

VILNIUS GEDIMINAS TECHNICAL UNIVERSITY

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TENSION STIFFENING MODEL FOR REINFORCED CONCRETE BEAMS

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VILNIAUS GEDIMINO TECHNIKOS UNIVERSITETAS

Aleksandr SOKOLOV

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Abstract

Modelling of behaviour of cracked tensile concrete is a complicated issue. Due to bond with reinforcement, the concrete between cracks carries a certain amount of tensile force normal to the cracked plane. The concrete adheres to reinforcement bars and contributes to overall stiffness of the structure. The phenomenon, called tension-stiffening, has significant influence on the results of short-term deformational analysis. The main objective of this PhD dissertation is to propose a free-of-shrinkage tension-stiffening law for bending RC members subjected to short-term loading.

Dissertation consists of introduction, four Chapters and general conclusions.

Reasons for investigation, main objective and tasks of the work, scientific novelty and originality, basic statements to be defended, list of publications by the author and scope of the scientific work are considered in the introduction.

The first Chapter presents literature review on deformational models of RC members. Furthermore design code and numerical methods for determining deflections of reinforced concrete members are reviewed. Conclusions are formulated at the end of the Chapter.

The second Chapter introduces shrinkage influence on tension-stiffening and stress-strain state of RC members subjected to short-term loading. A numerical procedure has been analysed for deriving free-of-shrinkage tension-stiffening relationships using test data of bending RC members. The procedure combines direct and inverse techniques of analysis of RC members. Conclusions are formulated at the end of the Chapter.

The third Chapter presents an experimental investigation of tension-stiffening and short-term deformations of reinforced concrete beams. Also derivation of free-of-shrinkage tension-stiffening law for bending RC members subjected to short-term loading are presented. Conclusions are formulated at the end of the Chapter.

The fourth Chapter presents statistical analysis on short-term deflections of RC bending members. The comparative study was based on the predictions made by design codes (*Eurocode 2*, *ACI 318*, *SP 52-101*), *Hsu*, *Vecchio & Collins* and proposed models. Conclusions are formulated at the end of the Chapter.

10 scientific articles were published on the topic of the dissertation: 3 – in reviewed scientific journals, included in international databases; 2 – in *THOMSON ISI Proceedings* and 5 – in national conference material.

The author has made 7 presentations in scientific conferences (2 international and 5 republican).

Reziumė

Adekvatus supleišėjusio tempiamojo betono įtakos įvertinimas yra bene svarbiausia ir sudėtingiausia problema, nustatant trumpalaikę apkrova veikiamų gelžbetoninių elementų deformacijas. Plyšio vietoje betonas negali atlaikyti tempimo įtempių, todėl visą įrąžą atlaiko armatūra. Kadangi plyšyje ir gretimuose pjūviuose armatūra praslysta betono atžvilgiu, kontakto zonoje atsiranda tangentiniai įtempiai. Šie įtempiai perduodami betonui, todėl jis atlaiko tempimo įtempius. Armatūros ir betono sąveika ruožuose tarp plyšių standina gelžbetoninį elementą. Supleišėjusio betono gebėjimas atlaikyti tempimo įtempius vadinama *tempimo sustandėjimu* (angl. *tension-stiffening*).

Disertaciją sudaro įvadas, keturi skyriai, darbo pabaigoje pateikiami pagrindiniai darbo rezultatai ir išvados, literatūros sąrašas ir autoriaus publikacijos disertacijos tema.

Įvadiniamame skyriuje nagrinėjamas problemos aktualumas, formuluojamas darbo tikslas bei uždaviniai, aprašomas mokslinis darbo naujumas, pristatomi autoriaus pranešimai ir publikacijos, disertacijos struktūra.

Pirmajame disertacijos skyriuje apžvelgiami analiziniai-empiriniai ir skaitiniai lenkiamų gelžbetoninių elementų, paveiktų trumpalaikę apkrova, deformatyvumo analizės metodai. Skyriaus pabaigoje formuluojamos išvados.

Antrajame disertacijos skyriuje analizuojamas G. Kaklauskio sukurtas algoritmas (atvirkštinis uždavinys), leidžiantis iš eksperimentinės momentų ir kreivių diagramos gauti tempimo sustandėjimo vidutinių įtempių ir deformacijų priklausomybę. Aptariamas V. Gribniak pasiūlytas gelžbetoninių elementų įtempių ir deformacijų nustatymo skaitinis metodas (tiesioginis uždavinys), leidžiantis įvertinti susitraukimo įtaką. Skyriaus pabaigoje formuluojamos išvados.

Trečiajame disertacijos skyriuje pateikiami gelžbetoninių sijų eksperimentinių tyrimų rezultatai. Taip pat parodytas tempimo sustandėjimo modelio išvedimas, įvertinant betono susitraukimo įtaką. Skyriaus pabaigoje formuluojamos išvados.

Ketvirtajame disertacijos skyriuje tiriamas gautųjų modelių tikslumas, vertinant gelžbetoninių elementų deformacijas. Skyriaus pabaigoje formuluojamos išvados.

Disertacijos tema išspausdinti 10 mokslinių straipsnių: 3 mokslo žurnaluose, referuojamuose tarptautinėse duomenų bazėse, kurių sąrašą sudarė Lietuvos mokslo taryba; 2 straipsniai, konferencijos straipsnių rinkinyje, referuotame *THOMSON ISI Proceedings* duomenų bazėje; 5 straipsniai respublikinių mokslinių konferencijų medžiagoje.

Disertacijoje atliktų tyrimų rezultatai paskelbti 7 konferencijose (2 tarptautinėse ir 5 respublikinėse).

Notations

Symbols

A_c	– is the area of plain concrete net section;
A_{s1}	– is the area of tensile reinforcement;
A_{s2}	– is the area of compressive reinforcement;
E_c	– is the modulus of elasticity of concrete;
E_{cs}	– is the secant modulus of deformations for the peak point;
EI	– is the flexural stiffness;
F	– is the internal force;
I_{cr}	– is the moment of inertia for the fully cracked section at the yielding of reinforcement;
I_e	– is the moment of inertia for cracked concrete section;
I_g	– is the moment of inertia for uncracked concrete section ignoring reinforcement;
I_{red}	– is the reduced moment of inertia of fully cracked section;
I_{s1}	– is the moment of inertia of tensile reinforcement;
I_{s2}	– is the moment of inertia of compressive reinforcement;
L	– is the length of the element;
M	– is the bending moment;

M_{cr}	– is the cracking moment;
M_{sh}	– is the fictitious (shrinkage-induced) bending moment;
M_u, M_{ult}	– is the ultimate bending moment;
N	– is the axial force;
N_{cr}	– is the cracking force;
N_{sh}	– is the fictitious (shrinkage-induced) axial force;
W	– is the section modulus;
a_2	– is the cover depth of the compressive reinforcement;
b	– is the width of the section;
d	– is the effective depth of a section or diameter of reinforcement;
f'_c, f_{cyl}	– is the cylinder compressive concrete strength;
f_{cm}	– is the 28-day mean compressive cylinder strength of concrete;
f_{cp}	– is the prism compressive concrete strength;
f_{ct}	– is the tensile concrete strength;
$f_{ct,n}$	– is the characteristic tensile strength of concrete;
f_{cu}	– is the compressive cube strength of concrete at test;
f_r	– is the modulus of rupture;
f_y	– is the yield strength;
h	– is the height of a section;
i	– is the load increment;
k_2	– is the empirical factor [see Equation 1.5];
m_Δ	– is the sample mean of the relative error Δ ;
n	– is the number of test points in the statistical analysis;
p	– is the reinforcement ratio;
s	– is the coefficient that depends on the shape of the bending moment diagram;
s_Δ	– is the sample standard deviation of the relative error Δ ;
t_i	– is the thickness of the i -th layer;
w_c	– is the unit weight of concrete;
x_m	– is the mean depth of compressive zone of concrete;
y_c	– is the coordinate of centroid of plain concrete net section;
y_t	– is the distance from the neutral axis to the tension face of the element;
Δ	– is the relative error of prediction;
Θ	– is the rotation deviation;
α	– is the modular ratio;

β	– is the bond factor or coefficient taking account of the influence of the duration of the loading [see Equation 1.13];
β_c	– is the empirical factor [see Equation 1.4];
δ	– is the mid-span deflection;
ε_c	– is the strain of compressive concrete;
ε_{c1}	– is the maximum compressive strain of concrete;
ε_{cm}	– is the mean strain of compressive concrete;
$\varepsilon_{cr}, \varepsilon_{s,cr}$	– is the tensile strain capacity;
ε_s	– is the strain of steel;
$\varepsilon_t, \varepsilon_{ct}$	– is the strain of tensile concrete;
ε_{sh}	– is the shrinkage strain of concrete;
ε_{sm}	– is the mean strain of steel;
ε_u	– is the ultimate strain of compressive concrete;
η	– is the correction factor;
κ	– is the curvature;
μ_Δ	– is the expectation of the relative error Δ ;
ν	– is the coefficient characterized elastic properties of compressive concrete;
ξ	– is the distribution factor;
σ_c	– is the compressive concrete strength;
σ_{ct}	– is the stress of tensile concrete;
σ_s, σ_{sr}	– is the stress of steel;
σ_{sy}	– is the yielding stress;
τ	– is the age of specimen at loading;
φ	– is the creep coefficient;
χ	– is the ageing coefficient;
ψ_s	– is the tension-stiffening factor[see Equation 1.36];

Abbreviations

FE	– finite element;
FPZ	– fracture process zone;
HSC	– high strength concrete;
LEFM	– linear elastic fracture mechanics;
MSC	– medium strength concrete;
NSC	– normal strength concrete;
RC	– reinforced concrete;
RH	– relative humidity.

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Introduction

Reasons for Investigation

Modelling behaviour of cracked tensile concrete is a complicated issue. Due to bond with reinforcement, the cracked concrete between cracks carries a certain amount of tensile force normal to the cracked plane. Concrete adheres to reinforcement bars and contributes to overall stiffness of the structure. The phenomenon, called tension-stiffening, has significant influence on the results of short-term deformational analysis. Assumption of a tension-stiffening law has great influence on numerical results of load – deflection behaviour of reinforced concrete members subjected to short – term loading. Under wrong assumption of this law, errors in calculated deflections, particularly for lightly members, may exceed 100 %. Most known tension-stiffening relationships relate average stresses to average strains. However, some experimental and theoretical investigations have shown that tension-stiffening may be affected by other parameters. The scientific supervisor of the thesis has proposed a tension-stiffening model depending on reinforcement ratio. This model has been developed using experimental data reported in the literature. Besides, concrete shrinkage effect was not taken into account. The main objective of this PhD dissertation is to propose a tension-stiffening law for bending RC members subjected to short-term loading with eliminated concrete shrinkage effect.

Research Object

The object of present study is modelling tension-stiffening and deformation behaviour of reinforced concrete bending members subjected to short-term loading.

Main Objective and Tasks

The main objective is to propose a free-of-shrinkage tension-stiffening law for bending RC members subjected to short-term loading. In order to achieve the objective, the following problems had to be solved:

1. To review empirical and numerical techniques of deformation analysis of RC members as well as material models with the emphasis on tension-stiffening effects.
2. To collect test data on cracking and deformation behaviour of bending reinforced concrete elements published in literature.
3. To perform experimental investigation on cracking and deformation of lightly and normally reinforced bending RC elements.
4. To derive tension-stiffening relationships using experimental moment-curvature diagrams with eliminated concrete shrinkage effect.
5. To propose a new tension-stiffening model taking into account shrinkage effect.
6. For the collected test data set, to perform comparative statistical analysis of deflections prediction techniques using the proposed tension-stiffening and other models.

Research Methods

Seeking to achieve the aim of the work the research methods such as theoretical study, numerical research, experimental and statistical analysis were used.

Scientific Novelty and Originality

1. Lightly and normally reinforced RC beams subjected to short-term loading were tested. Free concrete shrinkage strain was measured from the start of specimen curing. The tests were performed in small load increments. Average strains were measured at four horizontal levels in the pure bending

zone of RC beams. Therefore, the test data was suitable for developing free-of-shrinkage tension-stiffening model.

2. Based on the newly obtained test data of RC beams and inverse technique, tension-stiffening model with eliminated concrete shrinkage effect has been proposed. Application of this model makes it possible to take into account free shrinkage strain as independent parameter.
3. Using test data reported in the literature, a statistical analysis of accuracy of deflection prediction techniques has been performed.

Practical Value

The proposed tension-stiffening model can be used both in standard finite element software and in Layer model which is very effective in terms of computer resources. Concrete shrinkage influence on the stress-strain state of reinforced concrete members can be accounted for by the proposed model.

Basic Statements to be Defended

1. The proposed free-of-shrinkage tension-stiffening law makes it possible to assess shrinkage, occurring prior to short-term loading, influence on stress-strain state of reinforced concrete members.
2. The proposed stress-strain tension-stiffening law depends not only on concrete tensile strength, but also on reinforcement ratio.
3. Deflections of lightly reinforced members using the proposed tension-stiffening model are calculated more accurately in regard to other models and techniques.

Participation in Research Projects

The author has participated in a research project:

Constitutive Model for Stress-Strain Analysis of Fibre Reinforced Concrete Members (Dispersiškai armuotų gelžbetoninių elementų įtempių ir deformacijų modelis), funded by the Lithuanian State Fund of Research and Studies, 2009.

Reporting Results at Scientific Conferences

The author has made 7 presentations at 6 scientific conferences:

- The Tenth International Conference *Modern Building Materials, Structures and Techniques*, Vilnius, Lithuania, 2010.
- The Twelfth Lithuanian Conference of Young Scientists *Science – Future of Lithuania*, Vilnius, Lithuania, 2009.
- The Ninth International Conference *Modern Building Materials, Structures and Techniques*, Vilnius, Lithuania, 2007.
- The Fourth International Conference *Strength, Durability and Stability of Materials and Structures*, Palanga, Lithuania, 2007.
- The Tenth Lithuanian Conference of Young Scientists *Science – Future of Lithuania*, Vilnius, Lithuania, 2007.
- The Ninth Lithuanian Conference of Young Scientists *Science – Future of Lithuania*, Vilnius, Lithuania, 2006.

The Scope of the Scientific Work

The dissertation consists of introduction, four Chapters, general conclusion and a list of references.

The total scope of dissertation – 91 pages, 88 formulae, 30 figures, 10 tables and 186 references.

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Literature Review on Deformational Models

Design for serviceability is the main task for structural engineers and code-writing bodies. It is also essential to users of the designed structures. Increased use of high strength concrete with reinforcing bars and prestressed reinforcement, coupled with more precise computer-aided limit state serviceability designs, has resulted in lighter and more material-efficient structural elements and systems. This in turn has necessitated better control of short-term and long-term behaviour of concrete structures at service loads. Deflection of one-way non-prestressed concrete flexural members is controlled by reinforcement ratio limitations, minimum thickness requirements, and span/deflection ratio limitations. Wide availability of personal computers and design software, plus the use of higher strength concrete with steel reinforcement has permitted more material efficient reinforced concrete designs producing shallower sections. More prevalent use of high-strength concrete results in smaller sections, having less stiffness that can result in larger deflections. Consequently, control of short-term and long-term deflection has become more critical. In many structures, deflection rather than stress limitation is the controlling factor. Deflection computations determine the proportioning of many of the structural system elements. Member stiffness is also a function of short-term and long-term behaviour of the concrete. Hence, expressions defining the modulus of rupture, modulus of elasticity,

creep, shrinkage, and temperature effects are prime parameters in predicting the deflection of reinforced concrete members.

With the improved knowledge of material properties and behaviour, emphasis has shifted to the use of high strength concrete components. Consequently, designs using load-resistance philosophy have resulted in smaller sections that are prone to smaller serviceability safety margins. As a result, prediction and control of deflections and cracking through appropriate design have become a necessary phase of design under service load conditions. Beams and slabs are rarely built as isolated members, but are a monolithic part of an integrated system. Excessive deflection of a floor slab may cause dislocations in the partitions it supports or difficulty in levelling furniture or fixtures. Excessive deflection of a beam can damage a partition below, and excessive deflection of a spandrel beam above a window opening could crack the glass panels. In the case of roofs or open floors, such as top floors of parking garages, pounding of water can result.

The materials related to Chapter 1, the author in co-authorship has reported in the following papers: Gribniak *et al.* (2010), Kaklauskas *et al.* (2008), Gribniak *et al.* (2007d), Kaklauskas *et al.* (2007), Kaklauskas *et al.* (2006), Sokolovas & Logunov (2006).

1.1. Constitutive Models for Reinforced Concrete

Deformational behaviour of cracked RC members is a complex process including a wide range of effects, such as, different strength and deformation properties of steel and concrete, concrete cracking, tension-softening and tension-stiffening, bond slip between reinforcement and concrete, etc. It should be noted that many constitutive models for concrete have been developed all over the world. Also, such kind of research has been performed in Lithuania, home country of the author. Some more recent publications can be noted: Jonaitis *et al.* (2007), Jonaitis *et al.* (2009), Marciukaitis (1978), Marciukaitis & Balevicius (2009), Marciukaitis *et al.* (2007a), Marciukaitis *et al.* (2006a), Marciukaitis *et al.* (2007b), Marciukaitis *et al.* (2006b).

1.1.1. Uniaxial Compression

The compressive strength of concrete is the property most often used in structural design calculation (Nejadi 2005). It can be readily determined in practice by simple test procedures. The response of concrete in uniaxial compression is usually obtained from cylinders with a height to diameter ratio of 2 to 1 or from cubes. A standard cylinder is 300 mm high by 150 mm diameter and the lower

characteristics value of its compressive strength at 28 days is termed f'_c . This is the value of compressive strength exceeded by 95 % of all standard cylinder specimens tested at 28 days under standard conditions (Gilbert 2002). Smaller cylinders, or cubes, (generally having a higher compressive strength) are also used for production control, the results of which can be converted into equivalent standard cylinder strength values using appropriate conversion factors.

Fig. 1.1 illustrates a typical average stress-strain curve for a concrete specimens subjected to uniaxial compression. Although the curve is non-linear throughout, the response may be taken as linear-elastic up to 30-40 % of the ultimate strength, after which the curve exhibits increasingly non-linear behaviour up to the peak stress f'_c because of the propagation of internal micro cracks. The concrete undergoes strain softening after the peak stress, as shown in Fig. 1.1.

Response under compression in the post-peak range follows a softening or descending branch until the concrete reaches its ultimate strain. Strain softening under compression is influenced by specimen size, strength of the concrete, (Kaufmann 1998) and the stiffness of the testing machine. RILEM TC 148-SSC (1997) presented an extensive test program which minimised the influence of specimen size and boundary restraint in measuring strain softening of concrete subjected to uniaxial compression.

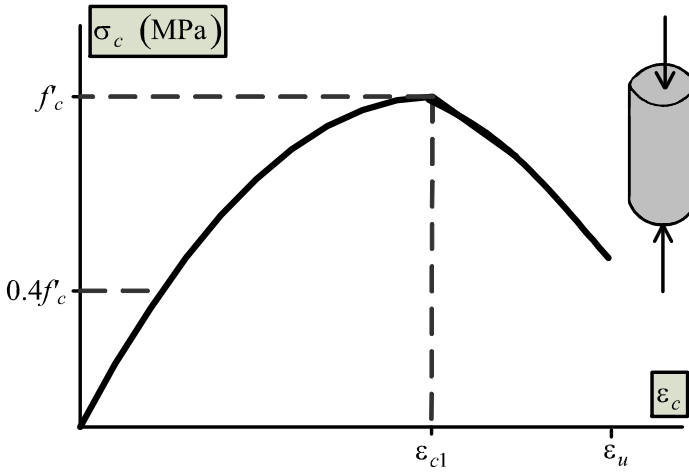


Fig. 1.1. Typical uniaxial stress-strain curves of concrete in compression

In early numerical simulations (Hand *et al.* 1973, Lin & Scordelis 1975) an ideally elasto-plastic diagram has been assumed for modelling of compressive concrete. It has been considered that plastic structural deformations were mainly due to cracking of tensile concrete and plastic steel strains but not due to plastic deformations of compressive concrete. However, soon importance of plastic

strains of the compressive concrete on total behaviour of structure has been realised and numerical analyses have employed a great number of stress-strain relationships based on uniaxial tests: Shah & Amad (1978), Hognestad *et al.* (1955), Dilger *et al.* (1984), CEN (2004), Wang *et al.* (1978), Smith & Young (1956), Barnard (1964), Park & Paulay (1975), CEB - FIP Model Code (1991), Szulczynski & Sozen (1961), Roy & Sozen (1961), Liebenberg (1962), Kent & Park (1990), Popovics (1970, 1973), Young (1960), Ros (1950), Saenz (1964), Mansur *et al.* (1997), Debernardi & Taliano (2001) etc. A few most widely used expressions of the curves are presented below:

$$\sigma_c = f'_c (2\eta - \eta^2), \text{ (Hognestad } et al. \text{ 1955)} \quad (1.1)$$

$$\sigma_c = f'_c \frac{k\eta - \eta^2}{1 + (k-2)\eta}, \quad \eta = \frac{\varepsilon_c}{\varepsilon_{c1}}, \quad k = \frac{1.05E_{cm}|\varepsilon_{c1}|}{f'_c}, \text{ (CEN 2004)} \quad (1.2)$$

Here σ_c and ε_c are the stress and strain of the compressive concrete, respectively; f'_c and ε_{c1} are the maximum compressive stress and the corresponding strain for standard cylinder test; E_{cm} is the secant modulus of elasticity of concrete.

Equations 1.1-1.2 describe the ascending branch of the curve while the descending part is often assumed as a horizontal or sloped line terminating at a limiting strain ε_u (Fig. 1.1). Although in flexural tests values from 0.0025 to 0.006 have been measured for ε_u , it is usually assumed within the interval of 0.003 and 0.004.

Carreira & Chu (1985) have proposed a complete stress-strain relationship of the serpentine curve:

$$\sigma_c = \frac{\beta_c f'_c (\varepsilon_c / \varepsilon_{c1})}{\beta_c - 1 + (\varepsilon_c / \varepsilon_{c1})^{\beta_c k_2}}. \quad (1.3)$$

Here β_c is an empirical factor.

Thorenfeldt *et al.* (1987) proposed to use Equation 1.3 for high strength concrete assuming:

$$\beta_c = 1 / (1 - f_c / \varepsilon_{c1} E_c). \quad (1.4)$$

Mansur *et al.* (1997) offered an expression for factor k_2 :

$$k_2 = (42.5 / f'_c)^{1.3}. \quad (1.5)$$

The initial slope of the curves indicating the modulus of elasticity, E_c , increases with an increase in compressive strength. E_c is affected by the modulus of elasticity of the cement paste and that of the aggregate. An increase in water-cement ratio increases the porosity of the paste, reducing its strength and

modulus of elasticity. Of equal importance is the modulus of elasticity of the aggregate. Normal weight aggregates have modulus of elasticity values ranging from 1.5 to 5 times that of cement paste. Therefore, the fraction of the total mix that is aggregate also affects E_c .

Literature review on assumption of the peak strain ε_0 has been performed by Nicolo *et al.* (1994). Constant value of ε_0 has been assumed by a number of researchers and codes: 0.0022 by CEN (2004) and CEB-FIP (1991) and 0.002 by Szulczynski & Sozen (1961), Roy & Sozen (1963), Liebenberg (1962), Kent & Park (1990), Popovics (1970), and Young (1960).

Numerous investigations (e.g., Parasonis 1971, Kudzyś & Notkusz 1978, Vecchio & Collins 1982) have been carried out for modelling deformational behaviour of the compressive concrete in reinforced concrete structures.

1.1.2. Uniaxial Tension

The tensile strength of concrete f_{ct} is only about 10 % of its compressive strength and is defined as the maximum stress that concrete can withstand when subjected to uniaxial tension. It becomes hard to measure direct uniaxial tensile strength because it is difficult to achieve true axial tension without secondary stresses caused by the holding devices. The most commonly used methods to determine tensile strength are the cylinder splitting or Brazilian test, and the flexural or modulus of rupture test.

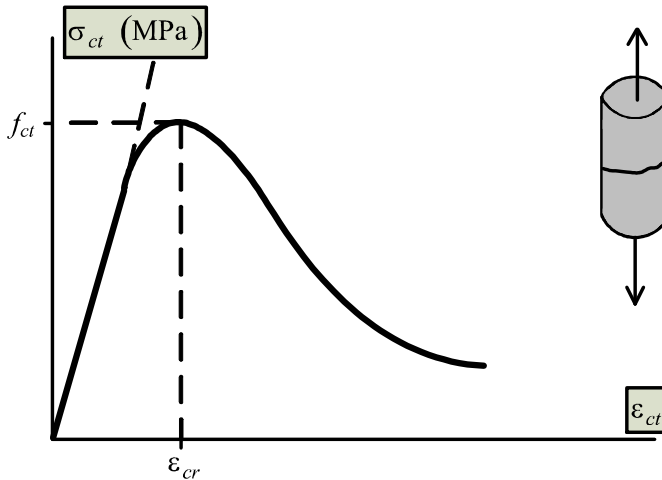


Fig. 1.2. Uniaxial tensile stress-strain curves of concrete

Strongly stochastic nature of test data also complicates interpretation of experimental results (Lemnitzer *et al.* 2008). Hughes & Chapman (1966) and Evans & Marathe (1968) have used modified direct tension testing machines to obtain stress-strain diagrams in tension with ascending and descending branches similar to those obtained in compression. The shape of the curve (Fig. 1.2) shows many similarities to the uniaxial-compression curve (Fig. 1.1), however, there are some differences. The curve shown in Fig. 1.2 is nearly linear up to a relatively higher stress level. The stress of about 60 % of the uniaxial tensile strength, f_{ct} , can be considered as a limit of elasticity. The interval of stable crack propagation is relatively short and the limit of unstable crack propagation is reached at about 75 % of f_{ct} (Evans & Marathe 1968). The failure in tension is caused by a few bridging cracks rather than by numerous cracks, as it is for the compressive states of stress.

The ratio between uniaxial tensile and compressive strength may vary considerably but usually ranges between 0.05 and 0.1. The modulus of elasticity under uniaxial tension is somewhat higher and the Poisson's ratio somewhat lower than in uniaxial compression.

In fracture mechanics approach, based on crack growth and localised effects, the shape of the strain softening curve in tension is adjusted from one problem to another, depending on fracture energy (e.g. Bazant & Oh 1983, Wittmann 2002, Liu *et al.* 2008, Darwin *et al.* 2001). Massicotte *et al.* (1988) performed a detailed analysis of 52 test results presented by Gopalaratnam & Shah (1985), Hilleborg (1983, 1985a, b), Guo & Zhang (1986), Bazant & Oh (1983) and Yankelevsky & Reinhardt (1987). Based on these results and as a reasonable compromise between accuracy and simplicity, a trilinear stress-strain curve was adopted (Massicotte 1988), with a linear ascending branch and a bilinear softening branch after cracking. The change of slope in the descending branch occurs at one-third of f_{ct} , as proposed by Hilleborg (1985).

1.1.3. Steel Reinforcement

Steel reinforcement is used to provide strength, ductility and serviceability to concrete structures. With regard to serviceability, reinforcement is used to reduce instantaneous and long-term deformations and provide crack control.

Structural concrete elements are generally designed such that failure will be governed by yielding of the reinforcement. The yield stresses typically amount to 400 - 600 MPa. The deformation capacity of structural concrete elements is an important aspect in the design of such structures. It mainly depends on the ductility of the reinforcement (Sigrist 1995). Therefore, ductility of the reinforcement is as essential to structural concrete as its strength.

Steel reinforcing bars are usually round with regularly spaced, rib-shaped deformations on the surface to provide a better bond between the concrete and steel. There are basically two different types of stress-strain characteristics of reinforcing steel; hot-rolled, low-carbon or micro-alloyed steel bars and cold-worked and high-carbon steel bars or wires (Kaufmann 1998). As shown in Fig 1.3a, a hot-rolled, low-carbon steel bar in tension, exhibits an initial linear elastic portion followed by a yield plateau in which the strain increases with little or no change in stress and a strain-hardening range which increases until rupture of steel bar at the tensile strength occurs. Cold-worked and high-carbon steel bars initially show a linear elastic response with a smooth change to the strain-hardening branch, without a distinct yield point (see Fig. 1.3b). In such case, the yield stress is taken as the stress at which a permanent strain of 0.2 % remains after unloading.

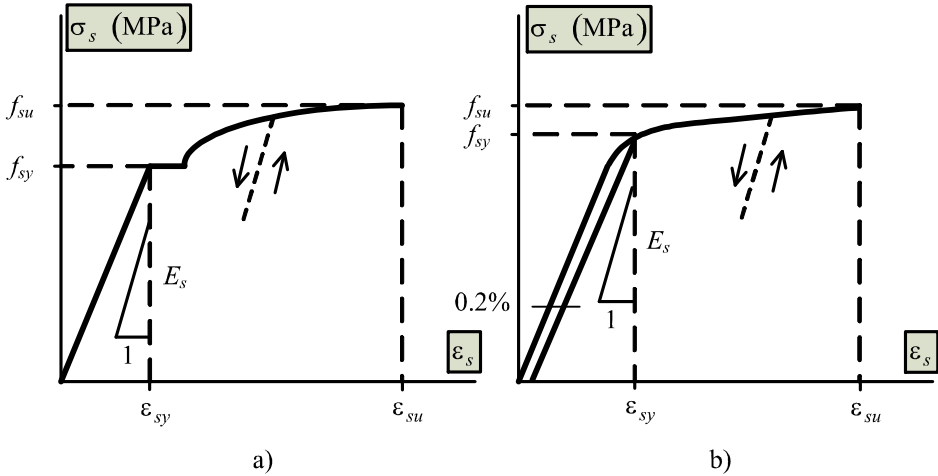


Fig. 1.3. Stress-strain characteristics of reinforcement in uniaxial tension: a) hot-rolled, heat-treated, low-carbon steel; b) cold-worked or high-carbon steel (Kaufmann 1998)

Reinforcing steel is generally assumed as elastic-plastic for design calculations and f_{sy} is taken to be the material strength, where the stress-strain curve is assumed horizontal when f_{sy} is reached. Under service loads steel stress is less than yield stress and its behaviour is linear-elastic; the stress-strain curve in compression is also assumed to be similar to that in tension.

1.1.4. Tension-Stiffening

A comprehensive literature review on tension-stiffening models and deformation behaviour of RC members has been performed by Kaklauskas (2001) and Gribniak (2009).

The first studies which defined the behaviour of concrete under tensile stress began towards the end of 19th century when, though the intense activity of skilful builders, the technique of RC reached certain stages that were fundamental for its subsequent success. Joly (1898a, 1898b) and Considere (1899a, 1899b) who made a decisive contribution to explaining the behaviour of RC flexural and tensile members performed first intuitions on the tension-stiffening. After the response of the elements to the applied actions had been explained, its behaviour was defined though mathematical models, necessary for the calculation theories according to Albenga (1945), were many and contradictory.

Cracking in a reinforced concrete member also causes a significant increase in deflection. This is a result of the reduction of bending stiffness at cracked section when the effect of tensile concrete below the neutral axis diminishes. However, at the sections between successive cracks, some tensile stress is restrained in the concrete around steel bars due to the actions of bond, contributing to the bending stiffness of the member. This is called tension-stiffening effect. If the tension-stiffening effect is neglected, calculated deflection may be overestimated by a large proportion. In simplified methods of deflection calculation the tension-stiffening effect is incorporated in a semi empirical manner by using the effective moment of inertia method. In analytical methods, the deflection is calculated using curvature values, evaluated by adopting a non-linear stress-strain relationship for tensile concrete. This allows concrete to retain some tensile stress beyond the cracking strain.

Many theoretical models of RC in tension have been proposed to predict cracking and deformations of RC members. Generally, these models may be separated into three main approaches:

- Average stress-average strain: simple approaches, extensively used in numerical analyses, based on smeared crack model.
- Bond stress transfer in the interface zone of concrete and reinforcement and fracture mechanics: these approaches aim at modelling bond between concrete and reinforcement steel and use the fracture mechanics principles to predict cracking behaviour of RC elements.
- Analytical-empirical: the earliest approaches were developed based on the analysis of test data. Such simplified calculation techniques are broadly presented in the design codes. The techniques on this approach in more detail are presented in Section 1.3.

1.1.4.1. Average Stress-Average Strain

This simple approach, extensively applied in numerical analyses, is based on use of average stress-strain tension-stiffening relationship. The approach introduced by Rashid (1968) is based on smeared crack model, i.e. cracks are smeared out in the continuous fashion and the cracked concrete is assumed to remain a continuum. Concrete becomes orthotropic with one of the material axes being oriented along the direction of cracking.

Differently from the discrete crack model, which is tracing individual cracks, smeared crack model deals with average strains and stresses. This model can handle single, multiple and distributed cracks in a unified manner. Thus, it can be used for both plain concrete and RC structures (Cervenka 1995). In FE analysis, smeared crack model has proven to be more flexible and more computationally effective concerning the discrete crack model since no topological constraints exist.

Tension-stiffening can be attributed either to tensile reinforcement (steel-related model) or to concrete (concrete-related model). In the latter approach, it may be assumed that tension-stiffening is effective either in the whole tension area or in the specified zone (close to reinforcement), called the effective area.

Gilbert & Warner (1978), Cervenka *et al.* (1990), Hofstetter & Mang (1995), Feenstra & Borst de (1995), Salys *et al.* (2009) have used the steel-related approach. It should be noted that this approach is relatively rarely applied.

In the effective area approach, the influence of tension-stiffening is limited to a volume of concrete in relatively close proximity to the bar (tension-stiffening zone). In *Model Code 90* (CEB-FIP 1991) tension-stiffening zone was limited to concrete area within 7.5 bar diameters from the reinforcement. Outside this zone, the second mechanism of post cracking prevails, that of tension-softening (Vecchio & Collins 1986, Stramandinoli & Rovere 2008).

Based on experimental investigation of reinforced concrete panels subjected to pure shear Vecchio & Collins (1986) in their modified compression field theory proposed the following relationship for the cracked concrete:

$$\sigma_t = \frac{f_{ct}}{1 + \sqrt{200\varepsilon_{ct}}}, \quad f_{ct} = 0.33\sqrt{f'_c}. \quad (1.6)$$

Here σ_t and ε_{ct} are the stress and strain of tensile concrete, respectively; f_{ct} is the cracking strength of concrete; f'_c is the cylinder strength of concrete in compression.

Hsu (1993) in his unified theory of reinforced concrete proposed the following relationship:

$$\sigma_t = f_{ct} \left(\frac{\varepsilon_{cr}}{\varepsilon_{ct}} \right)^{0.4}, \quad f_{ct} = 0.311 \sqrt{f'_c}, \quad \varepsilon_{cr} = 0.00008. \quad (1.7)$$

Carreira & Chu (1986) proposed the relationship of the same general form as the stress-strain relationship in compression, see Equation 1.3:

$$\sigma_t = \frac{\beta_t f_{ct} (\varepsilon_{ct} / \varepsilon_{cr})}{\beta_t - 1 + (\varepsilon_{ct} / \varepsilon_{cr})^{\beta_t}}. \quad (1.8)$$

Here β_t is an empirical factor.

Scott (1983) derived a stress-strain relationship for cracked tensile concrete using experimental beam data of Clark & Speirs (1978). An attempt was made by Prakhya & Morley (1990) to include several parameters affecting the tension-stiffening into the stress-strain curve of tensile concrete for analysis of flexural members. On the basis of simplified assumptions and by using some experimental data (Clark & Speirs 1978, Clark & Cranston 1979) they have applied Equation 1.8 (Carreira & Chu 1986) by modifying the empirical factor β_t :

$$\beta_t = \left(\frac{100A_s}{b(h - x_{nt})} \right)^{0.366} \left(\frac{b(h - x_{nt})}{n\pi d_b} \right)^{0.344} \left(\frac{c}{s} \right)^{0.146}. \quad (1.9)$$

Here A_{sI} is the cross-section area of tensile reinforcement; b is the width; h is the total depth; c is the clear cover to the reinforcement; d_b is the reinforcement bar diameter; n is the number of bars; s is the reinforcing spacing, and x_{nt} is the neutral axis depth neglecting tension in the concrete.

Most of the continuum-based FE methods incorporate tension-stiffening by the constitutive law of tensile concrete (Suidan & Schnobrich 1973, Lin & Scordelis 1975, Prakhya & Morley 1990, Barros *et al.* 2001, Ebead & Marzouk 2005, Gribniak & Kondratenko 2005, Gribniak & Girdzius 2005, Gribniak *et al.* 2005, 2006, 2007b, 2007d, Kaklauskas *et al.* 2007a, Kaklauskas *et al.* 2007c, Kaklauskas *et al.* 2008, Kaklauskas & Sokolov 2007, Sokolov 2007, Kaklauskas *et al.* 2006, Sokolovas & Logunov 2006 and Bacinskas *et al.* 2007). In present research, behaviour of RC member is modelled assuming a uniform tension-stiffening relationship over the whole tension area of concrete. Stress in the concrete is taken as the combined stress due to tension-stiffening and tension-softening, collectively called the tension-stiffening. Based on the above approach, a number of stress-strain constitutive relationships for cracked tensile concrete have been proposed. Kaklauskas (2001) and Bischoff (2001) have carried out a comprehensive review of the relationships. In the analysis of tension members, Bischoff (2001) has introduced bond factor β representing the ratio of average tensile stress in concrete and the cracking stress.

Most tension-stiffening relationships were derived using experimental data of tension (Bischoff 2001, Fields & Bischoff 2004) or shear (Vecchio & Collins 1986, Sato *et al.* 2004, Bentz 2005, Debernardi & Taliano 2006) RC members. These constitutive laws were applied for modelling flexural members, although their behaviour differs from tension or shear members.

The supervisor of the dissertation in co-authorship (Kaklauskas & Ghaboussi 2001) has proposed an alternative approach for deriving tension-stiffening relationships from test moment-curvature diagrams of RC members. Based on inverse technique, tension-stiffening relationships were computed from the equilibrium equations for incrementally increasing bending moment assuming portions of the relationships obtained from the previous increments. Due to stochastic distribution of test data, a probabilistic approach, based on *Monte Carlo* technique (Mosegaard & Tarantola 1995, Tarantola 2005, Kaklauskas *et al.* 2007b), can be assumed in the *inverse* problem.

Tension-stiffening relationship shown in Fig. 1.4 has been proposed by the Kaklauskas (2001). It was called Flexural as it has been derived on a basis of flexural experimental data. The descending branch of the relationship was expressed by the formula:

$$\sigma_t = 0.625 f_{ct} \left(1 - \frac{\overline{\varepsilon}_{ct}}{\beta} + \frac{1 + 0.6\beta}{\overline{\varepsilon}_{ct}} \right), \quad \overline{\varepsilon}_{ct} = \frac{\varepsilon_{ct}}{\varepsilon_{cr}}. \quad (1.10)$$

$$\beta = 32.8 - 27.6p + 7.12p^2. \quad (1.11)$$

Here p is the reinforcement ratio; f_{ct} is the tensile concrete strength.

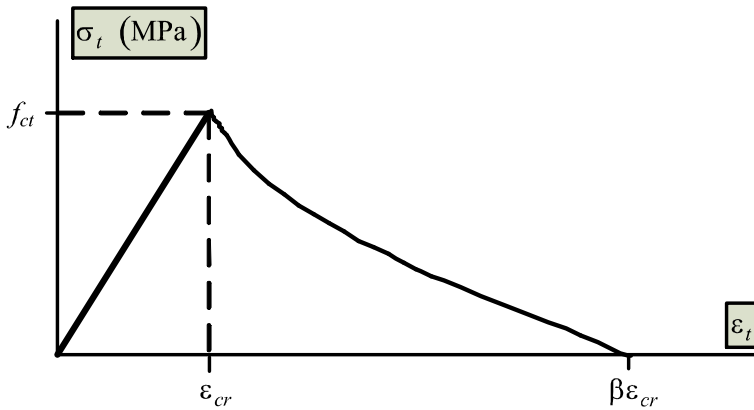


Fig. 1.4. Flexural tension-stiffening relationship

In the most cases, tension-stiffening relationships were derived using test data of RC members exposed to shrinkage. Therefore, derived constitutive rela-

tionships in an integrated manner include effects of tension-softening, tension-stiffening and shrinkage.

1.1.4.2. Bond Stress Transfer in the Interface Zone of Concrete and Reinforcement and Fracture Mechanics

This approach is based on bond-slip relationship, which models the bond-action between concrete and reinforcement. Saliger (1936) was the first one, who has published the basis for all theories using this approach. The bond-action at the interface of steel bars and surrounding concrete has great influence on the initiation and propagation of cracks in RC members. The bond-action is the main contributing factor to the tension-stiffening effect in concrete structures. In a cracked RC member, an increase in loading will result in an increase in steel strain, causing an extension of the reinforcing bar (Feenstra & Borst de 1995, Ngo & Scordelis 1967). Consequently, ribs in the bar will tend to move towards the nearest crack relative to the surrounding concrete, i.e. bond-slip is initialised. This will increase the rib bearing stress on concrete, contributing to the bond-slip.

In analytical investigations of RC members, the bond stress-slip relationship is of fundamental importance. Such relationships are analogous to the average stress-strain laws for concrete or steel (Nilson 1972, Edwards & Yannopoulos 1979). Unlike the constitutive laws for steel or concrete, a unique relationship for the bond stress-slip is not yet available despite the large number of investigations carried out. Nilson (1972), Mirza & Houde (1979), Ciampi *et al.* (1981), Jiang *et al.* (1984), Giuriani *et al.* (1991), Kankam (1997), Wu & Gilbert (2008) and many other researchers who have developed such relationships. Various aspects of stress transfer approaches has been investigated by Floegl & Mang (1982), Choi & Cheung (1996), Fantilli *et al.* (1998b), Manfredi & Pecce (1998), Polak & Blackwell (1998), Kwak & Filippou (1995), Monti & Spacone (2000), Kwak & Song (2002), Lackner & Mang (2003), Foster & Marti (2003), Borosnyoi & Balazs (2005), Eckfeldt (2005), Ruiz *et al.* (2007), Vollum *et al.* (2008) and etc. Piyasena (2002) and Leutbecher & Fehling (2009) have performed a comprehensive survey on the bond stress-slip relationships.

Stress transfer approach realistically models cracking, crack widths and deformations. However, accuracy of numerical results depends on the assumed bond stress-slip relationship. Besides, concerning smeared approach it is more complex and relatively rarely used in practice.

Initiated in 1960th by Kaplan (1961), studies of fracture mechanics has progressed by the turn of the century. Kesler *et al.* (1972) showed that LEFM of sharp cracks was inadequate for normal concrete structures. It was also supported by the results of Walsh (1972, 1976). Inspired by the softening and plas-

tic models of FPZ initiated in the works of Barenblatt (1959, 1962), Dugdale (1960), Hillerborg *et al.* (1976) have proposed the first nonlinear theory of fracture mechanics for concrete.

In the early 1980s, it was recognised that plain concrete is not a perfectly brittle material in the Griffith's sense, but it has some residual load-carrying capacity after reaching the tensile strength. This has led to the replacement of brittle crack model by tension-softening approach, in which a descending branch was introduced for modelling the gradually diminishing tensile strength of concrete upon further crack is opening. Such a descending branch also emerges in most tension-stiffening models causing much confusion while modelling tension-softening and tension-stiffening in RC (Borst de 2002).

The introduction of tension-softening in crack models was also motivated on theoretical grounds. It was observed that use of strength models (Cedolin & Bazant 1980) or the straightforward use of strain softening models led to an unacceptable and unphysical mesh sensitivity (Bazant 1976, Crisfield 1982). Recent developments in the application of fracture mechanics to concrete have made it possible to analyse effectively the post-cracking behaviour of plain concrete using the finite element (FE) method. These applications have incorporated tension-softening models to describe the gradual decay of stress/strain softening in plain concrete in tension as cracking propagates. Several researchers adopting a tension-softening model have obtained consistent results (Bazant & Oh 1983, Cornelissen *et al.* 1986, Hordijk 1991, Mier van 1991 and Carpinteri 1994).

Numerical modelling of plain and reinforced concrete started in the late 1960s with the landmark papers of Ngo & Scordelis (1967) and Rashid (1968) in which the discrete and smeared crack models were introduced. In the discrete crack model, cracking is assumed to occur as soon as the nodal force normal to FE boundaries exceeds the maximum tensile force that can be sustained and continuous re-meshing is required. In the smeared crack model, a cracked solid is imagined to be a continuum, describing the behaviour of cracked concrete by stress-strain relationships. This implies that the topology of the original FE mesh remains preserved. The latter approach leads to a straightforward computer implementation, and widespread by being used in practice (Li & Zimmerman 1998). With nonlinear fracture mechanics, the range of validity of both approaches was extended, leading to the fictitious crack model (Hillerborg *et al.* 1976) and to the crack band model (Bazant & Oh 1983). These methods have been well evaluated by Cope *et al.* (1979), Zimmerman (1986) and Elices & Planas (1989).

Bazant & Oh (1983) carried out a theoretical study on the spacing and width of cracks, using the energy criterion of fracture mechanics as well as the strength criterion. The strength criterion indicates whether the fracture formation can initiate, while the energy criterion indicates if the fracture can actually form. The

study suggested that the crack spacing depends mainly on the axial strain of steel bars, bar spacing, bar diameter fracture energy of concrete and its elastic modulus.

The fictitious crack model (Hillerborg *et al.* 1976) is a suitable and simple model for FPZ, which may be viewed as a specialisation of other more general approaches (Elices *et al.* 2002). For example, Broberg (1999) for materials that fail by crack growth and coalescence depicts the appearance of FPZ in a cross-section normal to the crack edge. He proposes to describe FPZ, in general, by decomposing it into cells. The behaviour of the single cell is defined by relationships between its boundary forces and displacements. This is very similar to the definition of FE in computations when these cells are assumed to be cubic (or prismatic) and to lie along the crack path, the resulting model is very similar to the smeared crack approach used for concrete, and, more specifically to the Bazant's crack band approach (Bazant & Planas 1998). The latter model was found to be in good agreement with the basic fracture data (Bazant & Oh 1983), and has been recognised convenient for programming. Based on the crack band model, two crack models were distinguished: fixed crack and rotating crack. In both models, a crack is initiated when the maximum principal stress violates the tensile strength of concrete and the initial orientation of the crack is normal to the maximum principal strain. In the fixed crack model, the crack plane is fixed during the total analysis process, whereas rotating crack model allows the crack plane to rotate (Cope *et al.* 1979). Nowadays it is the main concrete fracture model used in industry and commercial FE codes: *DIANA* (Rots 1988), *SBETA* (Cervenka & Pukl 1994, Cervenka *et al.* 1998) and *ATENA* (Cervenka *et al.* 2002).

It is commonly accepted that the consideration of tension-softening is indispensable in analysing the behaviour of concrete structures with relatively large un-reinforced areas. The fracture mechanics model is often used for modelling behaviour of RC structures in combination with other approaches. Feenstra & Borst de (1995) proposed a numerical model, which combines fracture mechanics concepts with tension-stiffening. It is assumed that the behaviour of cracked RC member can be obtained by superposition of the stiffness of plain concrete, a stiffness of reinforcement and additional stiffness due to interaction between concrete and reinforcement. The latter effect was simulated using model proposed by Cervenka *et al.* (1990). Fantilli *et al.* (1998a) modelled behaviour of tensile RC members combining the fracture mechanics and the stress transfer approaches.

1.2. Numerical Methods

1.2.1. Finite Element Method

In recent years, the use of finite element (FE) analysis has increased due to progressing knowledge and capabilities of computer software and hardware. Numerical techniques have been rapidly progressing for decades and commercial FE packages (*ABAQUS*, *DIANA*, *ATENA*, etc) now offer very powerful and general analytical tool for analysis of RC structures (Peiretti *et al.* 1991, Argyris & Kacianauskas 1996, Kaklauskas *et al.* 2004). In FE approach, tension-stiffening effect and consequently deflection can be predicted rationally (Gribniak *et al.* 2004, 2007d).

Concrete models in *ATENA* software (Cervenka 1985, Cervenka *et al.* 2002) are based on smeared crack concept and damage approach. Concrete without cracks is considered as an isotropic material and concrete with cracks as an orthotropic material.

In the fixed crack model crack direction and material axes are defined by the principal stress direction at the onset of cracking when the principal stress exceeds the tensile strength. In further analysis, this direction is fixed and cannot be changed though direction of principal strains may vary. A rotation of principal strain axes generates a shear stress on the crack plane. Consequently, the model of shear in cracked concrete becomes important. In the model a variable shear retention factor according to Kolmar (1986) is used, in which the shear modulus on the crack plane reduces with the crack opening.

It should be noted that the tension-stiffening effect is not explicitly included as a constitutive law in the above model. However, the fracture mechanics based model reaches almost the same effect, where distinct cracks formed and a contribution of concrete between cracks generates a tension-stiffening effect, as was shown by Cervenka (2002). Of course, this model has its limits bound to numerical discretization and is objective only for sufficiently fine mesh sizes.

1.3. Design Code Methods

This Section briefly overviews deflection calculation techniques for short-term loading from well-known design codes: *Eurocode 2*, *ACI 318* and the *Russian Code (SP 52-101)*.

1.3.1. Eurocode 2

In *Eurocode 2* (CEN 2004) model, a reinforced concrete member is divided into two regions: region I, uncracked and region II, fully cracked. In region I, both the concrete and steel behave elastically, while in region II the reinforcing steel carries all the tensile force on the member after cracking. The average curvature is expressed as:

$$\kappa = (1 - \xi)\kappa_I + \xi\kappa_{II}. \quad (1.12)$$

Here κ_I and κ_{II} are curvatures calculated for the uncracked and fully cracked regions respectively; ξ is a distribution coefficient (allowing for tension-stiffening at a section) given by:

$$\xi = 1 - \beta \left(\frac{\sigma_{sr}}{\sigma_s} \right)^2. \quad (1.13)$$

Here β is a coefficient taking into account the influence of duration of the loading or repeated loading on the average strain ($\beta=1.0$ for a single short-term loading and $\beta=0.5$ for sustained loads or many cycles of repeated loading); $\xi=0$ for uncracked sections; σ_s is the stress in the tensile reinforcement calculated on the basis of a cracked section; σ_{sr} is the stress in the tensile reinforcement calculated on the basis of a cracked section under the loading conditions causing first cracking. Note that ratio σ_{sr}/σ_s may be replaced by M_{cr}/M for flexure or N_{cr}/N for pure tension, where M_{cr} is the cracking moment and N_{cr} is the cracking force.

The calculation of κ_I and κ_{II} may be obtained from the relation:

$$\kappa = \frac{M}{EI_I} \text{ or } \kappa = \frac{M}{EI_{II}}. \quad (1.14)$$

It is known that actual deformations may considerably differ from calculated values, due to the dispersion of the material properties, the environmental effects, the influence of loading stages or restraint conditions at the supports. Different models may be used for the calculation of deformations, depending on the degree of accuracy required. For practical purposes, in order to prevent the occurrence of damage due to deformations, a rough estimate of displacements will often be sufficient.

1.3.2 American Code (ACI 318)

The short-term deflection calculation procedure is described by following.

Uncracked members – gross moment of inertia I_g – when the maximum flexural moment at service load in a beam or a slab causes a tensile stress less

than the modulus of rupture, f_r , no flexural tension cracks develop at the tension side of the concrete element if the member is not restrained or the shrinkage and temperature tensile stresses are negligible. In such a case, the effective moment of inertia of the uncracked transformed section, I_t , is applicable for deflection computations. However, for design purposes, the gross moment of inertia, I_g , neglecting the reinforcement contribution, can be used with negligible loss of accuracy. The combination of service loads with shrinkage and temperature effects due to end restraint may cause cracking if the tensile stress in the concrete exceeds the modulus of rupture.

The elastic deflection for non-cracked members can thus be expressed in the following general form:

$$\delta = s \frac{ML^2}{E_c I_g}. \quad (1.15)$$

Here s is a factor that depends on support fixity and loading conditions; M is the maximum flexural moment along the span and E_c is the modulus of elasticity of concrete.

Cracked members – effective moment of inertia I_e – tension cracks occur when the imposed loads cause bending moments in excess of the cracking moment, thus resulting in tensile stresses in the concrete which are higher than its modulus of rupture. The cracking moment, M_{cr} may be computed as follows:

$$M_{cr} = \frac{f_y I_g}{y_t}. \quad (1.16)$$

Here y_t is the distance from the neutral axis to the tension face of the beam, and f_r is the modulus of rupture of the concrete.

Simply supported beams - ACI 318 (2008) requires to use the effective moment of inertia I_e , proposed by Branson (1963, 1977). This approach was selected as being sufficiently accurate to control deflections in reinforced and prestressed concrete structural elements. Branson's equation for the effective moment of inertia I_e , for short-term deflections is as follows:

$$I_e = \left(\frac{M_{cr}}{M_a} \right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a} \right)^3 \right] I_{cr} \leq I_g. \quad (1.17)$$

Here M_{cr} is cracking moment; M_a is maximum service load moment at the stage for which deflections are being considered; I_g is gross moment of inertia of section and I_{cr} is moment of inertia of cracked transformed section.

Deflections should be computed for each load level. Equation 1.17 can also be simplified to the following form:

$$I_e = I_{cr} + \left(\frac{M_{cr}}{M_a} \right)^3 (I_g - I_{cr}) \leq I_g. \quad (1.18)$$

Heavily reinforced members will have an I_e , approximately equal to I_{cr} , which in some cases may (flanged members) be larger than I_g of the concrete section alone. For most practical cases, the calculated I_e , will be less than I_g and should be taken as such in the design for deflection control, unless a justification can be made for rigorous transformed section computations.

Continuous beams - for continuous members, ACI 318 (2008) stipulates that I_e may be taken as the average value obtained from Equation 1.17 for the critical positive and negative moment sections. For prismatic members I_e may be taken as the value obtained at mid-span for continuous spans. The use of mid-span section properties for continuous prismatic members is considered satisfactory in approximate calculations primarily because the mid-span rigidity including the effect of cracking has the dominant effect on deflections (ACI 435 1978).

If the designer chooses to average the effective moment of inertia I_e , then according to ACI 318 (2008), the following expression should be used:

$$I_y = 0.5I_{e(m)} + 0.25(I_{e(1)} + I_{e(2)}). \quad (1.19)$$

Here the subscripts m , 1, and 2 refer to mid-span, and the two beam ends, respectively. Improved results for continuous prismatic members can be, however, obtained using a weighted average as presented in the following equations:

For beams continuous on both ends:

$$I_e = 0.7I_{e(m)} + 0.15(I_{e(1)} + I_{e(2)}). \quad (1.20)$$

For beams continuous on one end only:

$$I_e = 0.85I_{e(m)} + 0.15I_{e(1)}. \quad (1.21)$$

When I_e is calculated as indicated in the previous discussion, the deflection can be obtained using the moment - area method taking the moment-curvature into consideration or using numerical incremental procedures.

Based on geometry of the deflected shape, the two following expressions are derived:

$$\Theta_{AB} = \int_A^B \kappa dx. \quad (1.22)$$

$$\delta_{BA} = \int_B^A \kappa x dx. \quad (1.23)$$

Here κ is the curvature; Θ is the rotation and δ is the tangential deviation (deflection). These two expressions are generalizations of the familiar Mohr or moment-areas theorems, and are applicable whether sections are cracked or uncracked. κ can be replaced by M/EI , when the material is linearly elastic.

Based on these fundamental principles, the designer can calculate the curvature and rotation incrementally at any section as well as the deflection or camber of the prestressed beam at the critical sections. Short-term deflections are defined as those occurring instantaneously under the application of any internal or external force. The time element is assumed to be unimportant, no matter what is the rate of loading, provided the load is applied within a matter of hours.

1.3.3 Russian Code (SP 52-101)

The simplified method in new normative regulation has essential advantage compare to principles in *SNiP*. The simplified method is based on general rules of strength of material and structural mechanics.

The curvature κ of uncracked and cracked reinforced concrete elements is determining:

$$\kappa = \frac{M}{D}. \quad (1.24)$$

Here M is bending moment caused by external loading relatively to axis, normal to moment action plane and passing through the centroid of transformed cross-section of element, D is the stiffness of element cross-section.

Bending stiffness for uncracked member:

$$D = E_{c1} I_{red}. \quad (1.25)$$

Here E_{c1} is the modulus of elasticity of concrete; I_{red} is moment of inertia of transformed cross-section.

Modulus of elasticity E_{c1} for elements without cracks in tension zone is equal:

$$E_{c1} = 0.85 E_c. \quad (1.26)$$

Here E_c is the tangent modulus of elasticity of concrete calculated according SP 52-101 (2006).

Moment of inertia I_{red} of transformed cross-section for elements in tension zone without cracks is equal:

$$I_{red} = I + \alpha I_{s1} + \alpha I_{s2}. \quad (1.27)$$

Here I is moment of inertia of concrete section relative to centroid of transformed cross-section of element, I_{s1} and I_{s2} are moment of inertia of tensile and compressive reinforcement respectively, relative to centroid of transformed cross-section of element.

$$I_{s1} = A_{s1}(d - y_c)^2. \quad (1.28)$$

$$I_{s2} = A_{s2}(y_c - a_2)^2. \quad (1.29)$$

α is modular ratio of reinforcement:

$$\alpha = \frac{E_s}{E_c}. \quad (1.30)$$

Here y_c is distance between extreme compressive fibre of concrete and centroid of transformed cross-section of element.

Stiffness of reinforced concrete members in tension zone with cracks is determined taking into account following regulations:

- Sections after deformation remain plain;
- Stress of concrete in compressive zone is represented by dual line diagram;
- Action of tensile concrete in section with normal crack isn't taken into account;
- Action of tensile concrete between adjacent cracks is taken into account by coefficient ψ_s .

Modulus of elasticity E_{c1} , with accepted regulation for elements with cracks, is taken as transformed modulus of elasticity $E_{c,red}$:

$$E_{c,red} = \frac{2000}{3} f_{cp}. \quad (1.31)$$

Here f_{cp} is compressive prism strength of concrete taken according SP 52-101 (2006).

Moment of inertia of transformed cross-section for elements with cracks in tensile zone is determined by following formula:

$$I_{red} = I_c + \alpha_{s2} I_{s1} + \alpha_{s1} I_{s2}. \quad (1.32)$$

Here I_{s1} and I_{s2} are moment of inertia of tensile and compressive reinforcement respectively, relative to centroid of transformed cross-section of element, I_c is moment of inertia of concrete section relative to centroid of transformed cross-section of element, α_{s1} and α_{s2} are modular ratios of reinforcement respectively to compressive and tensile concrete.

Moment of inertia of concrete I_c for rectangular cross-section is determined by formula:

$$I_c = \frac{bx_m^3}{3}. \quad (1.33)$$

Here x_m is mean depth of compressive zone of concrete taken according to SP 52-101 (2006).

Modular coefficient of reinforcement:

- For compressive reinforcement:

$$\alpha_{s1} = \frac{E_s}{E_{c,red}}. \quad (1.34)$$

- For tensile reinforcement:

$$\alpha_{s2} = \frac{E_{s,red}}{E_{c,red}}. \quad (1.35)$$

Here $E_{s,red}$ is transformed deformational modulus of tensile reinforcement, determined taking into account action of tensile concrete between the cracks:

$$E_{s,red} = \frac{E_s}{\psi_s}. \quad (1.36)$$

Coefficient ψ_s in Equation 1.36 which takes into account work of concrete in tension zone between cracks and represent relation of average deformation of tensile reinforcement in zone between cracks with deformation of “free” steel in cracked section. Coefficient ψ_s can be calculated by:

$$\psi_s = \frac{\varepsilon_{sm}}{\varepsilon_{s,cr}} = 1 - 0.8 \frac{M_{cr}}{M}. \quad (1.37)$$

$$M_{cr} = \frac{f_{ct,n} I_{red}}{y_t}. \quad (1.38)$$

Here ε_{sm} is the mean strain in tensile reinforcement; $\varepsilon_{s,cr}$ is the steel strain in the cracked section; $f_{ct,n}$ is characteristic tensile strength of concrete according to SP 52-101 (2006); I_{red} calculated using Equation 1.27.

1.4. Other Methods

In general, the displacements are determined by integration of curvatures, setting the boundary conditions. In that case, we assume the mean curvature for the zone subjected to cracking and the section curvature for the uncracked zones. Having determined both mean curvature and axial deformation along the axis or in the sections into which the structure has been subdivided, the calculation of displacements is only a geometrical problem which can be solved in various ways.

1.4.1 Murashev Theory

When studying the stiffness of reinforced concrete elements Murashev (1950) accept the following regulation:

- Flexural reinforced concrete members in service loads works in state 2, i.e. with cracks in tension zone;
- Concrete is assumed as elasto-plastic material;
- The work of concrete tensile zone between cracks is assumed;
- For average deformations plane section hypothesis is taken;

Using this regulations prof. Murashev wrote the expression for determination of curvature in pure bending zone:

$$\kappa = \frac{\varepsilon_{sm}}{d - x_m} = -\frac{\varepsilon_{cm}}{x_m}. \quad (1.39)$$

Here ε_{sm} and ε_{cm} are average deformations of tensile reinforcement and concrete extreme fibre in compression zone respectively. x_m is mean depth of concrete in compression, taking from expression:

$$x_m = \frac{\varepsilon_{cm}}{\varepsilon_{sm} + \varepsilon_{cm}} d. \quad (1.40)$$

Curvature of reinforced concrete element can be obtained by average deformation of tensile reinforcement or by average deformation of extreme fibre of concrete in compression zone. Equation 1.39 can be written in following form:

$$\kappa = \frac{\sigma_s}{E_{s,red}(d - x_m)} - \frac{\sigma_c}{E'_c x_m}. \quad (1.41)$$

Here $E_{s,red}$ is average elasticity modulus of reinforcement in concrete see Equation 1.36. E'_c is elasto-plastic modulus of concrete in compression, equal:

$$E'_c = \nu E_c . \quad (1.42)$$

Coefficient ν in Equation 1.42 characterizes elastic properties of compressive concrete and represents relation of elastic deformation of compressive concrete to its full deformation.

Section modulus of reinforced concrete element is equal:

$$W_t = \frac{M}{\sigma_s}; W_c = \frac{M}{\sigma_c} . \quad (1.43)$$

and transforming Equation 1.41, Murashev (1950) got expression for determination of stiffness of flexural element:

$$B = \frac{E_s}{\psi_s} A_{s1} (d - x_m) (d - 0.5x_m) . \quad (1.44)$$

Parameters ψ_s and x_m characterize change of stiffness of element. It works in state 2.

1.4.2 Rozenbliumas Method

Rozenbliumas (1964) accepts that in a cracked cross-section under short term loading there exist linear-elastic relationship between stresses and deflections (stress diagram in a compressed concrete is triangular). In the case of long-term loading the compressive stresses are described by a trapezoidal diagram so, that the maximal meaning doesn't exceed the concrete compressive strength for bending σ_R ; the tensile stresses in concrete between the neutral axis and the crack's top are described by trapezoidal diagram with maximal ordinate R_{ct} . The external moments M and axial forces N are analysed about the axis which passes through the concrete tensile layer. Reinforced concrete cracked and uncracked beam section curvatures are calculated by the same formulae:

$$\kappa = \kappa_0 (1 - k) + \kappa_I k . \quad (1.45)$$

Here κ_0 and κ_I are cracked and full cross-section curvatures respectively; k is coefficient, less than one, describes concrete σ_c and reinforcement σ_s stress diagram between cracks. Coefficient k is related with parameters φ_s and φ_c via the following expressions:

$$\varphi_s = (1 - k) + k \frac{\sigma'_s}{\sigma_s} . \quad (1.46)$$

$$\varphi_c = (1 - k) + k \frac{\sigma_c'}{\sigma_c}. \quad (1.47)$$

Curvatures κ_0 and κ_I are calculated as follows:

$$\kappa = \left(\frac{\sigma_s}{n} + \sigma_c \right) \frac{1}{E_c d}. \quad (1.48)$$

Here σ_c and σ_s are stresses calculated according to author's formulae, which corresponds to accepted computational scheme.

Analysing the change of curvature, A. Rozenbliumas defined that it is enough to calculate curvature of cracked element's part of the length only twice – where bending moment M is maximal and minimal. In other plain cross-sections curvature is calculated by an interpolation between these two ultimate meanings, assuming, that the change of curvature is proportional to change of moment M .

1.4.3 Method of Elastic Weights

According to Mohr's theorem, a beam's rotation and deflection can be determined respectively as shear forces and bending moments acting on a conjugate beam subjected to fictitious load corresponding to the value of the curvatures κ in the actual beam subjected to the actions being considered. As it is known for simply supported beams the conjugate beam coincides with the real beam; for continuous beams the conjugate beam is a beam of the same length but having different restraint conditions.

1.4.4 Virtual Work Method

The diagram of mean curvatures due to the actions acting on the structure is associated with a diagram of the load-effects rising from a suitable balanced system of fictitious forces. This system is such that it performs the work necessary for the displacement being sought and it consists of either a unit force, if it is needed to determine a displacement, or a unit couple, if it is desirable to determine a rotation. To the statically determined structure these forces can also be obtained from the initial structure by removing redundant restrains, and then the load-effects of the virtual system are calculated. Displacements are given by the relationship:

$$a = \int M_v \kappa_m dz. \quad (1.49)$$

Having subdivided the structure into discrete elements, the integral can be easily calculated as the area of function $M_v(\kappa_m)$.

1.4.5 Method of Global Coefficient

The displacement is estimated from the basic deflection a_c (calculated on the basis of the stiffness of the concrete section alone, $E_c I_c$), which is then corrected by means of a global coefficient, K , taking into account the effects of the tension and compression reinforcement, of cracking and creep:

$$a = K a_c . \quad (1.50)$$

Here K is determined through parametric studies, by taking into account only the permanent load and introducing certain simplifications. It depends mainly on the following parameters: d/h is ratio between effective and overall section depths; $E_s/E_c p$ – where p is the percentage of tension reinforcement: $p=A_{s1}/(bd)$; p/p' is the ratio between compression and tension reinforcement; ϕ is the creep coefficient.

The global coefficient can be written as follows:

$$k = \left(\frac{h}{d} \right)^3 \eta (1 - 20 p'). \quad (1.51)$$

Here η stands for the influence of cracking which depends on the load-effect level M_{sr}/M , in the determinant section, and the percentage of tension reinforcement, p . The value for η is given in Table 1.1 as a function of p for a pre-determined value $M_{sr}/M=0.5$.

Table 1.1. Correction factor η

p	0.15	0.20	0.30	0.50	0.75	1.00	1.50
η	10	8	6	4	3	2.5	2

Deflection values may now be determined through the following expressions:

$$a = (1 + \phi) a_c \text{ for } M < M_{sr} . \quad (1.52)$$

$$a = \left(\frac{h}{d} \right)^3 \eta (1 - 20 p') a_c \text{ for } M \geq M_{sr} . \quad (1.53)$$

To take into account the arrangement of the reinforcement along the beam, one can evaluate a mean percentage of the reinforcement in the tension zone, p_m .

1.4.6 Piyasena Method

Before calculating the deflection under a particular load, the locations of flexural cracks within the cracked region are determined (Piyasena 2002). It was concluded that all cracks in a varying moment region are developed at a regular spacing S_0 . After cracks are located, the curvature calculations are performed.

Calculation of curvature at a cracked section assumes that the plane sections before cracking remain plain after cracking. As a result, the strain distribution at the section is linear. The curvature, κ at the section is calculated as:

$$\kappa = \frac{\varepsilon_c + \varepsilon_s}{d}. \quad (1.54)$$

The values of ε_c and ε_s corresponding to a given bending moment M are determined by considering the translational and rotational equilibrium of the section.

Calculation of curvature values at sections between adjacent cracks requires the strain distribution across the height of the beam at each section. The concrete strains at various sections are evaluated by analysing the concrete block between adjacent cracks using the finite element method. The corresponding steel strain is determined by considering the equilibrium of part of the steel bar between a section and the nearest cracked section.

After curvature values at various sections along the length of a particular span are evaluated, the deflection is calculated as follows:

$$\delta = \int_A^C \kappa x dx. \quad (1.55)$$

Here κ is the curvature at a section at distance x from the point C where deflection is calculated. In a simply supported beam subjected to symmetrical loading, this point is taken at the mid-span. In a continuous beam, the maximum deflection within a span is determined by calculating the deflection at various points until the calculated value reaches the maximum.

1.4.7 Modification of Branson's Formula

There have been several attempts by different investigators to modify the Branson's formula aiming to improve the accuracy of the predicted deflection. These are briefly discussed below.

Modification by Al Zaid et al

Branson's formula has been developed using the results of test beams subjected to uniformly distributed loads. Based on test results, Al Zaid *et al.* (1991) suggested that the value of I_e calculated using Branson's formula may be increased by 20 % for beams subjected to central point loads.

Modification by Al Shaik & Al Zaid

Results of experiments carried out by Al Shaik & Al Zaid (1993) indicated that the value of β in Equation 1.17 decreases with the amount of tensile reinforcement ratio p . It was proposed to calculate the value of β using the following equation:

$$\beta = 3 - 0.8p. \quad (1.56)$$

Grossman (1981) carried out a parametric study by computing the effective moment of inertia using Branson's Equation 1.17 for various beams having different material and section properties. Based on the results, Grossman developed the following formula for calculation of I_e , which eliminates the requirement of computing I_{cr} .

$$\frac{I_e}{I_g} = \left(\frac{M_{cr}}{M_a} \right)^2 \leq 1.0 \text{ if } \frac{M_a}{M_{cr}} \leq 1.6. \quad (1.57)$$

$$\frac{I_e}{I_g} = 0.1 \left(\frac{M_a}{M_{cr}} \right) K \text{ if } \frac{M_a}{M_{cr}} > 1.6. \quad (1.58)$$

K in above formula provides adjustments for shape, material properties and the strength of the section and is determined as follows:

$$K = \left(\frac{d/h}{0.9} \right) \left[\frac{\sqrt{2330/w_c}}{0.4 + (1.4M_a/M_u)(f_y/690)} \right]. \quad (1.59)$$

Here w_c is unit weight of concrete (kg/m^3) and f_y is yield strength of steel (MPa).

Modification by Rangan

Rangan (1982) computed the effective moment of inertia, using Branson's Equation 1.17, for a number of simply supported rectangular beams and under service load. The mid span moment M_a in Equation 1.17 was replaced by the service moment M_s , which was calculated as the moment of resistance when the steel stress is equal to 60 % of the yield stress. Based on these results following equations were proposed.

$$\frac{I_e}{bd^3} = 0.1599\sqrt{\alpha p} \text{ if } \alpha p > 0.045. \quad (1.60)$$

$$\frac{I_e}{bd^3} = 0.0019/\alpha p \text{ if } \alpha p \leq 0.045. \quad (1.61)$$

Here α is modular ratio.

The above equation for I_e has been used by the Standard Association of Australia (2001) in the development of the formula to calculate the span-to-effective depth ratio for controlling deflections of beams.

1.5. Conclusions for Chapter 1

Literature review on methods of analysis of flexural reinforced concrete structures has indicated the following:

1) With the present trend of use of higher strength concrete and reinforcement, leading to longer spans and smaller depths, deformations are often the governing design criterion. Accurate deformation estimations are needed not only for deflection calculation, but also for more accurate numerical simulation of load carrying capacity of some structures. An adequate modelling of concrete cracking and, particularly, post-cracking behaviour, as one of the major sources of nonlinearity, is the most important task of deformational analysis.

2) Design codes of different countries (CEN 2004, ACI 318 2008 and the Russian Code (SP 52-101) 2006) are based on different assumptions and methods for strength, cracking and deformation analysis. Although these methods ensure safe design, they do not reveal the actual stress-strain state of cracked structures and often lack physical interpretation.

3) Numerical methods which were rapidly progressing are based on universal principles and can include all possible effects such as material nonlinearities, concrete cracking, creep and shrinkage, reinforcement slip, etc. However, the progress is mostly related to the development of mathematical apparatus, but not material models or in other words the development was rather qualitative than quantitative. Constitutive relationships are often too simplified and do not reflect complex multi-factor nature of the material. Existing constitutive relationships for concrete in tension do not assure higher statistical accuracy of deflection estimates for flexural reinforced concrete members in comparison to those obtained by empirical code methods.

Deformational Model of RC Flexural Member: Theoretical Background

This Chapter introduces shrinkage influence on tension-stiffening and stress-strain state of RC members subjected to short-term loading. A numerical procedure proposed by Kaklauskas *et al.* (2009) and Gribniak (2009) has been analysed for deriving free-of-shrinkage tension-stiffening relationships using test data of bending RC members. The procedure combines direct and inverse techniques of analysis of RC members. To eliminate shrinkage effect, a reverse shrinkage (expansion) strain was taken in the direct technique.

The materials related to Chapter 2, the author in co-authorship has reported in the following papers: Gribniak *et al.* (2010), Kaklauskas *et al.* (2008), Kaklauskas *et al.* (2007), Kaklauskas & Sokolov (2007), Kaklauskas *et al.* (2006).

2.1. Assumptions and Approaches

The numerical procedure combines direct and an inverse techniques. In the direct technique, a moment-curvature diagram is calculated for the assumed material stress-strain relationships. The inverse technique aims at determining an average stress-average strain tension-stiffening relationship from a moment-

curvature diagram of RC member. The techniques are based on the following approaches and assumptions of behaviour of shrunk RC members subjected to short-term load:

1. The constitutive model is based on a smeared crack approach, i.e. average stresses and strains are used.
2. The Bernulli Hypothesis of beam bending is adopted implying a linear distribution of strain within the depth of the beam section.
3. Perfect bond between reinforcement and concrete is assumed. Reinforcement slippage occurring at advanced stress-strain states is included into σ - ε diagram of tensile concrete.
4. Uniform shrinkage within the section is assumed.
5. Shrinkage-induced stresses do not exceed tensile strength of concrete f_{ct} .
6. All fibres in the tensile concrete zone follow a uniform stress-strain law.

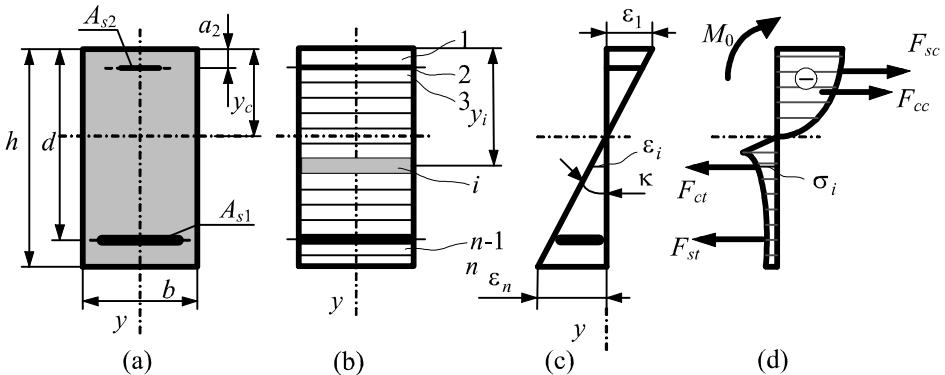


Fig. 2.1. Reinforced concrete section subjected to bending moment: a doubly reinforced section (a); layered section (b); strain compatibility (c); internal forces and external bending moment (d)

2.2. Basic Geometrical and Equilibrium Equation

Prof. G. Kaklauskas in co-authorship (Kaklauskas 2001, Kaklauskas & Ghaboussi 2001) has proposed a method for determining average stress-average strain relationships for concrete in tension from experimental moment-curvature diagrams of flexural RC members. The method considers a non-prestressed doubly reinforced concrete member subjected to bending. A cross-section for such member is presented in Fig. 2.1.

The sign convention adopted is that tension and distance below the neutral axis are positive. From equilibrium (see Fig. 2.1d):

$$F_{cc} + F_{sc} + F_{ct} + F_{st} = 0. \quad (2.1)$$

$$M_{cc} + M_{sc} + M_{ct} + M_{st} - M_0 = 0. \quad (2.2)$$

Here F are internal forces, M are internal moments with respect to the neutral axis, and M_0 is the external bending moment.

The first subscript corresponds to either c for concrete or s for steel and the second subscript refers to compression (c) or tension (t).

2.3. Inverse Analysis Using Layer Section Model: Derivation of a Tension-Stiffening Relationship from Test Data of Flexural RC Members

A Layer model (see Fig. 2.1b) can be conveniently employed for computation of internal forces in the cross-section. Stresses and internal forces in the reinforcing bars can be determined from the longitudinal strains using reinforcement (σ - ϵ) curves. Since the external moment M_0 is known, average stresses for the tensile and compressive concrete zones are two unknowns in the equilibrium Equations 2.1 and 2.2. However, there are many layers in both tensile and compressive concrete zones and the variation in stresses in each of these zones is not known.

A simple concept, based on the assumption of constant stress-strain diagrams allows the reduction of the number of unknowns to one for each zone. The equilibrium equations are solved for concrete stresses at extreme layers. Since the extreme layers have the highest strains, all other layers have lower strains, and therefore fall within the portion of the stress-strain diagram which has already been determined.

Computation is performed for incrementally increasing load. During the first load stage, tensile stresses corresponding to the strains in the extreme layers are computed. These stresses are then used in the equilibrium equations for the second load stage when new stresses corresponding to larger extreme layer strains are determined. In this way, stress-strain curves for the tensile concrete are progressively obtained from all previous stages and used in the next load stage.

The proposed method is illustrated in Fig. 2.2. It is assumed that the experimental moment-curvature diagram is given, as in Fig. 2.2a. Circled points in these curves correspond to experimental data. Computation is performed for an

assumed number of load increments equal to n . In order to avoid oscillations in the computed material (σ - ε) curves, n has to be sufficiently large. For most practical cases, $n \geq 50$ is sufficient.

The size of moment increment is:

$$\Delta M = M_{\max} / n . \quad (2.3)$$

Moment at load increment i is:

$$M_i = i \Delta M . \quad (2.4)$$

Here $M_{\max}$ is the maximum moment value for the M - κ diagram.

Curvatures corresponding to moment are determined numerically for the smoothed curves (see Fig. 2.2a):

$$\kappa_i = \kappa_1, \kappa_2, \dots, \kappa_{n-1}, \kappa_n . \quad (2.5)$$

$$M_i = M_1, M_2, \dots, M_{n-1}, M_n . \quad (2.6)$$

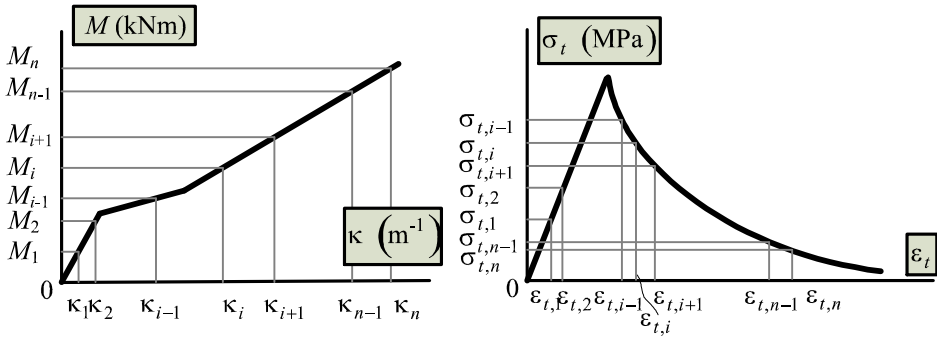


Fig. 2.2. Experimental moment-curvature diagram (a) and computed tension-stiffening relationship for concrete in tension (b)

A *Layer* model with a constant number of layers could be employed for this problem. However, for visualisation purposes and simplification of the solution, an approach based on an increasing number of concrete layers was introduced in the present work. The number of concrete layers in both compressive and tensile zones was assumed to be equal to the load increment number i . The thickness of the concrete layers is assumed to vary in such a way that the top fibres of layer j in the compressive zone have strains $\varepsilon_{c,j} = \varepsilon_{c,i}$ and the top fibres of layer j in the tensile zone have strains $\varepsilon_{t,j} = \varepsilon_{t,i}$ where $j \leq i$. Then the thickness of the layer for the compressive zone is taken as shown in Equation 2.7 and for the tensile zone as shown in Equation 2.8:

$$t_{c,j,i} = y_{c,i} \left(\frac{\varepsilon_{c,j} - \varepsilon_{c,j-1}}{\varepsilon_{c,i}} \right). \quad (2.7)$$

$$t_{t,j,i} = (h - y_{c,i}) \left(\frac{\varepsilon_{t,j} - \varepsilon_{t,j-1}}{\varepsilon_{t,i}} \right). \quad (2.8)$$

Here $y_{c,i}$ is the depth of the compressive zone at load increment i and the variation in layer numbers is due to $j \leq i, j=1, 2, \dots, i$ and $i=1, 2, \dots, n$.

In Equations 2.7 and 2.8 the subscript c refers to compressive concrete, t - to tensile concrete, i - to the number of the load increment, and j - to the layer number.

Since the number of layers increases with each load increment, the thickness of each layer decreases. This assumption for the number of concrete layers in a cross-section together with the corresponding strains and stresses is presented in Fig. 2.3. For load increment $i=1$ (see Fig. 2.3b), one layer is assumed for each concrete zone. The extreme fibre strains for the compressive and tensile concrete layers are $\varepsilon_{c,1}$ and $\varepsilon_{t,1}$ respectively. From the equilibrium equations, the corresponding stresses $\sigma_{c,1}$ and $\sigma_{t,1}$ are computed. For load increment $i=2$, (see Fig. 2.3c), two layers are assumed for each concrete zone. Both strains and stresses are known for the top fibres of the first layers ($j=1$) since $\varepsilon_{c,1}$, $\sigma_{c,1}$ and $\varepsilon_{t,1}$ and $\sigma_{t,1}$ are the same as for $i=1$. The top fibre strains for the second layers ($j=2$) are $\varepsilon_{c,2}$ and $\varepsilon_{t,2}$. The corresponding stresses $\sigma_{c,2}$ and $\sigma_{t,2}$ are computed from the two equilibrium equations.

The case for load increment i is shown in Fig. 2.3d. For computation of $\sigma_{c,i}$ and $\sigma_{t,i}$ at load increment i , equilibrium Equations 2.1 and 2.2 are rearranged so that values for F_{cc} , F_{ct} , M_{cc} , and M_{ct} , corresponding to the internal forces and moments for the concrete, are divided into two components each:

$$F_{cc,i-1} + \Delta F_{cc,i} + F_{sc} + F_{ct,i-1} + \Delta F_{ct,i} + F_{st} = 0. \quad (2.9)$$

$$M_{cc,i-1} + \Delta M_{cc,i} + M_{sc} + M_{ct,i-1} + \Delta M_{ct,i} + M_{st} - M_0 = 0. \quad (2.10)$$

Here $F_{cc,i-1}$, $F_{ct,i-1}$, $M_{cc,i-1}$, and $M_{ct,i-1}$ are internal concrete forces and moments due to all corresponding zone concrete layers except the extreme layers ($j=1,2,\dots,i-1$); and $\Delta F_{cc,i}$, $\Delta F_{ct,i}$, $\Delta M_{cc,i}$ and $\Delta M_{ct,i}$ are the forces and moments for the extreme layers $j=i$.

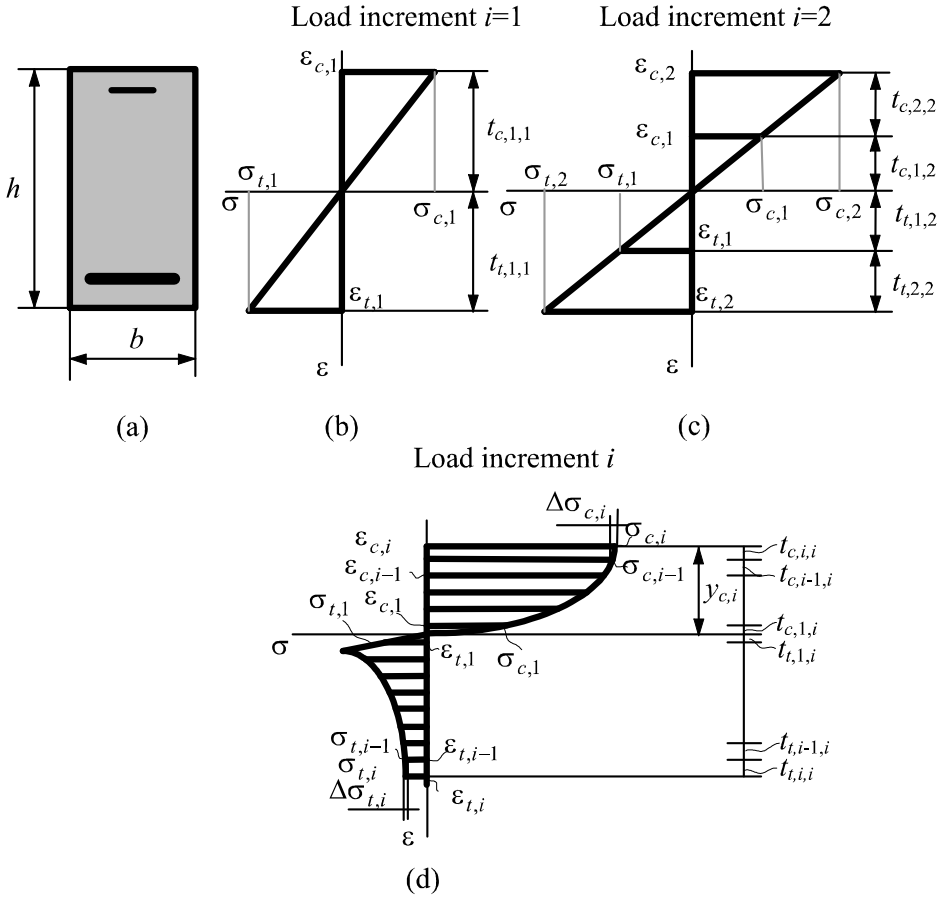


Fig. 2.3. Stresses and strains and concrete layer assumption in a cross-section for different load stages

At load increment i , components $F_{cc,i-1}$, $F_{ct,i-1}$, $M_{cc,i-1}$, and $M_{ct,i-1}$ can be fully determined since the compressive and tensile stresses in concrete layers $j=1,2,\dots,i-1$ are known. The remaining four concrete force and moment components can be expressed by stress increments $\Delta\sigma_{c,i}$ and $\Delta\sigma_{t,i}$ (see Fig. 2.3d) in the extreme fibres:

$$\Delta F_{cc,i} = bt_{c,i,i} \left(\sigma_{c,i-1} + \frac{1}{2} \Delta\sigma_{c,i} \right). \quad (2.11)$$

$$\Delta M_{cc,i} = bt_{c,i,i} \left[\sigma_{c,i-1} \left(y_{c,i} - \frac{1}{2} t_{c,i,i} \right) + \frac{1}{2} \Delta\sigma_{c,i} \left(y_{c,i} - \frac{1}{3} t_{c,i,i} \right) \right]. \quad (2.12)$$

$$\Delta F_{ct,i} = bt_{t,i,i} \left(\sigma_{t,i-1} + \frac{1}{2} \Delta \sigma_{t,i} \right). \quad (2.13)$$

$$\Delta M_{ct,i} = bt_{t,i,i} \left[\sigma_{t,i-1} \left(t - y_{c,i} - \frac{1}{2} t_{t,i,i} \right) + \frac{1}{2} \Delta \sigma_{t,i} \left(y_{c,i} - \frac{1}{3} t_{t,i,i} \right) \right]. \quad (2.14)$$

Equilibrium Equations 2.9 and 2.10 are explicitly solved for $\Delta \sigma_{c,i}$ and $\Delta \sigma_{t,i}$. Then concrete stresses $\sigma_{c,i}$ and $\sigma_{t,i}$ for load increment i are easily defined from:

$$\sigma_{c,i} = \sigma_{c,i-1} + \Delta \sigma_{c,i}. \quad (2.15)$$

$$\sigma_{t,i} = \sigma_{t,i-1} + \Delta \sigma_{t,i}. \quad (2.16)$$

Special arrangements can be made to assess net concrete area due to presence of reinforcement.

2.4. Direct Analysis Using Layer Section Model

This Section describes direct technique proposed by Kaklauskas *et al.* (2009) and Gribniak (2009). The method considers a doubly reinforced concrete member subjected to bending. A cross-section for such member is presented in Fig. 2.4. The member's cross-section is divided into a number of horizontal layers corresponding to either concrete or reinforcement (see Fig. 2.4e). Thickness of the reinforcement layer is taken from the condition of the equivalent area. The analysis needs to assume material laws for reinforcement and compressive and tensile concrete schematically shown in Figs. 2.5a and 2.5b.

Shrinkage in present analysis is modelled by means of fictitious axial force $N_{sh}(t, \tau)$:

$$N_{sh}(t, \tau) = \varepsilon_{sh}(t, \tau) E_{ea}(t, \tau) A_c. \quad (2.17)$$

$$E_{ea}(t, \tau) = \frac{E_c(\tau)}{1 + \phi(t, \tau) \chi(t, \tau)}. \quad (2.18)$$

Here $E_{ea}(t, \tau)$ is the age-adjusted effective modulus of concrete; $E_c(\tau)$ is the modulus of elasticity of concrete at time τ ; $\varepsilon_{sh}(t, \tau)$ is the mean free shrinkage strain of concrete taken negative; $\phi(t, \tau)$ is the creep factor; $\chi(t, \tau)$ is the ageing coefficient; A_c is the area of concrete net section; τ and t is the time of initiation of shrinkage and time under consideration, respectively.

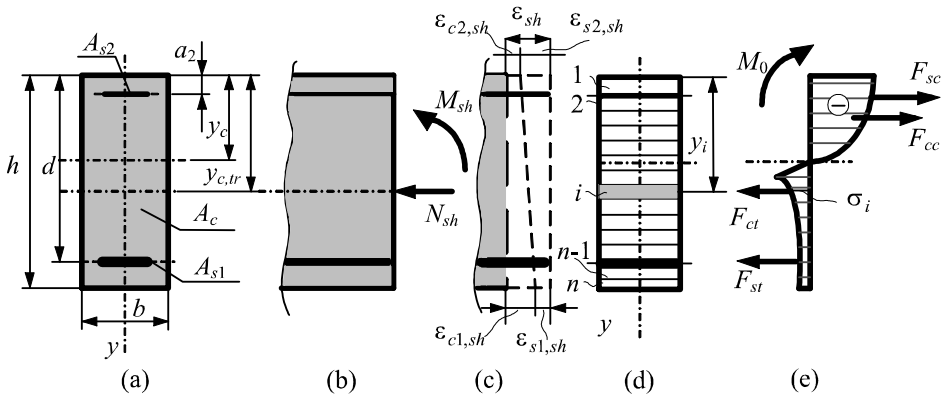


Fig. 2.4. Deformational behaviour of asymmetrically reinforced member due to shrinkage: (a) reinforced concrete section; (b) equivalent system of fictitious shrinkage force and bending moment in respect to the centroid of the transformed section; (d) distribution of deformations across the section; (e) layered section and (f) internal forces and stress distribution across the section due to external bending

The fictitious force $N_{sh}(t, \tau)$ is applied on the centroid of concrete section (see Fig. 2.4b). As shown in Fig. 2.4c, fictitious shrinkage force in a non-symmetrical section acts with eccentricity exerting bending upon the member:

$$M_{sh}(t, \tau) = N_{sh}(t, \tau)(y_{C,tr}(t, \tau) - y_C). \quad (2.19)$$

Here y_C and $y_{C,tr}$ are the centroid coordinates of plain and transformed sections, respectively.

Curvature and strain at any layer i (see Fig. 2.4d) can be calculated by the formulae:

$$\kappa(t) = \frac{\bar{M}_{sh}(t) + M}{E_c(t)I_{tr,sec}(t)}. \quad (2.20)$$

$$\varepsilon_i(t) = \frac{\bar{M}_{sh}(t) + M}{E_c(t)I_{tr,sec}(t)}[y_{C,tr}(t) - y_i] + \frac{N_{sh}(t)}{E_c(t)A_{tr}(t)}. \quad (2.21)$$

The analysis is performed iteratively until convergence of secant modulus at each layer is reached. Figs. 2.10d and 2.10e illustrate strain and stress distributions within the Layer section model.

2.5. Eliminating Shrinkage from a Tension-Stiffening Relationship

The method proposed by Kaklauskas (2001), Kaklauskas *et al.* (2009) and Gribniak (2009) for eliminating shrinkage from tension-stiffening relationships based on Layer approach combines direct and inverse techniques for analysis of RC members.

In the direct technique, moment-curvature diagrams are calculated for assumed material stress-strain relationships. The inverse technique aims at determining tension-stiffening relationships for cracked tensile concrete from flexural tests of RC members. For given moment-curvature diagrams, the material stress-strain relation (including the descending branch) is computed from the equilibrium equations for incrementally increasing moment assuming portions of the relations obtained from the previous increments.

This Section presents an improved technique for deriving free-of-shrinkage tension-stiffening relations from test data of shrunk flexural members. The analysis is performed in the following steps sketched in Fig. 2.5:

Step 1. Using the test moment-curvature diagram shown in Fig. 2.5a, a tension-stiffening relationship is derived (see Fig. 2.5b) by the inverse technique.

Step 2. The tension-stiffening relationship obtained in Step 1 (shown also in Fig. 2.5c) is applied in the direct analysis assuming reverse (expanding) shrinkage strain $\varepsilon_{expn} = -\varepsilon_{cs}$ [see Gribniak (2009)]. The calculated free-of-shrinkage moment-curvature diagram is shown in Fig. 2.5d along with the experimental curve. It should be noted that due to the expansion of concrete, initial negative curvature was obtained. In absolute value, it is equal to the initial curvature (the positive one) due to shrinkage.

Step 3. As unloaded non-shrunk beam has no curvature, the free-of-shrinkage moment-curvature diagram obtained in Step 2, is shifted to zero point as shown in Fig. 2.5e. Using this diagram, a free-of-shrinkage tension-stiffening relation is obtained by the inverse analysis. This relationship is shown in Fig. 2.5f along with the one obtained from the test of shrunk member (see Fig. 2.5b).

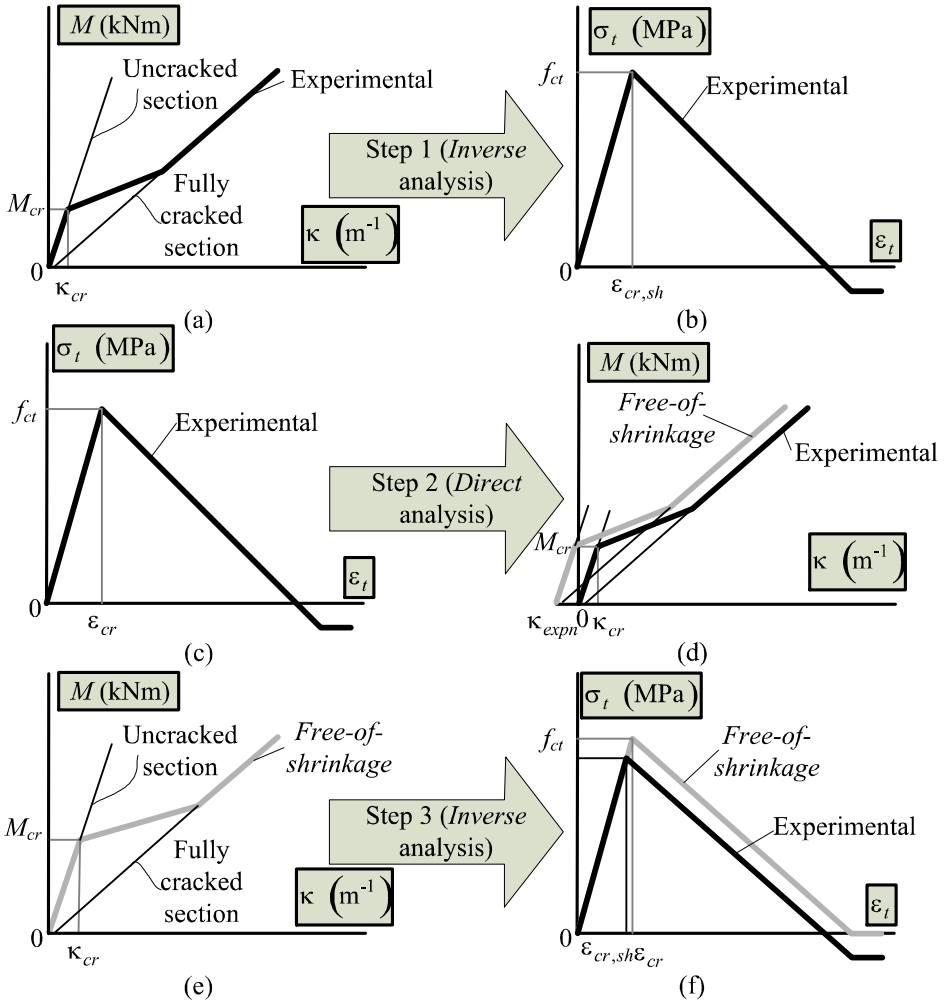


Fig. 2.5. Technique for deriving free-of-shrinkage tension-stiffening relationships from RC beam tests: deriving tension-stiffening relationship from test results (a) and (b); calculating free-of-shrinkage moment-curvature diagrams (c) and (d); deriving free-of-shrinkage tension-stiffening relationships (e) and (f)

2.6. Conclusions for Chapter 2

This Chapter presents a numerical procedure for deriving free-of-shrinkage tension-stiffening relationships using test data of bending RC. Two numerical tech-

niques, direct and inverse, have been analysed. In the direct technique, moment-curvature diagrams are calculated for the assumed material stress-strain relationships. The inverse technique aims at determining tension-stiffening relationships for cracked tensile concrete from flexural tests of RC members. The techniques are based on the following approaches and assumptions:

- smeared crack approach;
- Layer section model;
- linear strain distribution within the depth of the section implying perfect bond between reinforcement and concrete;
- all concrete fibres in the tension zone follow a uniform stress-strain tension–stiffening law.

A numerical procedure has been used for deriving free-of-shrinkage tension-stiffening relationships using test data (moment-curvature relationships) of bending reinforced concrete members. The procedure combines direct and inverse techniques. To eliminate shrinkage effect, a reverse shrinkage (expansion) strain was assumed in the direct technique.

Experimental Study of RC Beams and Derivation of Tension-Stiffening Model

This Chapter presents derivation of a new tension-stiffening law based on the techniques described in Section 2 and test data of twelve RC beams. Six beams were tested by the author. The remaining beams were tested by the research team guided by the supervisor of the PhD project (Kaklauskas *et. al.* 2008).

The materials related to Chapter 3, the author in co-authorship has reported in the following papers: Kaklauskas & Sokolov (2007), Sokolov (2007), Gribniak *et. al.* (2010), Sokolov *et. al.* (2010).

3.1. Experimental Program

3.1.1. Specimen Layout and Test

A series of experimental beams were produced in the *Laboratory of Civil Engineering Faculty of Vilnius Gediminas Technical University*. This project was supported by the LSSS foundation. Six beams were tested under a four-point loading scheme. All these beams had nominal length 3280 mm (span 3000 mm). The beams had the same nominal cross-section parameters and similar material

characteristics (see Fig. 3.1 and Table 3.1), but different reinforcement ratio ($p = 0.3, 0.6$ and 1%). Top reinforcement of beams $S1-3$, $S2-3$, $S3-2-2$ and $S3-2-3$ was $2\varnothing 6$ mm, for beam $S1-3R$ - $3\varnothing 18$ mm and for beam $S2-3R$ - $3\varnothing 14$ mm. Reinforcement grade S500 was used for tensile reinforcement: $3\varnothing 18$ mm for beams $S1-3$ and $S1-3R$, $3\varnothing 14$ mm for beams $S2-3$ and $S2-3R$ and $3\varnothing 10$ mm for beams $S3-2-2$ and $S3-2-3$.

Table 3.1. Main characteristics of the test beams

Beams	h	b	d	a_2	A_{s1}	A_{s2}	p	f_{cyl}	E_c
	mm				mm ²		%	MPa	
$S1-3$	299	283	268	23	755	57.3	1.00	47.0	35009
$S1-3R$	300	275	269	31	755	755	1.00	48.2	35266
$S2-3$	300	282	272	29	466	57.3	0.60	47.0	35009
$S2-3R$	299	280	272	21	466	466	0.60	47.0	35009
$S3-2-2$	298	283	274	32	232	57.3	0.30	55.8	36857
$S3-2-3$	298	284	271	32	232	57.3	0.30	53.2	36323

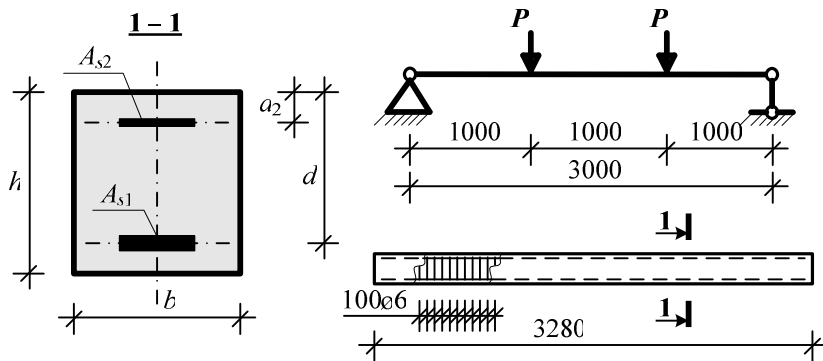


Fig. 3.1. Cross-section and loading scheme of experimental beam

3.1.2. Construction of Specimens and Loading Details

The beams were cast into steel formwork (see Fig. 3.2) and vibrated for approximately 20 seconds. The experimental beams and concrete specimens were cured under the laboratory conditions at average relative humidity (RH_m) 75.3 % and average temperature 14.5 °C (see Fig. 3.3).

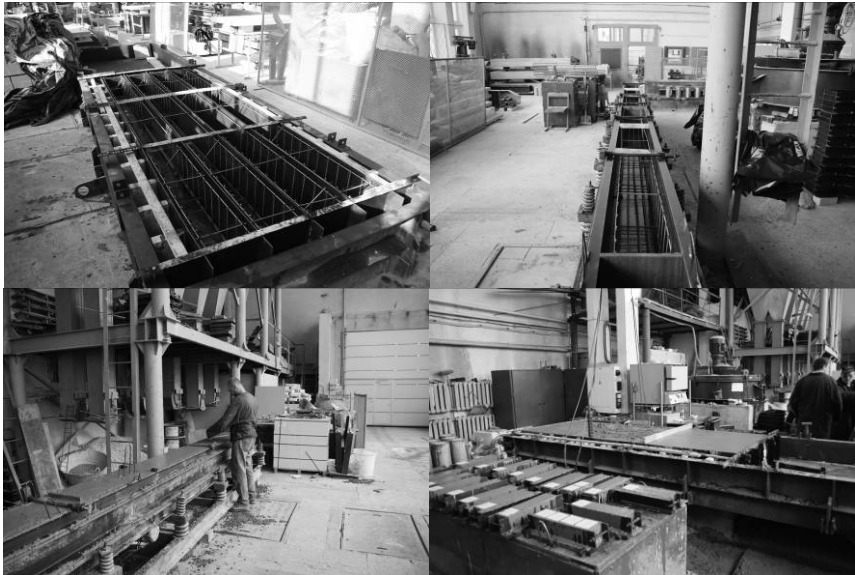


Fig. 3.2. Casting of test beams and specimens

Cubes, prisms and cylinders were produced (see Fig. 3.4) together with beams, for determining physical and mechanical properties of concrete. Cubes ($150 \times 150 \times 150$ mm) and cylinders ($d=150$ mm) were used for determining compression strength of concrete at different time moments, and prisms (in bending) – for concrete tensile strength. Also prisms were used for measurement of free concrete shrinkage strain, creep coefficient and modulus of elasticity of concrete. All the specimens were produced at the same day and from the same concrete as the beams.

Reinforced concrete beams were tested at workbench using *IIP-1000* testing machine. Load was increased in small increment 2 kN and paused for short period (about 2 minutes) to take readings of the gauge and to measure crack development. Value of the load was fixed using digital 500 kN dynamometer. Concentrated load was applied to the beam through steel traverse (length of traverse – 1250 mm). 10 mm steel plates were glued at the load application place due to decrease stress concentration.

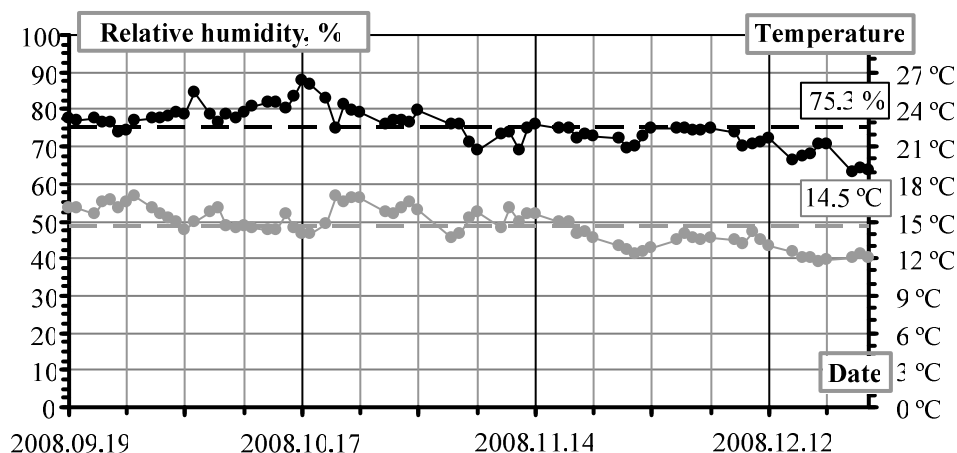


Fig. 3.3. Variation of relative humidity and temperature

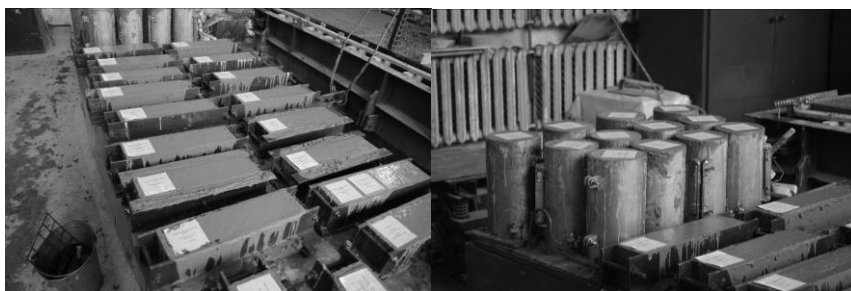


Fig. 3.4. Casting of test specimens: cubes, cylinders and prisms

Concrete surface strain was measured using 20 mechanical micrometers with circular scale (indicator), which accuracy - ± 0.003 mm. Gauge length - 200 mm. Concrete surface strains were measured in pure bending zone at the four different levels and at the five sections of every level. The readings of indicators were taken at every load stage.

Deflections of the beam were measured with 8 linear differential transducers ALMEMO T50, which accuracy - ± 0.15 %. Linear differential transducers were arranged in a such order: 1 near each support, 2 near the application points of external concentrated load (1 m from support) and 2 in the midspan of the beam (1.5 m from support). Linear differential transducers together with dynamometer were directly connected to personal computer through signal processing equipment ALMEMO 25 90-9 and the readings were taken every 1 s. Arrangement of the testing equipment are shown in Fig. 3.5.

During the test, occurrence and development of the cracks were observed and denoted. Cracks were numbered and the crack widths were fixed every sec-

ond load step using the 24 time magnification microscope. Cracks width was measured in the centroid level of tensile reinforcement.

The cracking moment M_{cr} of every beam was determined using linear differential transducers and indicators as well as visual examination of the occurrence of the first crack. Ultimate moment M_u was measured at the end of the test. Testing results were analysed in Section 3.3.

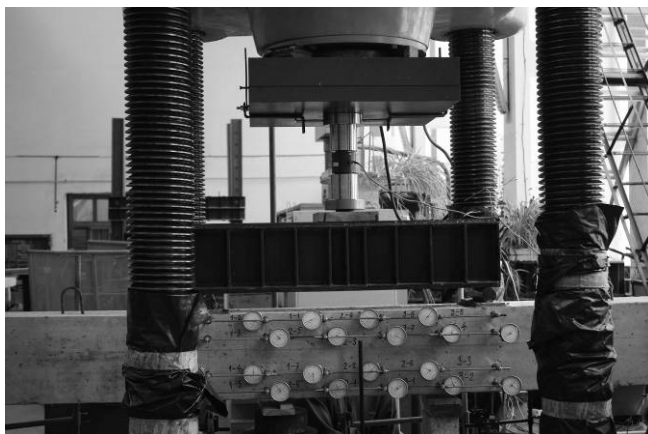


Fig. 3.5. Arrangement of testing equipment

3.2. Material Properties

3.2.1. Concrete

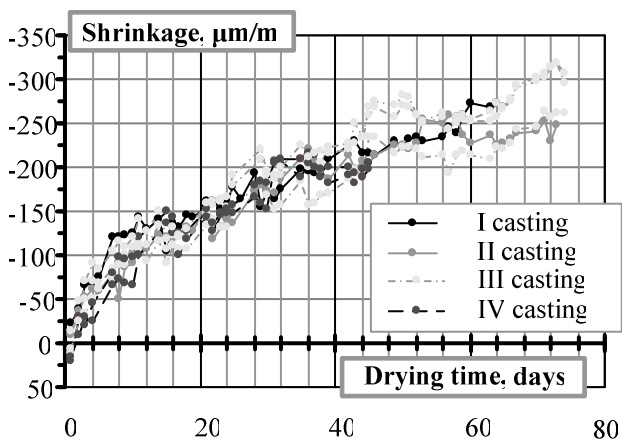
Ready-mixed concrete manufactured in UAB “Markuciai” was used for producing experimental specimens, which concrete grade was more or equal than C35/45. Cubes (100x100x100 mm and 150x150x150 mm), prisms and cylinders ($d=150$ mm) were produced together with beams. Cubes (150x150x150 mm) and cylinders ($d=150$ mm) were used for determination of compression strength of concrete at different time moments, and prisms (in bending) – for concrete tensile strength. Variation of cube compressive strength in time is given in Table 3.2. Also prisms were used for measurement of free concrete shrinkage strain, creep coefficient and modulus of elasticity of concrete. All the specimens were produced at the same day and from the same concrete as the beams.

Table 3.2. Compressive strength of concrete (150 mm cubes)

Beams	f_{cu} (MPa)				Age of the test
	7 (days)	14 (days)	28 (days)	At test	(days)
S1-3	44.2	50.4	48.7	53.5	67
S1-3R				54.9	68
S2-3				53.2	66
S2-3R	50.7	59.7	66.3	68.0	75
S3-2-2				67.1	47
S3-2-3					

3.2.2. Shrinkage and Creep of Concrete

Concrete prisms (100×100×400 mm and 280×300×400 mm) and cylinders ($d=150$ mm) were used for determining shrinkage (see Fig. 3.7) deformations. Measurement of shrinkage deformations was performed using digital DEMEC strain gauge (see Fig. 3.7) with gauge length of 200 mm.

**Fig. 3.6.** Concrete free shrinkage variation within time

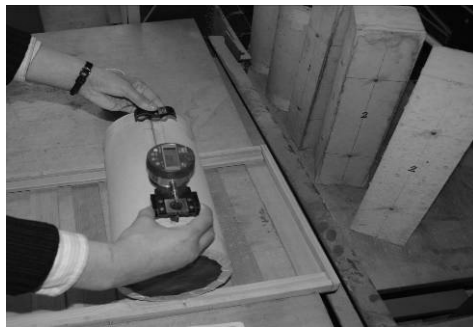


Fig. 3.7. Measurement of concrete shrinkage deformation

Also prisms $100 \times 100 \times 400$ mm are used for measurement of creep deformations (see Fig. 3.8). Creep coefficient was determined by loading prisms with spring-loaded press (14 days after concrete casting). Concrete stresses reached $0.3f_{cm}(t_0)$ at loading time, where $f_{cm}(t_0)$ is average concrete prism strength at loading time. Values of shrinkage and creep deformations were taken every day until the end of the tests. The concrete free shrinkage deformations (for specimens $280 \times 300 \times 400$ mm) and concrete creep coefficient variations within time are shown in Fig. 3.6 and Fig. 3.8, respectively.

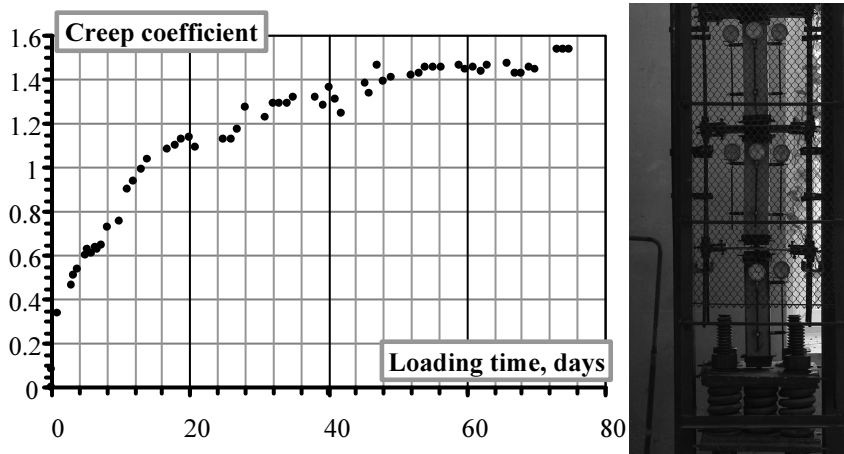


Fig. 3.8. Concrete creep coefficient variation within time

3.2.3. Reinforcement

Reinforcement bars supplied by UAB “Markuciai” were used in producing reinforcement cages for experimental beams. Three specimens of each diameter

were tested for determining material characteristics (modulus of elasticity, yielding stress). The length of each specimen was about 500 mm. Tensile testing machine was used for reinforcement specimen testing. Load was increasing by constant rate. Reinforcement deformations were fixed by linear differential transducers ALMEMO T50, which were connected to personal computer through signal processing equipment ALMEMO 25 90-9 (see Fig. 3.9). Full stress-strain relationships (see Fig. 3.10) of experimental reinforcement bars were determined after testing of reinforcement specimens. Experimental reinforcement characteristics are presented in Table 3.3.

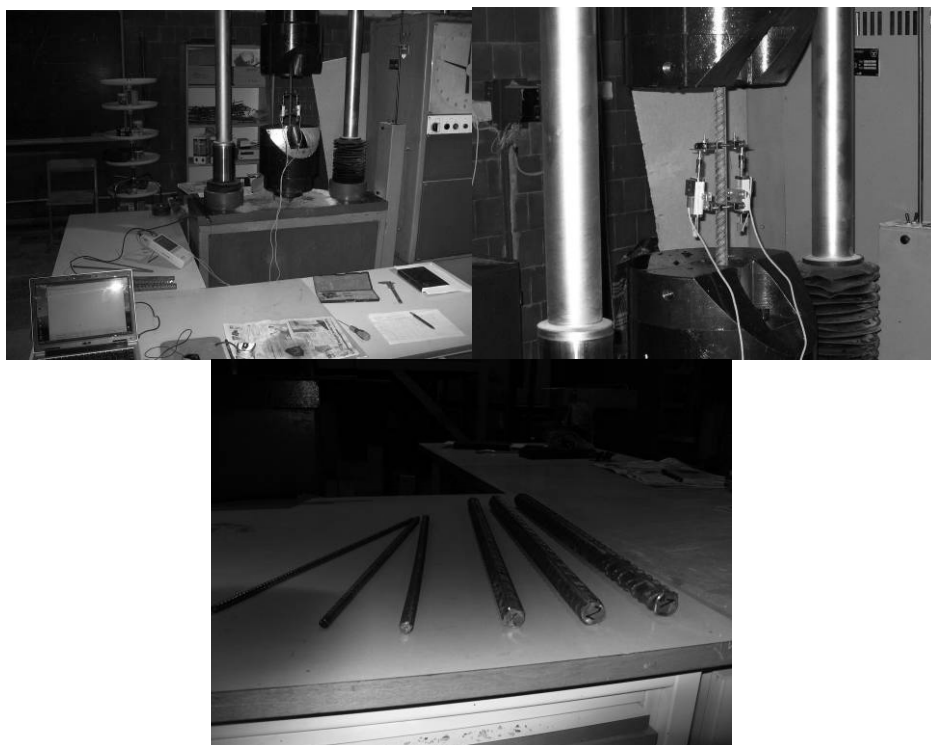


Fig. 3.9. Testing of reinforcement specimens

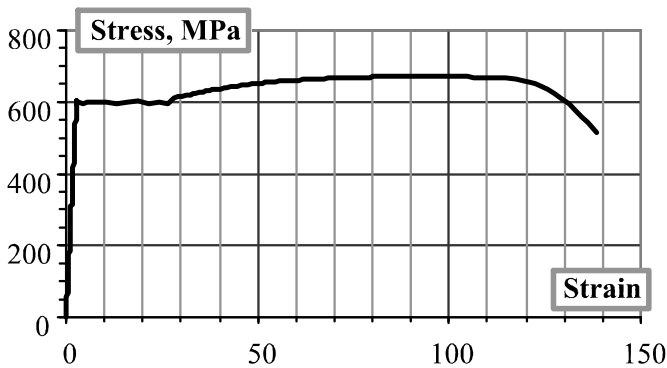


Fig. 3.10. Stress-strain relationship ($d = 10$ mm)

Table 3.3. Experimental reinforcement characteristics

Beams	Bottom reinforcement				Top reinforcement			
	$\varnothing$	A_{s1}	E_s	f_y	$\varnothing$	A_{s2}	E_s	f_y
	mm	mm ²	GPa	MPa	mm	mm ²	GPa	MPa
S1-3	18	755	207.3	582	6	57.3	223.5	585
S1-3R	18	755	207.3	582	18	755	207.3	582
S2-3	14	466	210.5	632	6	57.3	223.5	585
S2-3R	14	466	210.5	632	14	466	210.5	632
S3-2-2	10	232	209.9	578	6	57.3	223.5	585
S3-2-3	10	232	209.9	578	6	57.3	223.5	585

3.3. Short-Term Test Results and Discussion

Concrete surface strains were measured in pure bending zone at four different levels and at five sections of every level. Deflections of the beam were measured with linear differential transducers ALMEMO T50. Moment – curvature diagrams were obtained by both methods: a) from average surface strains and b) from deflections over pure bending zone. Moment – curvature diagrams of all experimental beams are shown in Fig 3.11. From Fig 3.11 can be seen, that moment – curvature diagrams obtained by both methods are in good agreement. Further analysis will be based on data derived from the average strains.

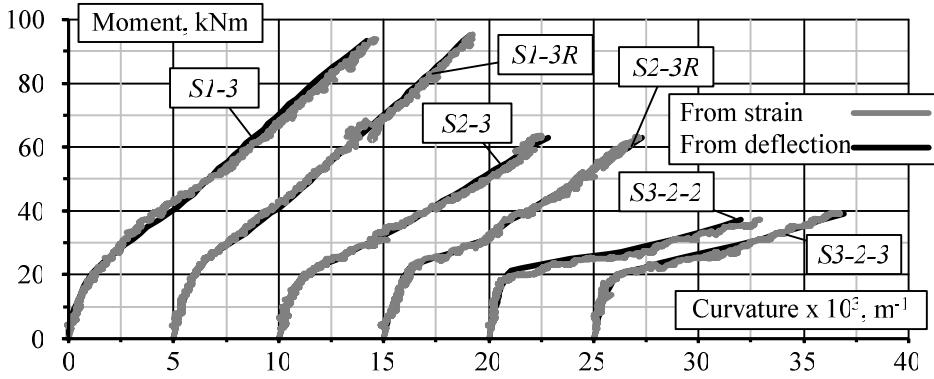


Fig. 3.11. Moment curvature relationships of experimental beams

3.4. Deriving Tension-Stiffening Relationships from Beam Tests

In this Section, tension-stiffening relationships were derived from the above test data using inverse procedure discussed in Chapter 2. The proposed method has been applied to present test data and free-of-shrinkage tension-stiffening relationships were derived for each beam.

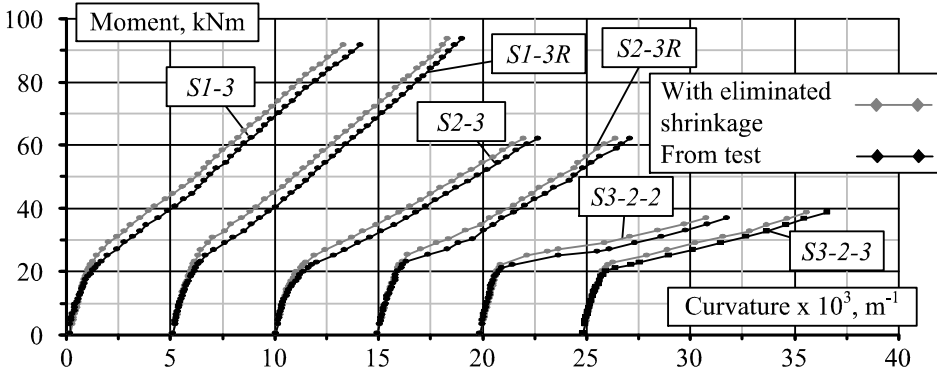


Fig. 3.12. Moment-curvature diagrams of present test data and free-of-shrinkage

The moment-curvature relationships of test data and *free-of-shrinkage* beams are shown in Fig. 3.12. The relative stress ($\sigma_{ct}/f_{ctm,EC2}$) and strain (in relative term of $\varepsilon_{cr}/\varepsilon_{cr,EC2}$) diagrams are shown in Fig. 3.13 where $f_{ctm,EC2}$ and $\varepsilon_{cr,EC2}$ are the tension strength and cracking strain, respectively, calculated according to

Eurocode 2. It can be concluded that the relationships obtained for each element were in good agreement.

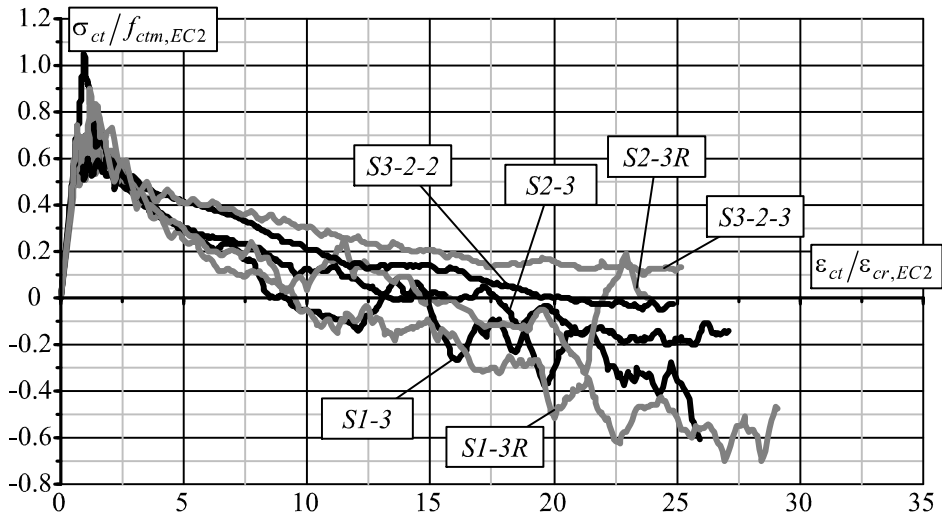


Fig. 3.13. Normalised tension-stiffening diagrams derived from the test data

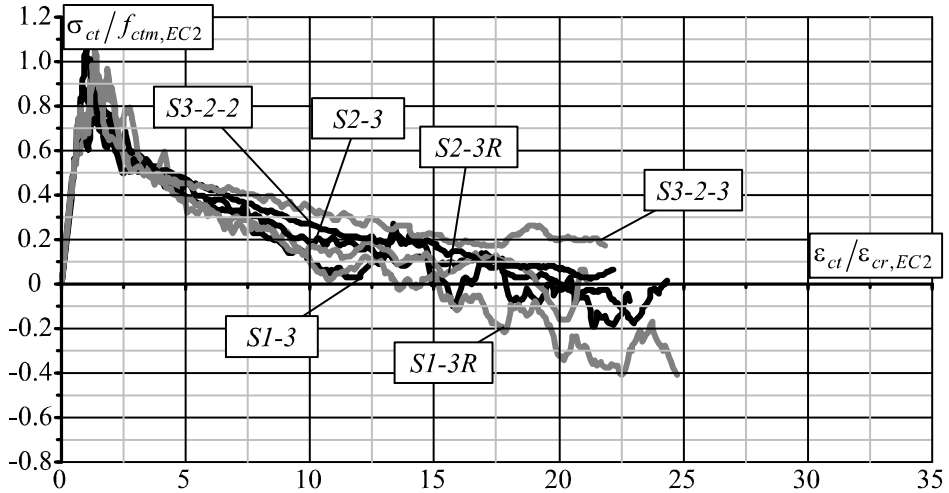


Fig. 3.14. Normalised tension-stiffening diagrams derived from the test data eliminating shrinkage effect

3.5. Other Experimental Data Employed for Constitutive Modelling

Six beams were tested by the research team guided by the supervisor of the PhD project (Kaklauskas *et. al.* 2008). The experimental program consisted of testing lightly and moderately reinforced concrete beams ($p = 0.30 \%$, 0.60% and 1%) by short-term loading. The beams with various reinforcement ratios and diameters (Table 3.4) were tested under a four-point loading system which gave a constant moment zone of 1.0 m and two shear spans of 1.0 m each. The results of all concrete control specimen tests are given in Table 3.4.

Table 3.4. Main characteristics of test beams

Beams	h	d	b	a_2	A_{s1}	A_{s2}	f_{cyl}	E_c	f_y	p
	Mm				mm ²		MPa			%
S3-2-1	304	279	278	40	232	57	47.7	35165	578	0.30
S1-2	300	273	284	29	777	57	49.4	35523	632	1.00
S1-4	300	267	280	24	760	57	49.4	35523	551	1.02
S3-2-6	303	277	279	49	232	57	50.9	35839	578	0.30
S3-2-9	298	273	285	21	232	57	44.2	34366	578	0.30
S3-2-10	298	271	281	21	232	57	44.6	34455	578	0.30

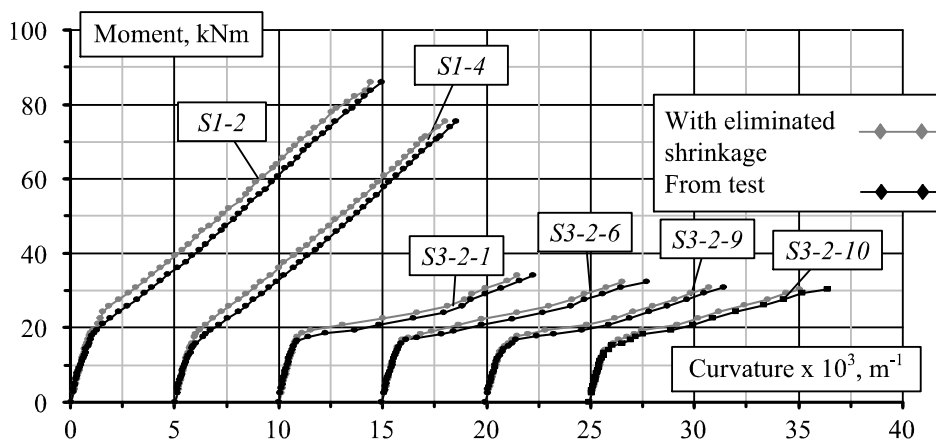


Fig. 3.15. The original and free-of-shrinkage experimental moment-curvature diagrams

Experimental results are presented in terms of moment-curvature ($M-\kappa$) diagrams for each of the specimen. The original and free-of-shrinkage experimental moment-curvature diagrams are shown in Fig. 3.12 and Fig. 3.15.

3.6. Derivation of Tension-Stiffening Model

The tension-stiffening relationships were obtained from moment-curvature diagrams for the all specimens applying the algorithms presented in Chapter 2. The tension-stiffening relationships obtained for the original test data are shown in Fig. 3.16. The tension-stiffening relationships with eliminated shrinkage are presented in Fig. 3.17. The relationships were normalised by dividing stresses by the tensile strength ($\sigma_{ct}/f_{ctm,EC2}$) and strains by the cracking strain(in relative term of $\varepsilon_{ct}/\varepsilon_{cr,EC2}$).

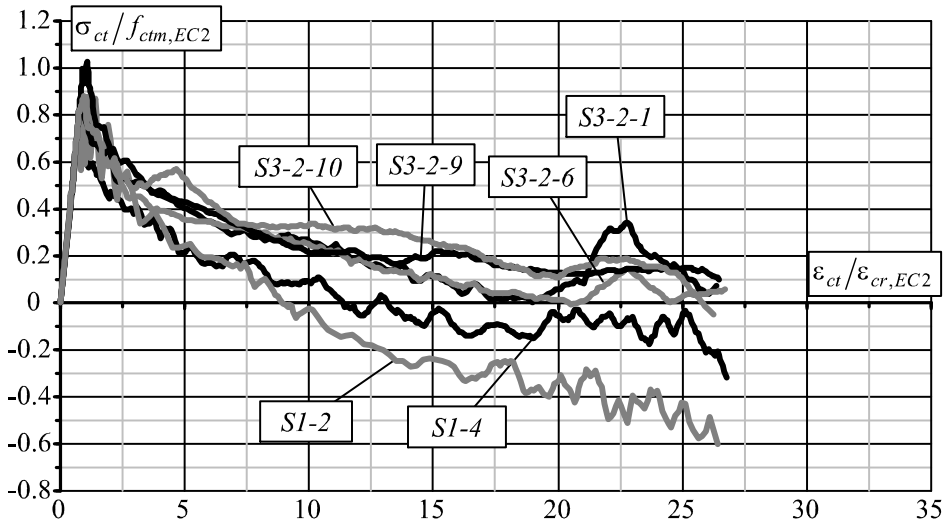


Fig. 3.16. Normalised tension-stiffening diagrams derived from the test data

It should be noted that some tension-stiffening relationships shown in Fig. 3.13, Fig. 3.14, Fig. 3.16 and Fig. 3.17 had significant portions of negative stresses, particularly for the specimens with higher reinforcement ratio. This can be due to the following: a) the assumption of perfect bond between tensile reinforcement and concrete is not accurate. Reinforcement slippage may occur at advanced stress-strain states, particularly for beams reinforced with bars of larger diameters. Such beams have relatively small total bar perimeter and respective bond surface leading to larger bond stresses. The reinforcement slippage is accompanied by increase in curvature. Calculation of strains and the resultant

force of the tensile reinforcement from this curvature, using the assumption of linear strain distribution within the section depth, gives overestimated values which in equilibrium have to be compensated by decreased (even negative) stresses of the tensile concrete; b) due to shrinkage, significant tensile concrete stresses were already present prior to the test. Therefore, the ascending branch of the calculated tension-stiffening relationship had a missing part of the initial stresses corresponding to the shrinkage stresses. With the shorter ascending branch, the zero point moves up and the descending branch enters the negative stress zone.

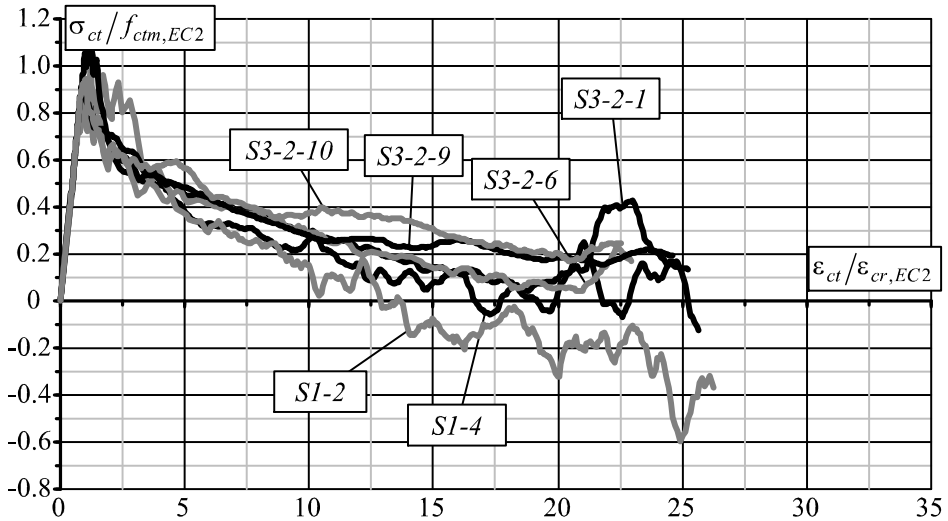


Fig. 3.17. Normalised tension-stiffening diagrams derived from the test data eliminating shrinkage effect

The tension-stiffening law was derived using the free-of-shrinkage tension-stiffening relationships shown in Figs. 3.14 and 3.17. Analysis has shown that the descending branch of the relationships depends on reinforcement ratio. The proposed model is shown in Fig. 3.18. It consists of a linear ascending and a non-linear descending branches. The shape for the descending branch has been chosen from a number of fitting curves, as a compromise between accuracy and simplicity. The proposed free-of-shrinkage tension-stiffening model has the form:

$$\sigma_{ct} = \begin{cases} E_c \varepsilon_{ct} & \varepsilon_{ct} \leq \varepsilon_{cr,EC2} \\ f_{t,EC2} \left(1 - 0.27 \ln \left(\frac{\varepsilon_{ct}}{\varepsilon_{cr,EC2}} \right) - 0.21p \right) & \varepsilon_{ct} > \varepsilon_{cr,EC2} \end{cases} \quad (3.1)$$

Here E_c is modulus of elasticity of concrete; $f_{t,EC2}$ is tensile strength of concrete according to *Eurocode 2*; $\varepsilon_{cr,EC2}$ is concrete cracking strain and p is reinforcement percentage.

This free-of-shrinkage tension-stiffening model makes it possible to take into account free concrete shrinkage strain as independent parameter.

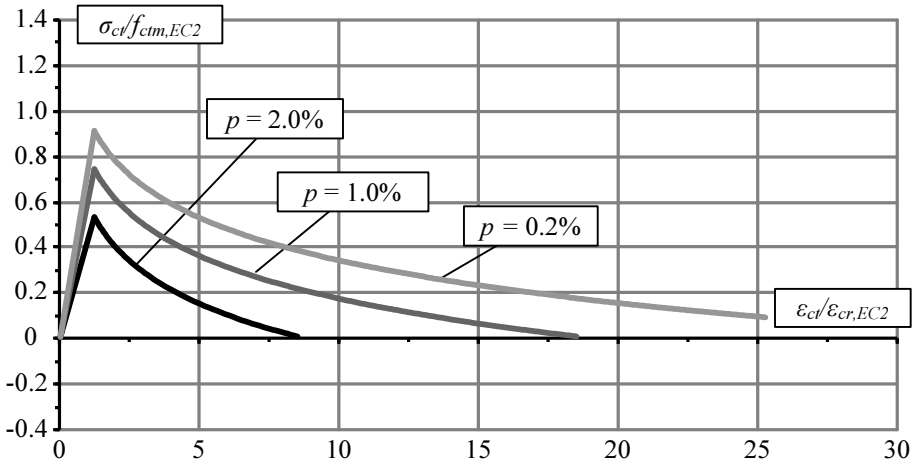


Fig. 3.18. Free-of-shrinkage tension-stiffening model

3.7. Conclusions for Chapter 3

The following conclusion can be drawn:

1. Experimental investigations carried out by the author have shown that shrinkage occurring prior to the short-term loading has a significant effect on the derived tension-stiffening model.
2. Based on the newly obtained test data of RC beams and inverse technique, a tension-stiffening model with eliminated concrete shrinkage effect has been proposed. Application of this model makes it possible to take into account free shrinkage strain as independent parameter.

3. The proposed stress-strain tension-stiffening law depends on concrete tensile strength and reinforcement ratio. Tension-stiffening reduces with increase in reinforcement ratio.
4. The proposed tension-stiffening model can be used both in standard finite element software and in Layer model. The latter is very effective in terms of computer resources, i.e. the calculation time decreases hundreds of times in comparison to standard non-linear finite element programs.

Comparative Statistical Analysis of Deflection Prediction Techniques

This Chapter presents statistical analysis (Gribniak 2009) on short-term deflections of RC bending members. The analysis has employed data of six experimental programs. The comparative study was based on the predictions made by design codes (*Eurocode 2*, *ACI 318*, *SP 52-101*) and tension-stiffening relationship proposed by the author (*Proposed model* see Equation 3.1), Hsu (1993) and Vecchio & Collins (1986).

The materials related to Chapter 4, the author in co-authorship has reported in the following paper: Sokolov *et. al.* (2010).

4.1. Experimental Data Employed for the Comparison

In this Section, experimental data of 40 reinforced concrete beams reported by six investigators is briefly described. All beams were subjected to a four-point bending. All beams had a rectangular cross-section. Main characteristics of the beams indicating variations in span, cross-section parameters and concrete strength are presented in Table 4.1. The tests covered wide ranges of reinforcement ratios. Experimental data of Clark & Speirs (1978), Ashour (2000) and

Nejadi (2005) can be categorized as beams having average and high reinforcement ratios. However, experimental data of Figarovskij (1962), Gushcha (1968) and Kaklauskas *et. al.* (2005) should be dealt separately, since most of the beams had a very low reinforcement ratio. Lightly reinforced beams is an extreme case of bending analysis, because the stress-strain state as well as curvatures and deflections are significantly influenced by effects of cracked tensile concrete. Since tensile strength is a highly dispersed value, it is very difficult to predict deflections accurately at loads just above the cracking loads, particularly for lightly reinforced members. The tests employed have also covered a wide range of concrete compressive strength (Table 4.1).

Table 4.1. Main characteristics of the beams

Author	Total number of beams	Span, (m)	Height, (mm)	Width, (mm)	Reinforcement percentage, (%)	Cube concrete strength, (MPa)
<i>Clark & Speirs</i> (1978)	14	3.20	200-500	200	0.44-1.98	23.0-39.6
<i>Kaklauskas et. al</i> (2005)	7	3.00	300	280	0.30	40.7-54.6
<i>Gushcha</i> (1967)	4	3.60	300	150	0.30-0.80	30.9-42.0
<i>Figarovskij</i> (1962)	7	3.00	250	180	0.40-0.90	29.4-37.1
<i>Ashour</i> (2000)	6	3.08	250	200	1.20-2.40	60.8-98.1
<i>Nejadi</i> (2005)	2	3.50	333-348	250	0.50	45.0
Total:	40	3.00-3.60	200-500	150-280	0.30-2.40	23.0-98.1

4.2. Calculation Methods Employed for Comparative Deflection Analysis

In this Section, calculation methods employed for comparative deflection analysis of flexural reinforced concrete members are listed. The following techniques are used:

1. *Eurocode 2* (CEN 2004) (see Section 1.3.1);
2. *ACI 318* (ACI Committee 318 2008) (see Section 1.3.2);

3. Russian code *SP 53-101* (see Section 1.3.3);
4. *Proposed* model (see Section 3.6);
5. Vecchio & Collins (1986) (see Equation 1.6);
6. Hsu (1993) (see Equation 1.7).

4.3. Statistical Analysis of the Predicted Deflections

The experimental mid-span deflections were compared with those predicted by the techniques described above. The analysis was based on introducing ten levels of loading intensity M taken in relative terms between the cracking and ultimate bending moment:

$$M' = (M - M_{cr}) / (M_{ult} - M_{cr}); M' = \{0.1; 0.2; \dots; 0.9; 1\}. \quad (4.1)$$

Here M_{ult} is the ultimate bending moment calculated for each member under assumption of yielding strength of tensile reinforcement 400 MPa; M_{cr} is the cracking moment (CEB-FIP 1991):

$$M_{cr} = I_{el} f_{ct} / y_t; f_{ct} = 0.3 \sqrt[3]{f_c'^2}; f_c' = f_{cu} / 1.25. \quad (4.2)$$

Here f_{cu} is the compressive 150 mm cube strength of concrete (see Table 4.1). Thus, $M'=1$ corresponds to failure of the RC element.

Accuracy of the predictions was estimated by means of a relative error $\Delta_{i,k}$ calculated at each level M' for each of 40 experimental members:

$$\Delta_{i,k} = \delta_{calc} / \delta_{obs}, \quad i = 1; 2; 3; \dots; 10, \quad k = 40. \quad (4.3)$$

Here δ_{calc} and δ_{obs} are the mid-point deflections interpolated at the level M' from calculated and original test data, respectively. It should be noted that not all specimens contained eleven output points as their testing was terminated before reinforcement reached 400 MPa. The transformation resulted in 344 output points covering post-cracking stage (compare to 899 measured points).

In practical application of the procedure, it was intended at each normalised load level to define reinforcement ratio intervals with normal probability distribution of relative error Δ valid for all the methods under consideration. Performed analysis has resulted in such three intervals:

$$1: p < 0.5\%; \quad 2: 0.5 \leq p \leq 1.0\%; \quad 3: p > 1.0\%. \quad (4.4)$$

Under the assumption of the above intervals, m_Δ and s_Δ were calculated at each normalised load level. The statistics for each of the deflection calculation method are presented in Tables 4.2 and 4.3.

Table 4.2. Basic statistics (mean and standard deviation) for analytical deflection calculation techniques grouped by reinforced ratio

<i>M'</i>	<i>n</i>	<i>Eurocode 2</i>		<i>ACI 318</i>		<i>SP 52-101</i>	
	Pts.	<i>m_A</i>	<i>s_A</i>	<i>m_A</i>	<i>s_A</i>	<i>m_A</i>	<i>s_A</i>
<i>p</i> < 0.5 %							
0.1	13	1.853	0.739	0.807	0.378	2.036	0.758
0.2	13	1.573	0.731	0.760	0.421	1.635	0.735
0.3	13	1.372	0.566	0.726	0.367	1.377	0.539
0.4	13	1.210	0.399	0.695	0.305	1.190	0.363
0.5	13	1.091	0.214	0.672	0.233	1.061	0.185
0.6	12	1.037	0.146	0.678	0.220	1.003	0.124
0.7	12	1.009	0.120	0.695	0.218	0.971	0.104
0.8	11	0.990	0.111	0.706	0.227	0.948	0.100
0.9	11	0.978	0.092	0.721	0.219	0.934	0.082
1	11	0.970	0.082	0.737	0.212	0.925	0.072
<i>0.5</i> ≤ <i>p</i> ≤ 1 %							
0.1	13	1.432	0.383	0.957	0.255	1.465	0.380
0.2	12	1.208	0.293	0.938	0.219	1.209	0.283
0.3	12	1.078	0.157	0.916	0.150	1.076	0.145
0.4	11	1.034	0.110	0.920	0.127	1.034	0.098
0.5	10	1.019	0.080	0.934	0.109	1.022	0.074
0.6	9	1.000	0.075	0.947	0.094	1.011	0.068
0.7	9	0.970	0.065	0.937	0.069	0.986	0.055
0.8	8	0.956	0.079	0.939	0.047	0.981	0.059
0.9	8	0.936	0.083	0.929	0.041	0.965	0.063
1	6	0.922	0.115	0.908	0.054	0.942	0.075
<i>p</i> > 1 %							
0.1	14	1.036	0.136	0.997	0.137	1.042	0.157
0.2	14	0.976	0.087	0.976	0.086	0.972	0.103
0.3	14	0.949	0.075	0.967	0.071	0.952	0.094
0.4	14	0.936	0.076	0.961	0.061	0.945	0.087
0.5	14	0.920	0.062	0.948	0.050	0.936	0.075

0.6	14	0.899	0.068	0.929	0.057	0.920	0.073
0.7	12	0.894	0.074	0.921	0.064	0.909	0.077
0.8	11	0.891	0.072	0.917	0.062	0.903	0.075
0.9	9	0.896	0.052	0.917	0.046	0.894	0.075
1	8	0.881	0.066	0.896	0.057	0.862	0.066

Table 4.3. Basic statistics (mean and standard deviation) for analytical deflection calculation techniques grouped by reinforced ratio

<i>M'</i>	<i>n</i>	<i>Proposed model</i>		<i>Vecchio & Collins</i>		<i>Hsu</i>	
	Pts.	<i>m_A</i>	<i>s_A</i>	<i>m_A</i>	<i>s_A</i>	<i>m_A</i>	<i>s_A</i>
<i>p</i> < 0.5 %							
0.1	13	0.958	0.459	0.887	0.380	1.268	0.534
0.2	13	0.920	0.366	0.805	0.408	1.273	0.650
0.3	13	0.971	0.381	0.749	0.337	1.179	0.522
0.4	13	0.978	0.337	0.702	0.260	1.072	0.369
0.5	13	0.950	0.214	0.666	0.169	0.985	0.197
0.6	12	0.940	0.153	0.664	0.141	0.954	0.134
0.7	12	0.947	0.126	0.671	0.131	0.937	0.109
0.8	11	0.948	0.117	0.678	0.130	0.930	0.098
0.9	11	0.956	0.092	0.688	0.119	0.924	0.081
1	11	0.963	0.078	0.699	0.111	0.921	0.074
<i>0.5</i> ≤ <i>p</i> ≤ 1 %							
0.1	13	1.141	0.323	0.837	0.192	1.039	0.243
0.2	12	1.064	0.242	0.783	0.159	1.004	0.224
0.3	12	1.009	0.125	0.759	0.085	0.949	0.126
0.4	11	0.999	0.085	0.768	0.065	0.939	0.092
0.5	10	1.005	0.061	0.787	0.056	0.945	0.071
0.6	9	1.005	0.057	0.806	0.049	0.945	0.064
0.7	9	0.988	0.050	0.806	0.039	0.928	0.054
0.8	8	0.986	0.066	0.820	0.031	0.928	0.066
0.9	8	0.974	0.074	0.820	0.034	0.918	0.069

1	6	0.965	0.104	0.812	0.048	0.907	0.096
$p > 1 \%$							
0.1	14	1.053	0.125	0.788	0.084	0.893	0.103
0.2	14	0.999	0.079	0.789	0.059	0.892	0.079
0.3	14	0.981	0.065	0.805	0.049	0.894	0.065
0.4	14	0.974	0.066	0.821	0.045	0.899	0.065
0.5	14	0.964	0.053	0.830	0.033	0.897	0.050
0.6	14	0.949	0.058	0.829	0.038	0.887	0.054
0.7	12	0.948	0.063	0.832	0.043	0.888	0.059
0.8	11	0.951	0.059	0.841	0.038	0.892	0.054
0.9	9	0.954	0.045	0.847	0.028	0.897	0.042
1	7	0.942	0.054	0.839	0.031	0.885	0.048

Results of the analysis are summarised in Table 4.2, Table 4.3, Table 4.4 and Table 4.5 and Figure 4.1, Figure 4.2, Figure 4.3. Tables 4.2 and 4.3 present numerical values of means and standard deviations after data grouping was performed.

Graphical illustration of the analysis results is presented in Figure 4.1, Figure 4.2 and Figure 4.3 with shown 95% confidence intervals of expectation μ_{Δ} for the grouped data. Width of the confidence intervals characterises variation of the relative error of predictions (see Equation 4.4). It is clearly seen that the variations were different not only for different deflection prediction methods, but also for different reinforcement ratios and load intensities. Strikingly different results were obtained for the members with minimal reinforcement ratios. Another point of sharp contrast was significantly larger data scatter at early cracking stages. The latter effect is particularly clear for the members with small amounts of reinforcement due to increased share of concrete in resisting tension forces acting in the section.

$$\mu_{\Delta} \in \left[m_{\Delta} - t_{1-\alpha/2}(n-1) \times \frac{s_{\Delta}}{\sqrt{n}}; m_{\Delta} + t_{1-\alpha/2}(n-1) \times \frac{s_{\Delta}}{\sqrt{n}} \right]. \quad (4.5)$$

Here $t(n-1)$ is the t – statistics (*Student's*) having $(n-1)$ degrees of freedom and significance level $\alpha/2$; $1-\alpha$ is confidence coefficient.

Tables 4.4 and 4.5 present numerical values of means and standard deviations for analytical deflection calculation for the total data.

Reasonable results in terms of consistency and variation have been demonstrated using the *Proposed* model (see Section 3.6).

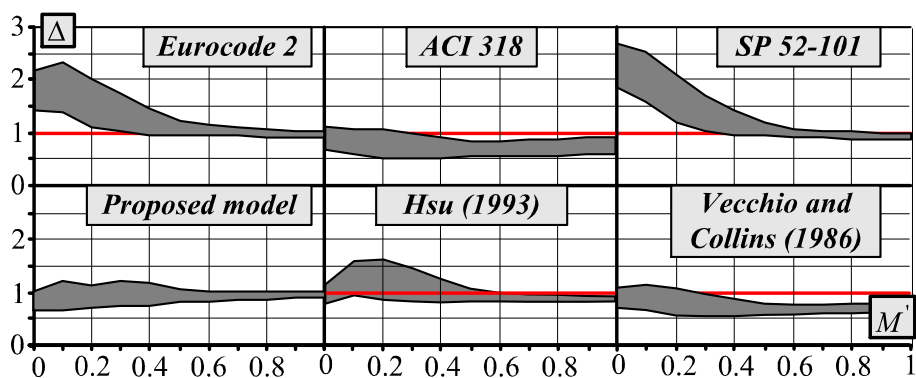
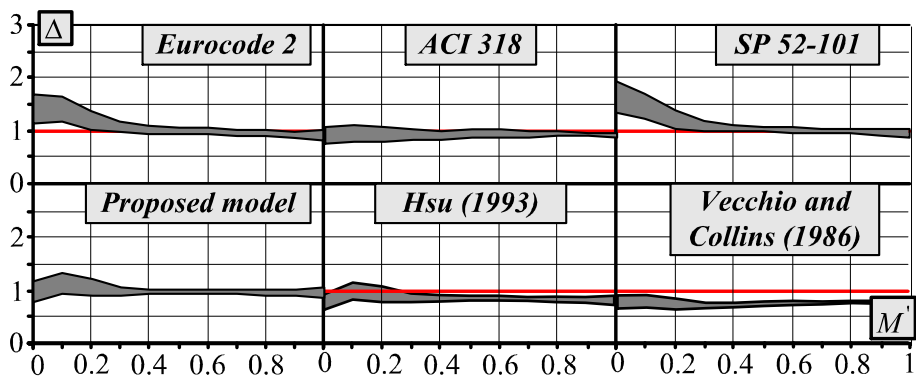
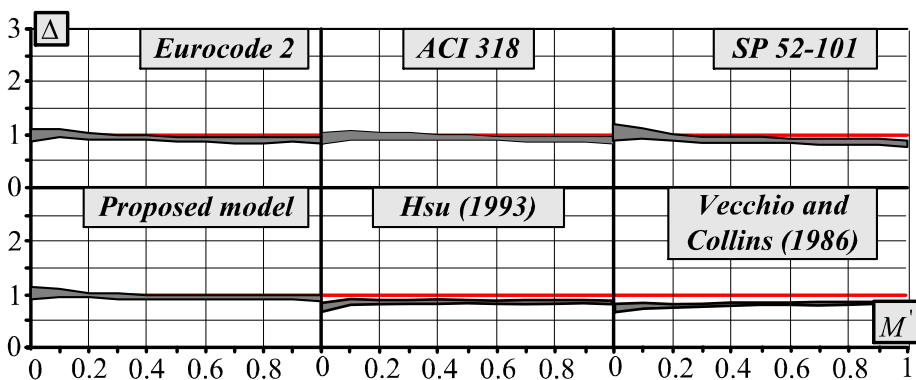
Fig. 4.1. 95 % confidence intervals ($p < 0.5 \%$)Fig. 4.2. 95 % confidence intervals ($0.5 \leq p \leq 1.0 \%$)Fig. 4.3. 95 % confidence intervals ($p > 1.0 \%$)

Table 4.4. Basic statistics (mean and standard deviation) for analytical deflection calculation techniques for the total data

M'	n	<i>Eurocode 2</i>		<i>ACI 318</i>		<i>SP 52-101</i>	
	Pts.	m_A	s_A	m_A	s_A	m_A	s_A
0.1	40	1.430	0.578	0.922	0.278	1.502	0.633
0.2	39	1.246	0.510	0.892	0.286	1.266	0.526
0.3	39	1.130	0.378	0.871	0.249	1.132	0.366
0.4	38	1.058	0.266	0.858	0.224	1.055	0.243
0.5	37	1.007	0.154	0.847	0.198	1.003	0.134
0.6	35	0.972	0.118	0.847	0.186	0.972	0.100
0.7	33	0.957	0.102	0.843	0.179	0.952	0.088
0.8	30	0.945	0.097	0.845	0.177	0.940	0.085
0.9	28	0.940	0.083	0.844	0.170	0.930	0.078
1	24	0.932	0.092	0.826	0.168	0.911	0.076
Total	383	1.111	0.389	0.867	0.224	1.144	0.447

Table 4.5. Basic statistics (mean and standard deviation) for analytical deflection calculation techniques for the total data

M'	n	<i>Proposed model</i>		<i>Vecchio & Collins</i>		<i>Hsu</i>	
	Pts.	m_A	s_A	m_A	s_A	m_A	s_A
0.1	40	1.051	0.328	0.836	0.211	1.063	0.281
0.2	39	0.993	0.254	0.793	0.209	1.058	0.318
0.3	39	0.986	0.228	0.772	0.169	1.009	0.251
0.4	38	0.983	0.201	0.765	0.135	0.970	0.185
0.5	37	0.971	0.133	0.762	0.095	0.940	0.107
0.6	35	0.960	0.102	0.765	0.090	0.926	0.083
0.7	33	0.958	0.088	0.768	0.085	0.916	0.074
0.8	30	0.959	0.085	0.776	0.085	0.916	0.069
0.9	28	0.960	0.073	0.780	0.079	0.912	0.058
1	24	0.957	0.076	0.776	0.076	0.903	0.063
Total	383	0.979	0.199	0.783	0.175	0.958	0.246

4.4. Conclusions for Chapter 4

This Chapter was devoted to statistical accuracy investigation of the proposed tension-stiffening model (see Equation 3.1). For that purpose, deflections and curvatures were calculated for 40 experimental reinforced concrete beams reported by six investigators. The beams were tested by a short-term loading under a four-point bending system. All the beams were subjected to a four-point bending. All the beams had a rectangular cross-section. The tests covered wide ranges in reinforcement ratio (0.3 to 2.4 %). For calculation of the stress-strain state at any point of the beam as well as prediction of curvatures and deflections, the proposed stress-strain relationship (see Equation 3.1) were incorporated into a simple iterative technique of deformational analysis based on classical expressions of strength of materials, extended to Layer approach. The accuracy results for the proposed relationship (see Equation 3.1) were compared with deflections estimates by other methods.

The following conclusion can be drawn:

1. Deflections of lightly reinforced members using the proposed tension-stiffening model are calculated more accurately in regard to other models and techniques. Accuracy of the predictions has been assessed using relative deflections $\Delta = \delta_{cal}/\delta_{obs}$, which for different methods varied in the following range: 0.92–1.14 (*the proposed model*); 0.67–1.00 (*ACI 318*); 0.89–1.85 (*Eurocode 2*); 0.86–2.04 (*SP 52-101*); 0.66–0.89 (*Vecchio & Collins*); 0.89–1.27 (*Hsu*).
2. Accuracy of the predictions varied significantly within different intervals of reinforcement ratio and load intensity. Significant errors (over 100 %) in terms of mean deviation were obtained for lightly reinforced beams ($p < 0.5$ %) at loading stage close to cracking. Whereas maximal error for the beams with average and high reinforcement ratio ($p > 1.0$ %) was below 15 %.

General Conclusions

1. Experimental investigations carried out by the author have shown that shrinkage occurring prior to the short-term loading has a significant effect on the derived tension-stiffening model.
2. Based on the newly obtained test data of RC beams and inverse technique, a tension-stiffening model with eliminated concrete shrinkage effect has been proposed. Application of this model makes it possible to take into account free shrinkage strain as independent parameter.
3. The proposed stress-strain tension-stiffening law depends on concrete tensile strength and reinforcement ratio. Tension-stiffening reduces with increase in reinforcement ratio.
4. The proposed tension-stiffening model can be used both in standard finite element software and in Layer model. The latter is very effective in terms of computer resources, i.e. the calculation time decreases hundreds of times in comparison to standard non-linear finite element programs.
5. Deflections of lightly reinforced members using the proposed tension-stiffening model are calculated more accurately in regard to other models and techniques. Accuracy of the predictions has been assessed using relative deflections $\Delta = \delta_{cal}/\delta_{obs}$, which for different methods varied in the following range: 0.92–1.14 (*the proposed model*); 0.67–1.00 (*ACI 318*);

0.89–1.85 (*Eurocode 2*); 0.86–2.04 (*SP 52-101*); 0.66–0.89 (*Vecchio & Collins*); 0.89–1.27 (*Hsu*).

6. Accuracy of the predictions varied significantly within different intervals of reinforcement ratio and load intensity. Significant errors (over 100 %) in terms of mean deviation were obtained for lightly reinforced beams ($p < 0.5$ %) at loading stage close to cracking. Whereas the maximal error for the beams with average and high reinforcement ratio ($p > 1.0$ %) was below 15 %.

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TENSION STIFFENING MODEL FOR REINFORCED CONCRETE BEAMS

Doctoral Dissertation

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Civil Engineering (02T)

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