c_\theta^4 L_H^2}{e_2} + \frac{2048 \Delta m_{21}^2 \Delta m_{31}^2 m_r^2 \pi^4 L_A L_h s_\theta^2}{e_2} \\
&+ \frac{2048 \Delta m_{21}^2 \Delta m_{31}^2 m_r^2 \pi^4 c_\theta^2 L_h L_H s_\theta^2}{e_2} + \frac{2048 \Delta m_{21}^2 \Delta m_{31}^2 m_r^2 \pi^4 c_\theta^2 L_A L_H}{e_2}
\end{aligned} \tag{88}$$

$$\begin{aligned}
e_1 =& L_h L_H c_\theta^4 + \left(2 L_h L_H s_\theta^2 + L_A L_h + L_H \left(3 L_z - 32\pi^2\right)\right) c_\theta^2 + \left(L_h s_\theta^2 + L_A\right) \left(L_H s_\theta^2 \right. \\
&\left. + 3 L_z - 32\pi^2\right)
\end{aligned} \tag{89}$$

$$\begin{aligned}
e_2 =& \big(-L_h L_H c_\theta^4 - 2 L_h L_H s_\theta^2 c_\theta^2 - L_A L_h c_\theta^2 + 32\pi^2 L_H c_\theta^2 - 3 L_H L_z c_\theta^2 - L_h L_H s_\theta^4 + 32\pi^2 L_h s_\theta^2 \\
&- L_A L_H s_\theta^2 - 3 L_h L_z s_\theta^2 + 32\pi^2 L_A - 3 L_A L_z\big)^2 c^4
\end{aligned} \tag{90}$$

$$\tag{91}$$

$$u_1 = -\frac{e_3}{e_4} \tag{92}$$

$$\begin{aligned}
e_3 =& c_\theta \cos(\mathrm{ph}) s_\theta \big(L_h - L_H\big) \big(L_A \big(2 c^4 c_\theta^2 L_h \big(L_H s_\theta^2 + 3 L_z - 32\pi^2\big) \\
&+ c^4 c_\theta^4 L_h^2 + 1024\pi^4 m_r^2 \sqrt{\Delta m_{21}^2 \Delta m_{31}^2} + c^4 L_H^2 s_\theta^4 \\
&- 64\pi^2 c^4 L_H s_\theta^2 + 6 c^4 L_z \big(L_H s_\theta^2 - 32\pi^2\big) + 9 c^4 L_z^2 + 1024\pi^4 c^4\big) \\
&+ c_\theta^2 \big(L_H \big(1024\pi^4 \big(m_r^2 \sqrt{\Delta m_{21}^2 \Delta m_{31}^2} + c^4\big) + c^4 L_h^2 s_\theta^4 - 64\pi^2 c^4 L_h s_\theta^2 \\
&+ 6 c^4 L_z \big(L_h s_\theta^2 - 32\pi^2\big) + 9 c^4 L_z^2\big) + c^4 L_H^2 s_\theta^2 \big(2 L_h s_\theta^2 + 3 L_z - 32\pi^2\big) \\
&+ c^4 L_h^2 \big(3 L_z - 32\pi^2\big) s_\theta^2\big) + c^4 c_\theta^6 L_h^2 L_H \\
&+ c^4 c_\theta^4 L_h L_H \big(2 L_h s_\theta^2 + L_H s_\theta^2 + 6 L_z - 64\pi^2\big) \\
&+ L_h s_\theta^2 \big(1024\pi^4 \big(m_r^2 \sqrt{\Delta m_{21}^2 \Delta m_{31}^2} + c^4\big) + c^4 L_H^2 s_\theta^4 - 64\pi^2 c^4 L_H s_\theta^2 \\
&+ 6 c^4 L_z \big(L_H s_\theta^2 - 32\pi^2\big) + 9 c^4 L_z^2\big)\big)
\end{aligned} \tag{93}$$

$$\begin{aligned}
e_4 =& 256\pi^4 c m_r^2 \big(L_A \big(c_\theta^2 (-L_h) - L_H s_\theta^2 - 3 L_z + 32\pi^2\big) \\
&+ c_\theta^4 (-L_h) L_H + c_\theta^2 L_H \big(-2 L_h s_\theta^2 - 3 L_z + 32\pi^2\big) - L_h s_\theta^2 \big(L_H s_\theta^2 + 3 L_z - 32\pi^2\big)\big)
\end{aligned} \tag{94}$$

$$\tag{95}$$

$$u_2 = \frac{e_5}{512 c^2} \tag{96}$$

$$
\begin{aligned}
e_5 =& L_H\left(\frac{\cos(2\text{ph})L_h c^4}{m_r^2\pi^4} + \frac{1024\sqrt{\Delta m_{21}^2\Delta m_{31}^2}L_H}{e_1}\right)c_\theta^4 \\
&+ \left(\frac{(\cos(2\text{ph})+2)L_h^2 s_\theta^2 c^4}{m_r^2\pi^4} + \frac{(\cos(2\text{ph})+2)L_H^2 s_\theta^2 c^4}{m_r^2\pi^4}\right. \\
&+ L_H\left(-\frac{32\cos(2\text{ph})c^4}{m_r^2\pi^2} + \frac{3\cos(2\text{ph})L_z c^4}{m_r^2\pi^4} + 4L_h s_\theta^2\left(\frac{512\sqrt{\Delta m_{21}^2\Delta m_{31}^2}}{e_1}\right)\right)\Bigg)c_\theta^2 \\
&+ L_h s_\theta^2\left(\frac{\cos(2\text{ph})L_H s_\theta^2 c^4}{m_r^2\pi^4} - \frac{32\cos(2\text{ph})c^4}{m_r^2\pi^2} + \frac{3\cos(2\text{ph})L_z c^4}{m_r^2\pi^4}\right. \\
&+ \left.\frac{1024\sqrt{\Delta m_{21}^2\Delta m_{31}^2}L_h s_\theta^2}{e_1}\right) + L_A\left(-\frac{32\cos^2(\text{ph})c^4}{m_r^2\pi^2} + \frac{32\sin^2(\text{ph})c^4}{m_r^2\pi^2}\right. \\
&+ \frac{\cos^2(\text{ph})L_H s_\theta^2 c^4}{m_r^2\pi^4} - \frac{\sin^2(\text{ph})L_H s_\theta^2 c^4}{m_r^2\pi^4} + \frac{3\cos(2\text{ph})L_z c^4}{m_r^2\pi^4} \\
&+ c_\theta^2\left(\frac{\cos(2\text{ph})L_h c^4}{m_r^2\pi^4} + \frac{2048\sqrt{\Delta m_{21}^2\Delta m_{31}^2}L_H}{e_1}\right) + \left.\frac{2048\sqrt{\Delta m_{21}^2\Delta m_{31}^2}L_h s_\theta^2}{e_1}\right) \\
&+ \frac{1024\sqrt{\Delta m_{21}^2\Delta m_{31}^2}L_A^2}{e_1}
\end{aligned} \tag{97}
$$

$$ \tag{98} $$

$$
u_3 = \frac{c_\theta\cos(\text{ph})c s_\theta\left(L_h - L_H\right)\left(L_A + c_\theta^2 L_H + L_h s_\theta^2\right)}{256\pi^4 m_r^2} \tag{99}
$$

$$ \tag{100} $$

$$
u_4 = \frac{\left(L_A + c_\theta^2 L_H + L_h s_\theta^2\right)^2}{1024\pi^4 m_r^2} \tag{101}
$$