

Self-calibratable absolute modular rotary encoder: a theoretical feasibility study

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Abstract

This paper introduces a self-calibratable absolute modular rotary encoder based on the equal division average method, designed to significantly enhance measurement accuracy and simplify installation. The proposed design integrates multiple optical sensors into the encoder stator Printed circuit board, enabling precise absolute position measurement through the averaging of sensors data. This multi-sensor approach compensates for alignment and installation inaccuracies, which are common issues in conventional modular encoders. To validate the design and predict encoder behaviour prior to manufacturing, a theoretical modelling of virtual optical sensors is performed. Based on experimentally collected cross-calibration data from a real optical encoder, this modelling framework enables to estimate the efficiency of self-calibration algorithm with sufficient accuracy and optimize the design and performance of the encoder. The obtained results confirm that the proposed measurement system significantly reduces error margin, improving the reliability and precision of position feedback. The initial position deviation of several hundreds of arcseconds might be reduced and kept below 8 arcseconds by using two or more additional optical sensors, even with the 0.5 mm misalignment of the encoder stator.

Keywords: modular rotary encoder, self-calibration, eccentricity error

1. Introduction

Optical modular (or ‘kit’) rotary encoders have a composite design. The assembly of the coded glass disc and metal hub, as well as the scanning unit (commonly referred to as the rotor and stator respectively) must be mounted and integrated directly into the application. The rotor is mounted on the host shaft, while the stator attaches to the support of the rotating system. Encoders of this configuration do not have an internal shaft and bearing system and are frameless in some

cases. Therefore, they have a comparatively small and compact size, low height, and minimum mass. There are no moving parts, so they are wear-free, which minimizes the risk of failure due to mechanical and thermal stresses, shock, or vibration. Furthermore, this design eliminates the need for a separate shaft coupling, which might affect the accuracy of the system. All of this makes kit encoders the first choice in servo drives, robotic joints and more sophisticated fields such as military and surveillance, medicine, and space technology.

The optical working principle ensures contactless operation. Encoders can be incremental or absolute and can operate on transmissive or reflective principles. Modern customized semiconductor optical sensors have integrated signal conditioners and interpolators, making them as a single-chip solution. Wide tolerance ranges of the most advanced optical-reflective encoder sensors allow them to work at air gaps from 1 mm to 3 mm and handle tangential and radial misalignment

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of ± 0.5 mm. However, this potential for simplifying encoder installation cannot be fully exploited. The accuracy of the encoder highly depends on the rigidity and stability of the mechanical assembly. Any axial or radial runout of the host shaft, as well as scale swash or bearing wander, can reduce the accuracy of the entire system. In practice, it is also impossible to avoid the eccentricity of the coded glass disc resulting from the fit tolerances between the rotor hub and the host shaft. This is usually the main source of error. For example, 5 μm eccentricity of a 50 mm mean diameter circular scale will cause a peak-to-peak deviation of ± 82.4 angular seconds. Thus, precise optical centering of the rotor is usually needed.

Several studies have been conducted to eliminate eccentricity errors through various means. Li *et al* propose an encoder that adapts a spider-web-patterned scale grating as a measurement standard [1]. Jia *et al* introduce a visual system and image processing technology to detect eccentricity [2]. Qin *et al* present a method of optical angular encoder calibration using a ring laser gyroscope for installation error elimination [3]. However, the implementation of these methods requires additional devices or knowledge of the size of the eccentricity and radial error motion. The integration of two optical sensors or the use of two separate scanning heads remains the simplest and most effective approach [4–6].

After eliminating the eccentricity error, which manifests itself as the first harmonic in a calibration chart (i.e. one period of sine shape per full circular scale revolution), the remaining part of the deviation is also significant and is related to the installation of the encoder, the manufacturing precision of the circular glass scale, properties of the optical sensor, interpolation error, etc. An effective compensation method of such residual deviation would enable the development of a high-precision modular rotary encoder suitable for high demanding application. The specific high-tech areas requiring accurate determination of angular position are often no longer a one-off production. Therefore, cost effective and easy-to-install optical measurement systems that do not require complex setup and calibration procedures are becoming increasingly necessary.

Measurement error compensation involves the correction or adjustment of deviations in the readings provided by an optical encoder. To achieve this, the encoder must be calibrated. Cross-calibration methods are common and have been studied in many articles [7–13]. These methods require a reference device that must be mechanically coupled with the encoder under the test to perform calibration at different relative angular positions. The most common approach is to use a high-accuracy angle encoder or a polygon as an external standard. This is a convenient practice used by encoder manufacturers, where produced devices are calibrated using specialized equipment called angular comparators. This is useful for closed-type rotary encoders because they can maintain a stable relative position between the optical components due to their mechanical design. In the case of open-type encoder kits, their calibration must be performed at the customer's location after installation. Additionally, an autocollimator should

be used for polygon measurement, and both instruments must be pre-calibrated. Furthermore, this method has measurement limitations concerning the pitch angle of the polygon. Although this limitation problem has recently been solved by Hsieh *et al* with the introduction of shift-angle [14] and two-autocollimators [15] methods these procedures are time-consuming and unsuitable for the efficient calibration of open-type encoders.

Self-calibration methods are self-referential and do not require an external standard. This offers a significant advantage in terms of efficiency and productivity. Lu *et al* proposed a new self-calibration method of on-axis rotary encoders based on pulse width measuring in the time domain [16, 17]. This time-measurement dynamic reversal method enables in-place calibration on the application rotary axis and provides an instant encoder error map for further compensation. Since the angular deviations are estimated from the spindle's free-response dynamics by turning off the spindle motor at a certain speed and letting it slow down freely, it is sensitive to bearing intrinsic torque. Moreover, not all applications have precision air bearings and can be calibrated in such a dynamic manner. Another method, which is insensitive to intrinsic torque of the bearing, is presented by Gou *et al* [18]. The coaxial sensors relative rotation method can self-calibrate angular sensors on their application axes and is suitable for operation in harsh environments. This principle requires modification of the machine so that two coaxial sensors can operate in locked and separate modes at constant speed. The position data sample from both sensors must cover angles from 0 to 2π . This is a disadvantage because not all applications require position determination of the angle in all 360 degrees, and due to its design, it cannot perform a complete turn.

The integration of multiple optical sensors seems to be one of the most reliable approaches. The appropriate selection of their number, positioning, and deviation calculation technique are essential factors that are studied and presented in many different methods. Ren *et al* analyse the error of circular gratings used in angle measurement systems [19]. Experiments using four evenly spaced reading heads demonstrate the possibility to eliminate mechanical installation errors. However, this approach cannot completely eliminate scale graduation errors. The equal division average (EDA) method proposed by Tsukasa Watanabe was a game-changing approach that has been widely adopted. The high-accuracy calibration system for angular encoders, which was installed at AIST (National Institute of Advanced Industrial Science and Technology), was introduced [20, 21]. Later, the self-calibratable rotary encoder SelfA, combined of five 'Heidenhain' ERO725 reading heads, was presented [22, 23]. The multicomination Watanabe method approach is introduced in SelfA34 encoder, which combines two sets of equally spaced 3 and 4 optical sensors [24], along with the super-high resolution and accuracy interferometer-type rotary encoder SelfA235 [25]. The employment of the EDA method can be found in other applications such as angular index table [26], angular velocity calibration system [27], precision angle comparator [28], and

portable real-time angle encoder [29]. Since this method uses uniform distribution, the number of required reading heads can lead to excessive increase in the cost and complexity of the measuring equipment. Non-uniform calibration methods that detect higher order components using fewer sensor heads have also been presented. Ishii *et al* propose a virtual equal division average method [30, 31]. This method allows for the use of two reading heads arranged at 180 degrees to achieve equivalent self-calibrating result of multiple sensors by virtually calculating error values of the other sensors. Unfortunately, to perform a complete self-calibration process, including virtual calculation, it is necessary to record a large amount of data during the full rotation of the encoder. Masuda and Kajitani describe an automatic calibration system for angular encoders that consists of six non-uniform distributed reading heads [32, 33]. Each head is placed in a position that divides the angular sector in which it is installed in half. The higher harmonic orders can be suppressed by comparing the total output signal of all reading heads with the output of a single head.

Recent work by Kiryanov *et al* analyses two self-calibration algorithms that differ by uniform or non-uniform arrangement. The proposed simulation method allows for the identification of the metrological error of the studied algorithm used by identifying the residual contribution of the distorting function to the total measurement error, with accuracy sufficient for practical applications [34]. The research reveals that the self-calibration technique based on uniform arrangement (EDA method) shows higher accuracy compared to the non-uniform arrangement technique (Masuda approach).

All discussed calibration and measurement error compensation methods are focused on high-accuracy systems such as angle comparators or closed-type angular encoders. However, such methods also have significant potential to increase the accuracy of modular rotary encoders by reducing the influence of mounting errors. A self-calibratable encoder kit using multiple evenly spaced sensors would ensure a simple installation procedure that does not require precise centering of the circular scale. Unlike the other methods, this approach does not require initial processing where the encoder has to be rotated at a constant speed or use the free-response dynamics of the application system to measure and compensate for errors. The EDA method can operate instantly once the encoder is powered on. In combination with advanced absolute optical sensors, such a metrological system could immediately output an angular position with high accuracy without the need of any additional rotation of the measured shaft.

In this paper, a self-calibratable design solution for a high-accuracy absolute rotary encoder kit is presented. The theoretical modelling is applied to test the efficiency of the self-calibration and to determine the optimal number of sensors and their alignment. The cross-calibration data of a real optical encoder kit is used to virtually simulate the angular deviations of the artificial sensors that might be placed in the respective positions. The information obtained during the experiment allows for the evaluation of the self-calibration capability to eliminate eccentricity and other mounting errors created in

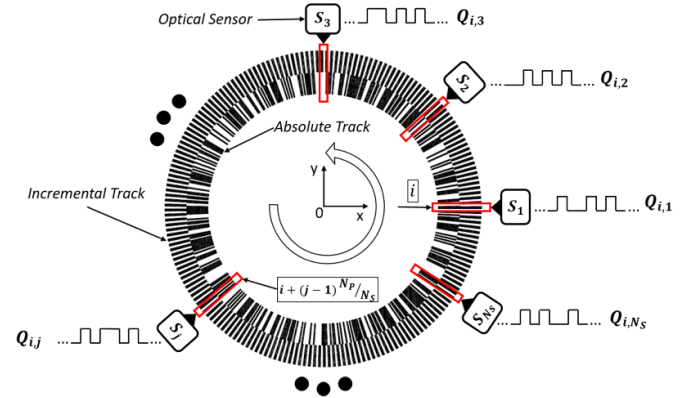


Figure 1. Relationship schematics of the optical sensors and absolute encoder circular scale.

real conditions. Moreover, the case when the encoder stator is installed non-concentrically with respect to the rotation axis of the circular scale is modelled by adding the calculated offset values to each of the artificial sensors.

2. Absolute rotary encoder based on EDA method

2.1. Basic principle

In general, the working principle of the absolute rotary encoder is based on the optical scanning of two tracks: incremental and absolute. The incremental track has equally spaced uniform lines, while the absolute track is usually composed of a pseudorandom binary sequence, such as a maximum length sequence. Properly aligned, these two tracks form a measuring scale that can tentatively present as many unique angular positions from 0 to 2π as there are lines in the incremental track. Furthermore, the analog sine and cosine signals could be electrically interpolated to obtain a higher resolution, reaching millions of nominal positions i ($i = 1, 2, \dots, N_p$) per revolution. Where N_p represents the overall number of absolute positions.

Several such optical sensors S_j are arranged around one scale at same angular intervals, as shown in figure 1. Where j ($j = 1, 2, \dots, N_s$), N_s indicates the total number of the optical sensors.

After receiving a request from the controller, all sensors read the unique positions of the scale and immediately transmit it via the specific digital protocol that implements a real-time interface. While the arbitrary sensor S_1 transmits the coded absolute position $Q_{i,1}$, the angular position transmitted by the sensor S_j could be expressed as:

$$Q_{i,j} = i + (j - 1)N_p/N_s. \quad (1)$$

In practical use, each position detected by the sensor S_j inevitably differs from the nominal position with respect to the reference. While the scale disc mounted on the application shaft turns one full revolution of 360° , this periodic angle deviation $\delta_{i,j}$ has broad frequency components. Although this deviation

cannot be determined while the encoder is operating in the real application because the reference values could not be detected, but it can be measured during the cross-calibration process.

The difference $\Delta_{i,(1,j)}$ between the transmitted data of the arbitrary and the j -th sensors could be obtained by the equation:

$$\Delta_{i,(1,j)} = \delta_i - \delta_{i+(j-1)N_P/N_S}. \quad (2)$$

The mean value represents the calibration curve of the encoder and can be expressed according to the law of the Fourier series, which mathematically proves the following about an arbitrary periodic curve:

‘An arbitrary periodic curve of 2π can be expressed by the Fourier series, and when n -number of curves with a phase shift of $2\pi/n$ are averaged, the averaged curve shows the sum of an integral multiple of n -order Fourier components of the original curve’ [24].

The mean value μ_i can be calculated as:

$$\mu_i = \frac{1}{N_S} \sum_{j=1}^{N_S} \Delta_{i,(1,j)} = \delta_i - \frac{1}{N_S} \sum_{j=1}^{N_S} \delta_{i+(j-1)N_P/N_S}, \quad (3)$$

$$Q_C = \frac{1}{N_S} \sum_{j=1}^{N_S} Q_{i,j}. \quad (4)$$

A self-calibrated position reading Q_C from multiple optical sensors can be instantly calculated and transmitted by the absolute encoder as soon as it is turned on, without the need for any angular motion.

2.2. Multi-combination of EDA method

To reduce the influence of more Fourier components without increasing the total number of optical sensors, a multi-combination EDA method could be adapted. A self-calibrated position could be calculated by averaging readings of multiple sets, with each set having a different number of optical sensors arranged according to the EDA method. One common sensor $Q_{i,1}$ must be used in each set.

$$Q_C = \frac{1}{N_{\text{Set}}} \sum_{n=1}^{N_{\text{Set}}} \left(\frac{Q_{i,1} + \sum_{j=1}^{N_S} Q_{i,j}}{N_S} \right), \quad (5)$$

where $n (n = 1, 2, \dots, N_{\text{Set}})$ is a set number and N_{Set} indicates the total number of the sets.

$$B = \prod_{n=1}^{N_{\text{Set}}} N_{S_n}. \quad (6)$$

The bandwidth B of restrained Fourier components is the product of the number of optical sensors in each set N_{S_n} [29].

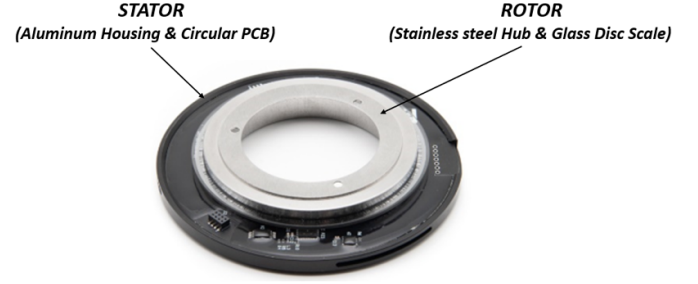


Figure 2. Absolute rotary encoder kit AR90 used for investigation.

3. Investigation of self-calibration feasibility

3.1. Device under the test

An absolute 22-bit resolution modular optical encoder AR90 is selected as the platform for self-calibration feasibility investigation. This encoder consists of a round stator mounted concentrically to the rotor, as can be seen in figure 2.

On the circular shape Printed circuit board (PCB), there are two optical reflectance sensors arranged at an angle of 180 degrees to each other. The combined position reading data from sensor #1 and sensor #2 is transmitted via a bidirectional serial synchronous protocol BiSS-C. Later, the position readings from each of the sensors can be easily separated and used for further analysis.

3.2. Cross-calibration procedure

A cross-calibration procedure is performed on the tested encoder to obtain actual position deviation values. These values will be used to virtually simulate the theoretical deviations of artificial optical sensors. The procedure is performed using the angular comparator, whose measurement error is calibrated and compensated by using a polygon and an autocollimator. This setup assures a measurement uncertainty of ± 0.65 angular seconds. The device under test is mounted on a specially designed mechanical interface and is connected to a high-accuracy reference angular encoder via the system of an air bearing and a precise shaft coupling. Both encoders are rotated simultaneously by a worm gear driven by an electric motor. The schematic view of the angular comparator is shown in figure 3.

The comparator is set to record the readings of both encoders at specific angular positions $k (k = 1, 2, \dots, N_{\text{Samp}})$, related to the counted impulses of the incremental reference encoder. The total number of recorded samples per turn is $N_{\text{Samp}} = 1440$. One sample is approximately equivalent to one-quarter of a degree. The sample $k = 1$ corresponds to the initial position, while $k = 1440$ is relatively close to the angular position of 360 degrees. The incremental impulses of the reference encoder are recorded using data acquisition device USB4 (produced by US DIGITAL), while the absolute positions from both optical sensors of the tested encoder are monitored and recorded using the iC-Haus BiSS-to-PC USB

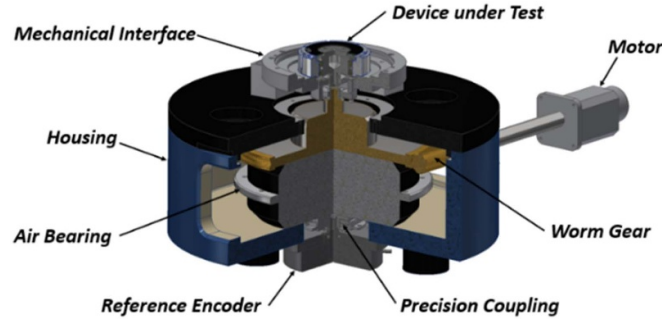


Figure 3. Schematic view of angular comparator used for cross-calibration procedure.

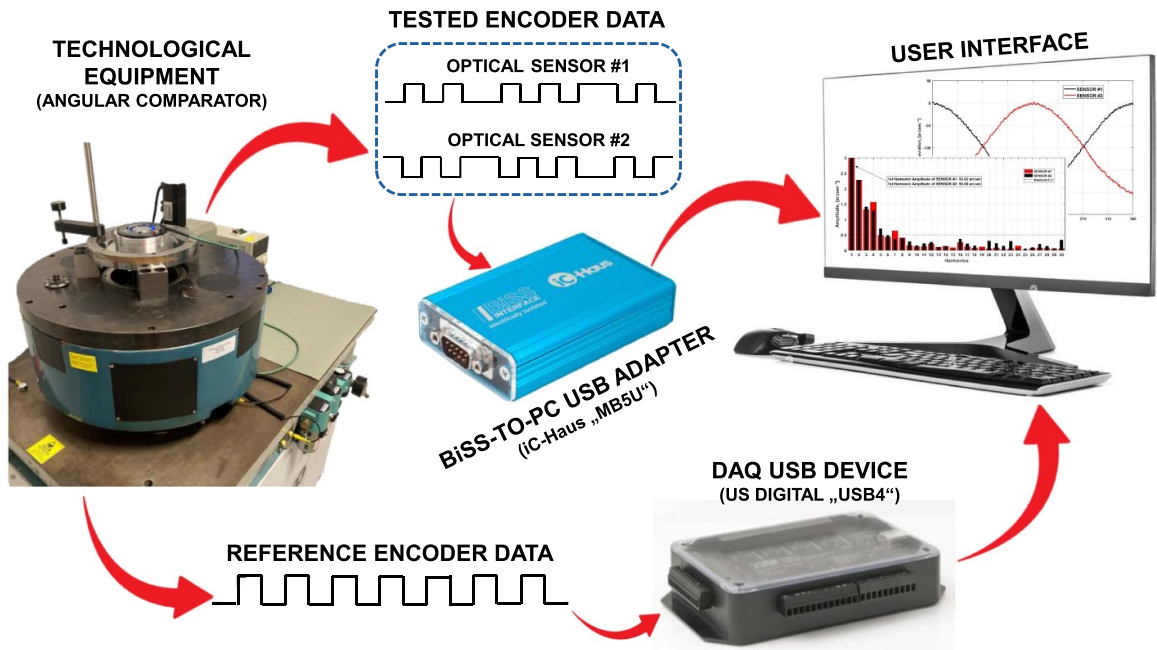


Figure 4. Schematic representation of cross-calibration procedure and setup used.

adapter MB5U. The collected data is then processed using appropriate computational and visualization software. The schematic representation of the performed cross-calibration procedure is shown in figure 4.

The differences between the reference encoder readings Q_{Rk} and the readings of the tested encoder sensors $Q_{k,1}$ and $Q_{k,2}$ are accepted as the measurement deviations:

$$\delta_{k,j} = Q_{k,j} - Q_{Rk}. \quad (7)$$

The obtained deviation plots of sensor #1 and sensor #2 are shown in figure 5 (top view). The deviation plot of sensor #2 is phase shifted by 180 degrees, with the maximum value reaching approximately 200 arcseconds. The largest part of the deviation consists of a clearly visible first harmonic (one sine period) caused by the eccentricity error. The discrete Fourier analysis performed shows that the amplitudes of the 1st harmonics reach approximately 98 arcseconds. Considering other harmonics that have a significant influence, the amplitudes of

the first seven of them exceed the margin of 0.5 arcsecond. The minimization of these harmonics is a priority during self-calibration.

After averaging the deviations of both sensors, the final error graph of the tested encoder is obtained and is shown in figure 6. This resulting data has a range of approximately 9.15 arcseconds. The harmonic analysis shows a drastically reduced influence of the first and all odd harmonics. However, the specific use of two sensors does not allow for the reduction of even harmonics. The amplitudes of 2nd and 4th harmonics respectively reach ~ 2.3 and ~ 1.3 arcseconds and are greater than the 0.5 arcsecond margin of interest. These harmonics may be caused by the non-circular shape of the grating or the deformation of the glass disc during the assembly process of the rotor.

The experimental study of the encoder reveals that the use of two optical sensors integrated at an angle of 180 degrees to each other is an effective way to eliminate the eccentricity error. The possibility of reducing the remaining harmonics by

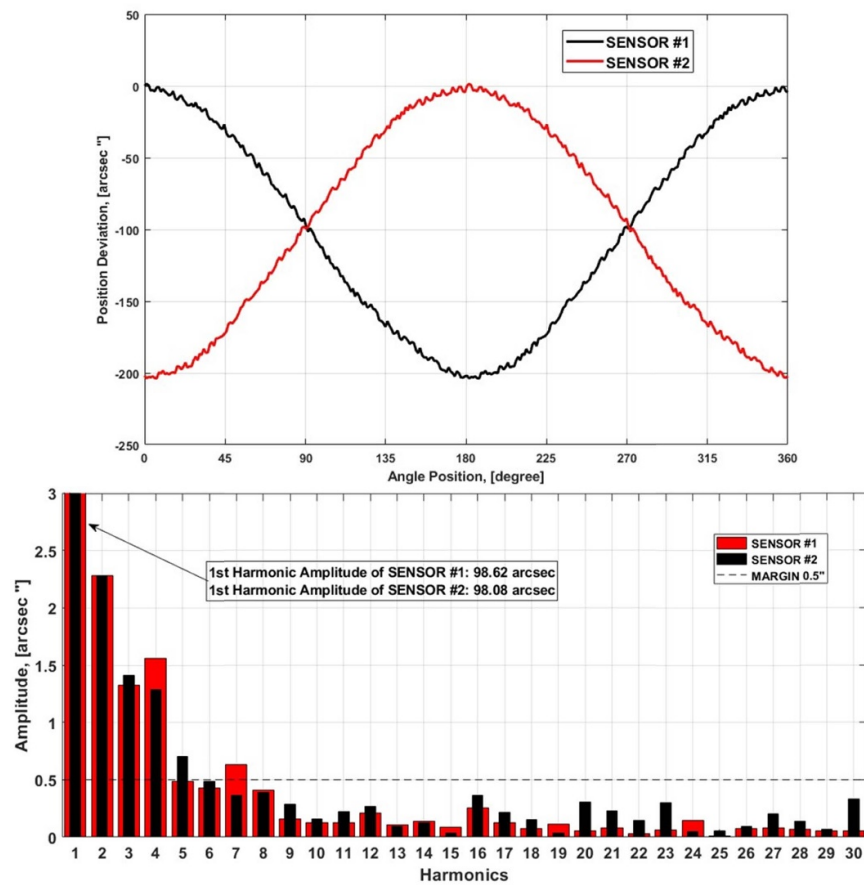


Figure 5. Cross-calibration values: deviations of sensors #1 and #2 of the tested encoder (top) and their Fourier components (bottom).

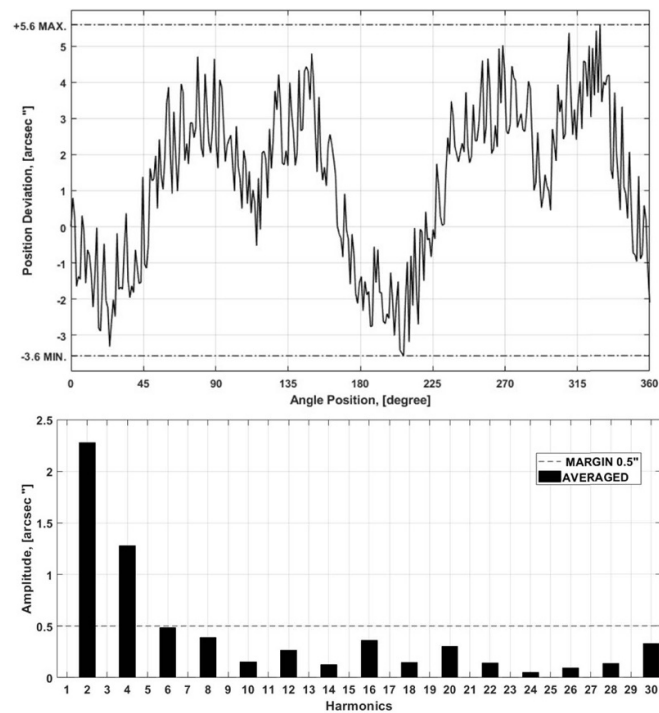


Figure 6. Averaged deviation of sensors #1 and #2 of the tested encoder (top) and its Fourier components (bottom).

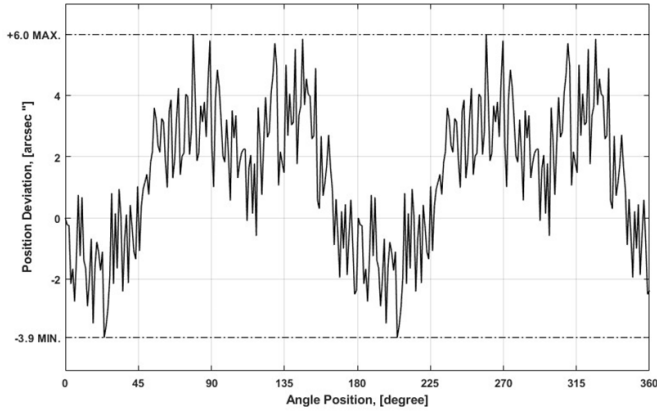


Figure 7. Theoretical averaged deviation plot of sensors #1 and artificial sensor respectively shifted by 180 degrees.

integrating several additional sensors is investigated through theoretical modelling.

3.3. Virtual simulation of the artificial optical sensors

The theoretical modelling is performed by manipulating the data collected during the cross-calibration. The data vector of calculated deviations from sensor #1 is considered. Assuming that all artificial optical sensors have similar operating characteristic and are placed in precise positions that evenly divide the 360-degree circular scale, the deviations of the artificial sensors are obtained by correspondingly shifting the data array. For example, in order to virtually simulate the deviation function of the artificial optical sensor placed at 180 degrees, the data array is phase shifted from the initial sensor #1 position $k = 1$ to the sample $k = N_{\text{Samp}}/2$, which corresponds to the reference angular position of 180 degrees. Such a procedure can be visually presented as:

$$\delta_{k,1} = \left\{ \begin{array}{c} \delta_{1,1} \\ \vdots \\ \delta_{(N_{\text{Samp}}/2)-1,1} \\ \delta_{(N_{\text{Samp}}/2),1} \\ \vdots \\ \delta_{(N_{\text{Samp}}),1} \end{array} \right\} \rightarrow \left\{ \begin{array}{c} \delta_{(N_{\text{Samp}}/2),1} \\ \vdots \\ \delta_{(N_{\text{Samp}}),1} \\ \delta_{1,1} \\ \vdots \\ \delta_{(N_{\text{Samp}}/2)-1,1} \end{array} \right\} = \delta_{k,2} \quad (8)$$

The new vector contains information that, when associated with the initial values of the reference encoder, defines the respective deviation of the artificial sensor. After calculating the average values of the newly obtained deviations and the cross-calibrated sensor #1 deviations, the theoretical error graph of the rotary encoder with two evenly placed optical sensors is calculated and shown in figure 7.

The range of the simulated deviation is approximately 9.89 arcseconds and differs from the cross-calibrated encoder with two sensors by about 8 %. Other statistical parameters of both deviation vectors are presented in table 1.

A statistical comparison shows a strong linear relationship between the experimental and theoretically modelled data, with a correlation coefficient of $r = 0.9303$. The low p -value

Table 1. Statistical data of deviation plots.

Deviation plot	Data statistics (arcsec.)				
	Range	Min.	Max.	Mean	Std.
Cross-calibrated	9.15	−3.6	5.6	1.26	2.16
Theoretical	9.89	−3.9	6	1.39	2.25

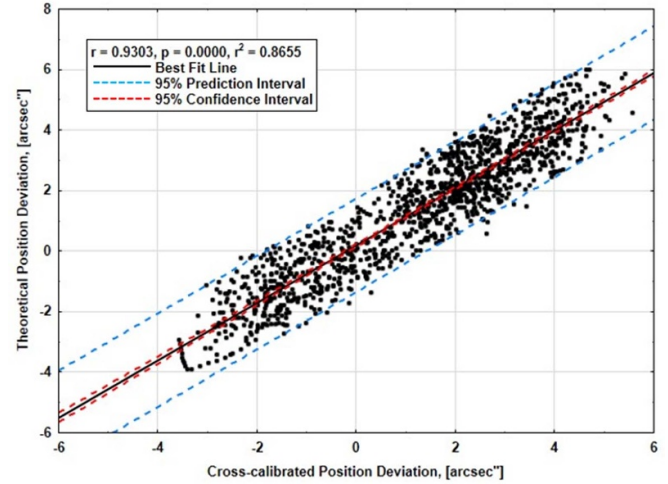


Figure 8. Comparison plot of theoretical and cross-calibrated deviation values.

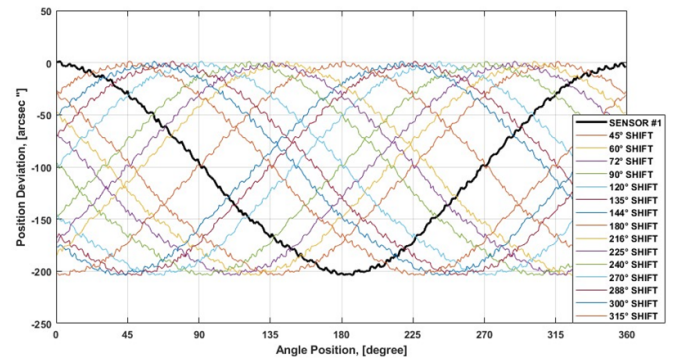


Figure 9. Deviation plot of virtually simulated artificial sensors in relation to experimentally determined sensor #1.

confirms that the applied linear regression model is statistically significant (p – value = $0.0000 < 0.05$). Comparison plot of both vectors is shown in figure 8.

The following phase-shifted deviation vectors of other artificial sensors might be derived in the same manner discussed earlier. For this theoretical research, seven different self-calibration alignments of the artificial sensors are investigated: equally spaced 3, 4, 5, 6 and 8 sensors, as well as multi-combinations of sets by 2 & 3 sensors and 3 & 4 sensors. In total, 15 new phase-shifted vectors are simulated to cover all these alignments. The modelled vectors have a phase shift of 45°, 60°, 72°, 90°, 120°, 135°, 144°, 180°, 216°, 225°, 240°, 270°, 288°, 300° and 315° from the initial position. The deviation plots of all artificial sensors are shown in figure 9.

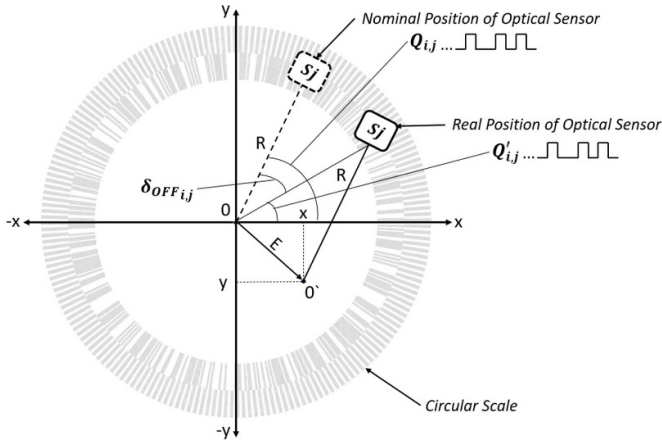


Figure 10. Schematic presentation of optical sensor misalignment in relation to circular scale.

3.4. Optical sensor misalignment

The general schematic model shown in figure 1 presents an ideal theoretical case in which the circular scale and all optical sensors are concentrically aligned with respect to the center O of the coordinates x and y . The center O of the coordinate system represents the theoretical rotation axis of the scale. In practice, this axis of rotation has an eccentricity, which can be observed as a radial runout of the scale. Another mounting error is related to the eccentricity of the optical sensors. In the considered modular encoder, all sensors are mounted on the same stationary PCB. The non-concentric mounting of the stator, as well as inaccuracies in PCB assembly, lead to inappropriate positioning of the sensors in relation to the circular scale. Such positional divergence is inevitable in practical applications. A wide range of mounting tolerances allows sensors to function properly. However, it impacts the self-calibration process. The schematic presentation of optical sensor misalignment is shown in figure 10.

When the optical sensor is displaced from the nominal position O by a distance $E = \sqrt{x^2 + y^2}$, the output position reading $Q'_{i,j}$ contains a constant offset δ_{OFF} , which can be calculated as:

$$\delta_{OFF_{i,j}} = \frac{E}{R} \sin(Q_{i,j} - Q'_{i,j}), \quad (9)$$

where R is the mean diameter of the circular scale. Different levels of self-calibration efficiency can be achieved depending on the arrangement of the optical sensors and the nature of their misalignment. For example, if two opposite sensors are displaced tangentially in the same clockwise direction, as shown in figure 11 (top), the angle between the devices will remain 180 degrees, and error compensation will be fully effective. Only an absolute position offset will be added to the system. However, if the sensors are displaced tangentially in opposite directions, as shown in figure 11 (bottom), the angle between them will deviate from 180 degrees, and the self-calibration will lose its effectiveness.

In order to evaluate the effect of optical sensors misalignment on the considered self-calibration process, a theoretical

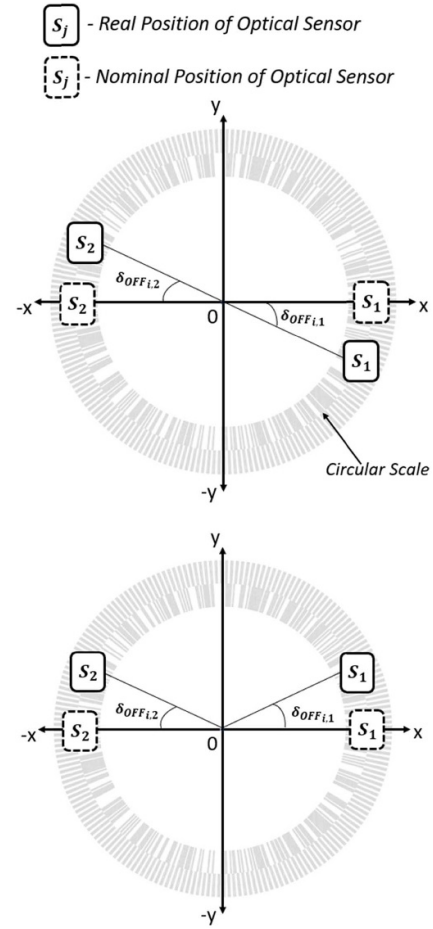


Figure 11. Schematic presentation of two opposite optical sensors misalignment: tangential in the same clockwise direction (top); tangential in opposite direction (bottom).

investigation is conducted for the case where the stator of the encoder is shifted by 0.5 mm in the positive x -axis direction. The offset values for each of the 15 simulated phase-shifted sensors are calculated according to equation (9) and added to their respective deviation vector values.

4. Results and discussion

4.1. Results of modelled self-calibration

Considering the specific artificial sensors included in each self-calibration arrangement, the averaged values of the simulated deviation vectors are calculated according to equation (3). The newly obtained data vectors represent the measurement error of the self-calibrating encoder. Their basic statistics and amplitudes of the first ten harmonics are provided in table 2.

As expected, an increased number of optical sensors improves the overall accuracy of the encoder, reducing the range of angle measurement deviation. The lowest error value is achieved by using eight sensors, where the range reaches approximately 3.11 arcseconds. In contrast, the largest range, about 6.85 arcseconds, is observed when using three

Table 2. Statistical data and harmonic amplitudes of obtained encoder deviation.

Deviation plot	Data statistics (arcsec.)										Harmonic amplitudes (arcsec.)									
	Range	Min.	Max.	Mean	Std.	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th					
3 Sensors	6.85	-4.43	2.42	-1.23	1.48	0.25	0.01	1.45	0.01	0.01	0.48	0.01	0.01	0.30	0.00					
4 Sensors	5.85	-3.42	2.43	-0.54	1.17	0.09	0.00	0.00	1.28	0.00	0.00	0.00	0.38	0.00	0.00					
5 Sensors	5.78	-2.49	3.29	0.25	0.95	0.42	0.01	0.01	0.02	0.69	0.01	0.00	0.00	0.01	0.15					
6 Sensors	3.92	-2.40	1.52	-0.90	0.75	0.30	0.01	0.00	0.01	0.01	0.48	0.01	0.01	0.00	0.00					
8 Sensors	3.11	-1.98	1.13	-0.48	0.60	0.25	0.00	0.01	0.00	0.01	0.00	0.01	0.38	0.01	0.00					
Sets of 2 & 3	7.11	-3.48	3.62	-0.02	1.38	0.12	1.07	0.73	0.64	0.00	0.48	0.00	0.19	0.15	0.08					
Sets of 3 & 4	5.69	-4.15	1.54	-1.26	0.99	0.15	0.01	0.73	0.64	0.00	0.24	0.00	0.19	0.15	0.00					

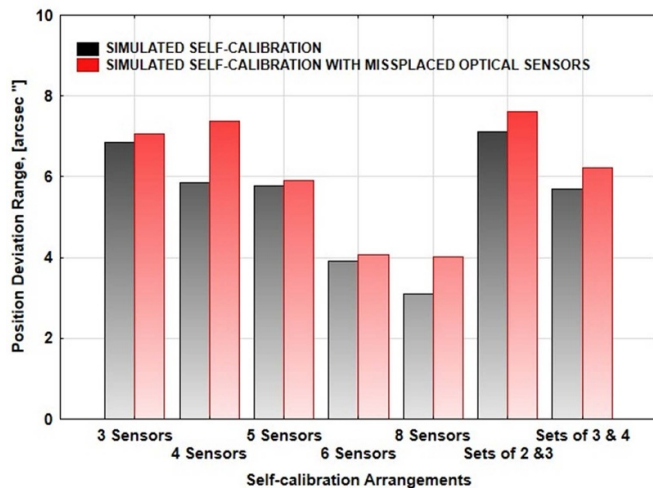


Figure 12. Schematic presentation of two opposite optical sensors misalignment: tangential in the same clockwise direction (left figure); tangential in opposite direction (right figure).

sensors. The theoretical modelling confirms that the averaged curve reflects the sum of integral multiple of n -order Fourier components of the original vector obtained through cross-calibration. The self-calibration arrangement with three equally spaced sensors could not eliminate the 3rd, 6th, and 9th harmonics, while four sensors are unable to reduce the 4th, 8th, and other multiple harmonics. The initial goal of reducing the amplitude values of the main first harmonics below 0.5 arcseconds can be achieved using six optical sensors. However, the best elimination of the eccentricity error (the 1st harmonic) is demonstrated by the encoder containing four sensors.

In terms of multi-combination arrangements, both investigated options performed worse than self-calibration with the same number of evenly spaced sensors. The deviation range of averaged sets of 2 & 3 sensors reaches approximately 7.11 arcseconds, while the combination of 3 & 4 is around 5.69 arcseconds. For comparison, the arrangements with four and six sensors reach about 5.85 and 3.92 arcseconds, respectively. Although both variants similarly reduce the 1st harmonic, the combination of 2 & 3 sensors showed the worst result of all tested cases due to the exceptionally high influence of the 2nd harmonic.

4.2. Results of modelled sensors misalignment

A response of a modelling encoder to a non-concentric stator installation, when all optical sensors are misplaced by 0.5 mm from the nominal rotation axis of the circular scale, is simulated. The measurement error function for each considered self-calibration arrangement is calculated averaging the simulated deviation vectors of the relatively misaligned artificial sensors. The impact on self-calibration efficiency is determined by comparing the ranges of the obtained encoder error functions. The column plot provided in figure 12 represents the position deviation ranges of the modelled self-calibratable encoder with and without misplaced optical sensors. The misalignment of the stator affects the overall encoder accuracy,

regardless of the arrangement of the optical sensors used. In terms of absolute value, the largest difference of approximately 1.53 arcseconds is observed when using four sensors. The least sensitive arrangement involves five optical devices, with a difference of about 0.13 arcseconds.

The modelled multi-combination arrangements demonstrate similar performance. The calculated difference between the simulated deviation ranges of the encoder with and without a misplaced stator is approximately 0.5 arcseconds.

5. Conclusions

A self-calibration feasibility study of a modular rotary encoder is presented. The proposed design solution is based on the EDA method and is applied to an absolute encoder. The theoretical modelling of certain encoder cross-calibration data is performed. The manipulation of the determined angular position measurement deviation allows to simulate different arrangements of optical sensors and estimate the efficiency of the self-calibration algorithm. The capability to eliminate circular scale eccentricity and other mounting errors, as well as the predicted misalignment of the optical sensors, is verified. After systemizing the obtained results, the following conclusions can be drawn:

1. The selected EDA method proves to be an effective approach for use in absolute modular rotary encoders. This method facilitates the elimination of mounting errors and allows for straightforward installation without the need for precise alignment or extensive initial processing. By enhancing the self-calibration capabilities of the encoder, the EDA method significantly improves overall accuracy, reducing the initial position deviation from several hundred arcseconds to below 8 arcseconds by using two or more additional optical sensors.
2. The presented theoretical simulation model is a convenient and accurate method that makes it possible to identify the parameters of artificial optical sensors and predict the total measurement error of the encoder with sufficient accuracy. The performed modelling of an arrangement with two optical sensors corresponds to the cross-calibration results by more than 90%.
3. Increasing the number of optical sensors reduces the overall measurement error of the encoder. The objective to minimize the amplitudes of the main harmonics to 0.5 arcseconds can be achieved by using at least six sensors.
4. The investigated multi-combination arrangements did not demonstrate better results compared to equally spaced optical devices.
5. The simulation of relatively misaligned optical sensors shows that the self-calibration method employing five sensors is the least sensitive to the studied case of a 0.5 mm shift (in the positive x direction) of the encoder stator.

While the theoretical analysis presented in the article provides a robust framework for understanding the behaviour of the encoder system, these predictions should ideally be

complemented by extensive experimental testing under various real-world conditions to confirm their accuracy and reliability. To address this, future work should focus on comprehensive empirical studies that validate the theoretical models. This could involve testing the encoder in diverse industrial environments, assessing its performance over time, and comparing its accuracy with other existing encoder systems. Such empirical evidence would strengthen the theoretical findings and provide a more complete understanding of the encoder's capabilities and limitations.

Data availability statement

The data cannot be made publicly available upon publication because they contain commercially sensitive information. The data that support the findings of this study are available upon reasonable request from the authors.

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