



Quantifying Residual Strength of Cement-Based Reinforced Composites Under Repeated Loads

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Abstract. Recent structural engineering trends revealed that fiber-reinforced polymers (FRP) define an efficient replacement for steel because of their lightweight and corrosion resistance. However, the unfavorable environment (elevated temperature and humidity) can affect the long-term resistance of those materials. Therefore, this study analyzes the adverse effects of environmental conditions and repeated mechanical loads on the composite performance of concrete beams reinforced with FRP materials. A new testing layout and a simplified analytical model are developed to quantify the flexural stiffness decay. This model relates the particular moment and curvature values, requiring neither iterative calculations nor the load history. Thus, this exceptional feature ensures the explicit quantification of the equivalent strength and comparative analysis of the residual efficiency of the composite systems with different reinforcement layouts and materials. This study compares several reinforcement schemes and considers the open atmosphere corresponding to the Nord Europe climatic region and the reference laboratory conditions. The analysis reveals that the outdoor environment specifically affected the stiffness of the external bonded reinforcement system under repeated loading. In contrast, the near-surface mounted reinforcement was resistant to this effect.

Keywords: Reinforced Composites · Stiffness · Tests · Analytical Model

1 Introduction

Current trends in structural engineering revealed that fiber-reinforced polymers (FRP) define an efficient replacement for steel because of their lightweight and corrosion resistance [1]. However, the unfavorable environment (elevated temperatures and humidity) affects these materials' long-term performance because of the fibers' swelling and delamination, cracking, and thermal degradation of the polymer matrix [2, 3]. Hence, this study analyzes the adverse effects of environmental conditions and repeated mechanical loads on the composite performance of concrete beams reinforced with FRP materials.

A new testing layout and a simplified analytical model were developed to quantify the flexural stiffness of the standardized samples [4, 5]. This analytical model relates the

particular moment and curvature values, requiring neither iterative calculations nor the load history. This exceptional feature enables explicit quantification of the equivalent strength of tensile concrete. Furthermore, this methodology is equally suitable for quantifying the residual strength of elements under monotonic and repeated mechanical loads and resistance estimation confidence comparable to measurement precision. Thus, this investigation employs this methodology for quantifying the adverse effects of repeated external factors, i.e., environmental conditions and mechanical loads.

This study compares the performances of several reinforcement schemes and considers the open atmosphere corresponding to the Nord Europe climatic region and the reference laboratory conditions. The analysis reveals an adverse effect of the outdoor environment on the beam samples with externally bonded reinforcement. However, this effect was less significant in the near-surface mounted reinforcement system.

2 Analysis Procedure

This section briefly explains the testing procedure and the proposed analytical model (Fig. 1). The article [4] provides an interested expert with the model verification details. The proposed simplification of the analytical model describes the scientific novelty of this work. The previous results [4–6] customized the geometry of the test specimen (Fig. 1a), which ensures the shape of the tensile stresses' distribution in concrete under bending (Fig. 1b) close to rectangular (Fig. 1c and 1d). Thus, this simplification allows the model to solve the tension-stiffening problem explicitly and makes it independent of the loading conditions.

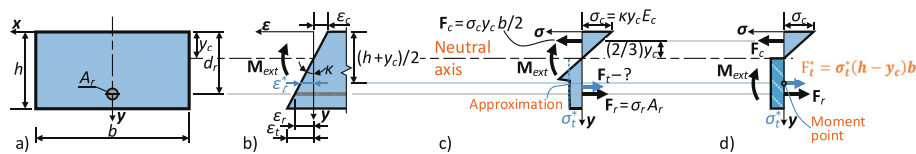


Fig. 1. Simplified stress distribution model [4]: a) transformed reinforcement; b) and c) the bending-induced strains and stresses; d) the equivalent stress approximation in the tension concrete.

2.1 Testing Layout

This study considers the $200 \times 100 \times 1000$ mm (width $\times$ height $\times$ length) beam subjected to a four-point-bending. Figure 2 shows the test geometry and setup, which ensures verification of the monitoring outcomes. Thus, this investigation can use three mutually independent indicators groups distributed in the pure bending zone: two linear variable displacement transducers (LVDT) arrays monitor the displacements (longitudinal and transversal); a digital image correlation (DIC) system captures the relative movements of pixels recognized at the side surface of the beam. The DIC also monitors crack development [7]. The LVDT and DIC devices monitor the opposite sides, denoted as “①” and “②” in Fig. 2a. Figure 2b shows the loading setup. The data logger ALMEMO 5690–2 collects the load cell and LVDT outputs every second.

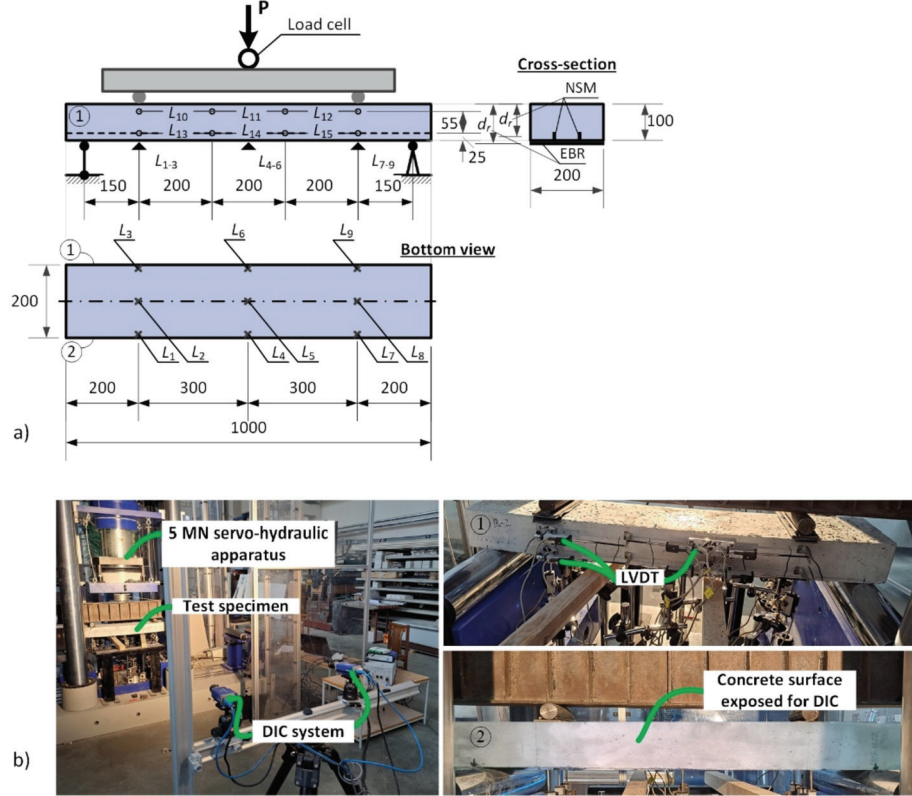


Fig. 2. Bending test samples with EBR and NSM reinforcement: a) loading scheme and geometry; b) test setup.

2.2 Simplified Analytical Model

The analysis is limited to the mechanical performance of the pure bending zone. The model employs the transformed reinforcement idea (Fig. 1a), combining all materials in a single component. The following formulas describe the transformed reinforcement parameters, i.e., the effective depth d_r , area A_r , and modulus of elasticity E_r :

$$d_r = \frac{\sum_{i=1}^n E_i A_i d_i}{\sum_{i=1}^n E_i A_i}, A_r = \frac{1}{E_r} \sum_{i=1}^n E_i A_i, E_r = E_1, \quad (1)$$

where n is the number of the reinforcement components; E_i , A_i , and d_i are the modulus of elasticity, area, and effective depth of the i -th reinforcement type, i.e., $i = 1 \dots n$.

In addition, the model uses the following assumptions:

- The Euler-Bernoulli hypothesis describes the strain distribution (Fig. 1b). This assumption was proved in the literature (e.g., [8]).
- The smeared crack model is valid for the tension concrete (Fig. 1c). Numerous literature examples (e.g., [9,10]) proved this concept.

- The mechanical behaviors of the reinforcement and the compressive concrete follow the perfectly elastic material laws. This assumption simplifies the mathematical expressions, ensuring a straightforward solution to the tension-stiffening problem.
- The model considers the equivalent rectangular stress distribution in the tensile concrete (Fig. 1d). This assumption defines the solution avoiding iterative calculations.

The following equilibrium equations of internal forces and bending moments for the centroid of the equivalent tensile stress diagram (Fig. 1d) form the theoretical model:

$$\mathbf{F}_t^* + \mathbf{F}_r - \mathbf{F}_c = 0; \mathbf{F}_r \left(d_r - \frac{h+y_c}{2} \right) + \mathbf{F}_c \left(\frac{h+y_c}{2} - \frac{y_c}{3} \right) - \mathbf{M}_{ext} = 0, \quad (2)$$

where $\mathbf{F}_t^*$ is the equivalent internal force acting on the tensile concrete; $\mathbf{F}_r$ and $\mathbf{F}_c$ are the force components corresponding to the tensile reinforcement and the compressive concrete; Fig. 1 describes other notations.

The system (2) relates the internal forces and corresponding stresses using the strain compatibility condition (Fig. 1b). Gribniak et al. [4] described the steps of the solution, which represents the third-order polynomial regarding the centroid coordinate y_c :

$$\sum_{i=0}^3 k_i (y_c)^i = 0, \quad (3)$$

with coefficients

$$k_3 = \frac{E_c b}{6E_r A_r}, \quad k_2 = 1 + \frac{E_c b h}{2E_r A_r}, \quad k_1 = h - 3d_r, \quad k_0 = 2d_r^2 - h d_r - \frac{2\mathbf{M}_{ext}}{\kappa E_r A_r}.$$

The above polynomial has three roots, and the $[0 < y_c \leq h]$ condition determines a single solution of the polynomial (3), describing the centroid position:

$$y_c = \frac{\bar{k}}{1.5k_3} \cos \left(\frac{1}{3} \cos^{-1} \left[-\frac{13.5k_3^2 k_0 - 4.5k_3 k_2 k_1 + k_2^3}{\bar{k}^3} \right] \right) - \frac{k_2}{3k_3}, \quad (4)$$

with

$$\bar{k} = \sqrt{k_2^2 - 3k_3 k_1}.$$

For a certain curvature κ , the following expressions describe the equivalent stress (Fig. 1d) and mean strain of the tensile concrete (Fig. 1b):

$$\sigma_t^* = \kappa \frac{y_c^2 b E_c - 2(d_r - y_c) E_r A_r}{2(h - y_c) b}, \quad \varepsilon_t^* = 0.5\kappa(h - y_c). \quad (5)$$

3 Experimental Program

3.1 Test Data

This manuscript exemplifies the proposed analysis procedure using the bending test results of 10 standardized beam samples (Fig. 2a). The tests were carried out employing a 5 MN servo-hydraulic apparatus (Fig. 2b) under displacement control and the

0.4 mm/min loading velocity. The beams underwent two testing stages repeated after half a year. After the first loading stage, the reference samples were stored under laboratory conditions; alternate samples were exposed to the open environment outside the laboratory building. Figure 3 describes the outdoor conditions; the laboratory specimens were kept at an average 20.5 °C temperature and 35% relative humidity.

The beams were produced from laboratory-mixed concrete with the following proportions for an m³: the cement CEM I 42.5 R = 356 kg, water = 163 l, the limestone powder = 177 kg, the 0/4 mm sand = 890 kg, the 4/16 mm crushed dolomite aggregates = 801 kg, a superplasticizer = 1.97% (by the cement weight), and the admixture SCP 1000 Optimiser = 3.5 kg. In addition, the mixture included 0.9 kg synthetic micro-fibers (13 mm × Ø22 µm, tensile strength = 380 MPa) and 4.2 kg synthetic macro-fibers (40 × Ø0.7 mm, tensile strength = 500 MPa; modulus of elasticity = 6.0 GPa).

This illustrative study considers two composite layouts, i.e., externally bonded reinforcement (EBR) using carbon FRP (CFRP) sheets and near-surface mounted (NSM) CFRP strips. Two unidirectional CFRP sheets (100 mm width and 0.164 mm equivalent thickness of the dry fabric) formed the wet lay-up EBR system. Two pultruded 10 × 1.4 mm CFRP strips adhesively installed in the 4 × 12 mm grooves produced the NSM system. Table 1 characterizes the beam samples, where f_{cm} is the Ø150 × 300 mm concrete cylinder strength at 87 days; t is the first and second testing stage ages; the producers specified the modulus of elasticity (E_r) and tensile strength (f_r) of the CFRP materials. The first letter of the notation describes the reinforcement type (“N” = NSM and “E” = EBR), the second letter specifies the storing conditions (“L” = laboratory and “O” = outdoor), and the number differentiates the identical samples.

Table 1. Geometry and main characteristics of the beam samples.

Beam	h [mm]	b [mm]	d [mm]	A_r [mm ²]	E_r [GPa]	f_r [MPa]	f_{cm} [MPa]	t [days]
NL-1	103	199	93	28.0	170	2800	48.14	120/274
NL-2	104	198	94	28.0	170	2800	48.14	119/274
NO-1	103	200	93	28.0	170	2800	60.5	116/273
NO-2	102	200	92	28.0	170	2800	60.5	117/270
NO-3	100	201	90	28.0	170	2800	60.5	116/270
EL-1	104	198	105	32.8	230	4830	48.14	119/274
EL-2	105	197	106	32.8	230	4830	60.5	116/271
EO-1	105	201	105	32.8	230	4830	48.14	120/274
EO-2	104	198	105	32.8	230	4830	48.14	119/276
EO-3	106	197	107	32.8	230	4830	48.14	119/273

The beam samples were subjected to five repeated load cycles to investigate the stiffness decay and ensure the reliability of the test measurements. The load cycles fluctuated around the service moment, nominally representing 55% of the ultimate load.

The previous tests [4–6] determined the load-bearing capacity of the considered beams, which was almost identical for the EBR and NSM reinforcement systems. Thus, the exact 3.710 kN service moment was assumed for all beams (Table 1); the lower and upper boundaries in the load cycles were 3.075 kN and 4.125 kN.

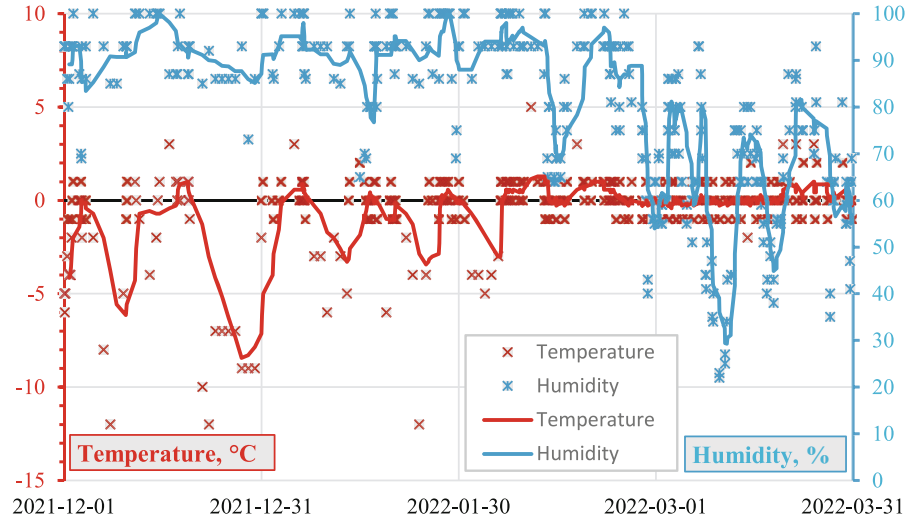


Fig. 3. Outdoor conditions.

3.2 Results and Discussion

As noted in Sect. 2.1, the measurement devices ensure the curvature estimation using three mutually independent datasets, i.e., the vertical displacement registered by the L_1 – L_9 LVDT devices and the surface deformation captured by the DIC and L_{10} – L_{15} LVDT (Fig. 2a). This analysis employs the vertical displacement monitoring results (u_i , $i = 1 \dots 9$); the following equation defines the corresponding curvature values:

$$\kappa = \frac{8 \cdot U}{l_b^2 + 4 \cdot U^2}, \quad U = \frac{u_4 + u_5 + u_6}{3} - \frac{u_1 + u_2 + u_3 + u_7 + u_8 + u_9}{6}, \quad (6)$$

where U is the deflection over the pure bending zone of length l_b ($= 600$ mm).

Figure 4 shows the moment-curvature diagrams of the beams listed in Table 1. The purple lines determine the first loading stage results; the yellow lines define the test outcomes from the second loading stage. The looping branches correspond to the repeated loading (generating the analysis datasets).

Remarkably, the equivalent stress σ_t^* results, Eq. (5) are independent of the loading history—the calculations only require a moment and curve pair. The service moment (red dashed line in Fig. 4) describes the reference load in this study, and the ascending branch curvature values in Fig. 4 form the database for the stiffness regression analysis. Thus, each loading stage generated five data points. However, the study skipped the first

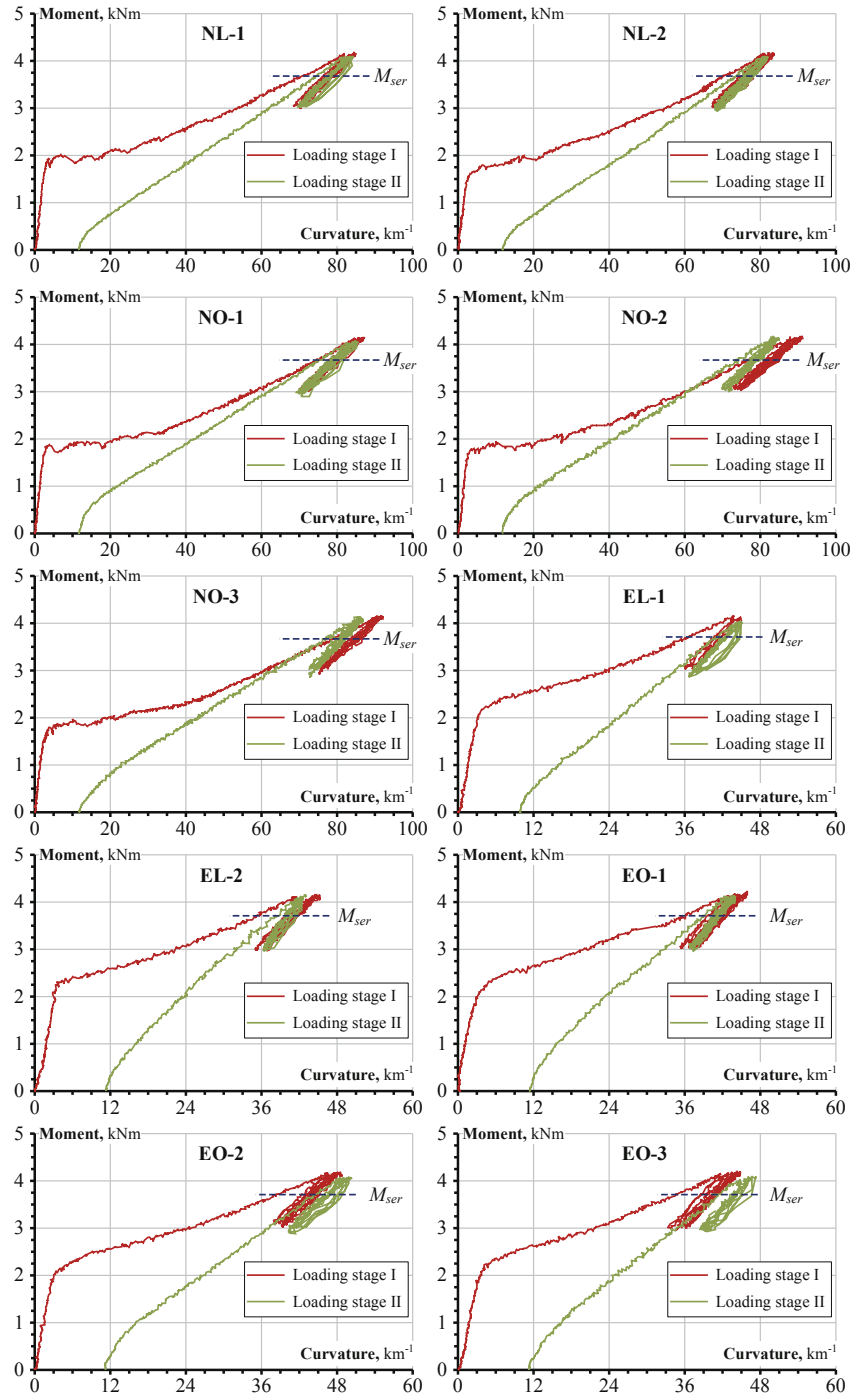


Fig. 4. Moment-curvature diagrams of the beams listed in Table 1.

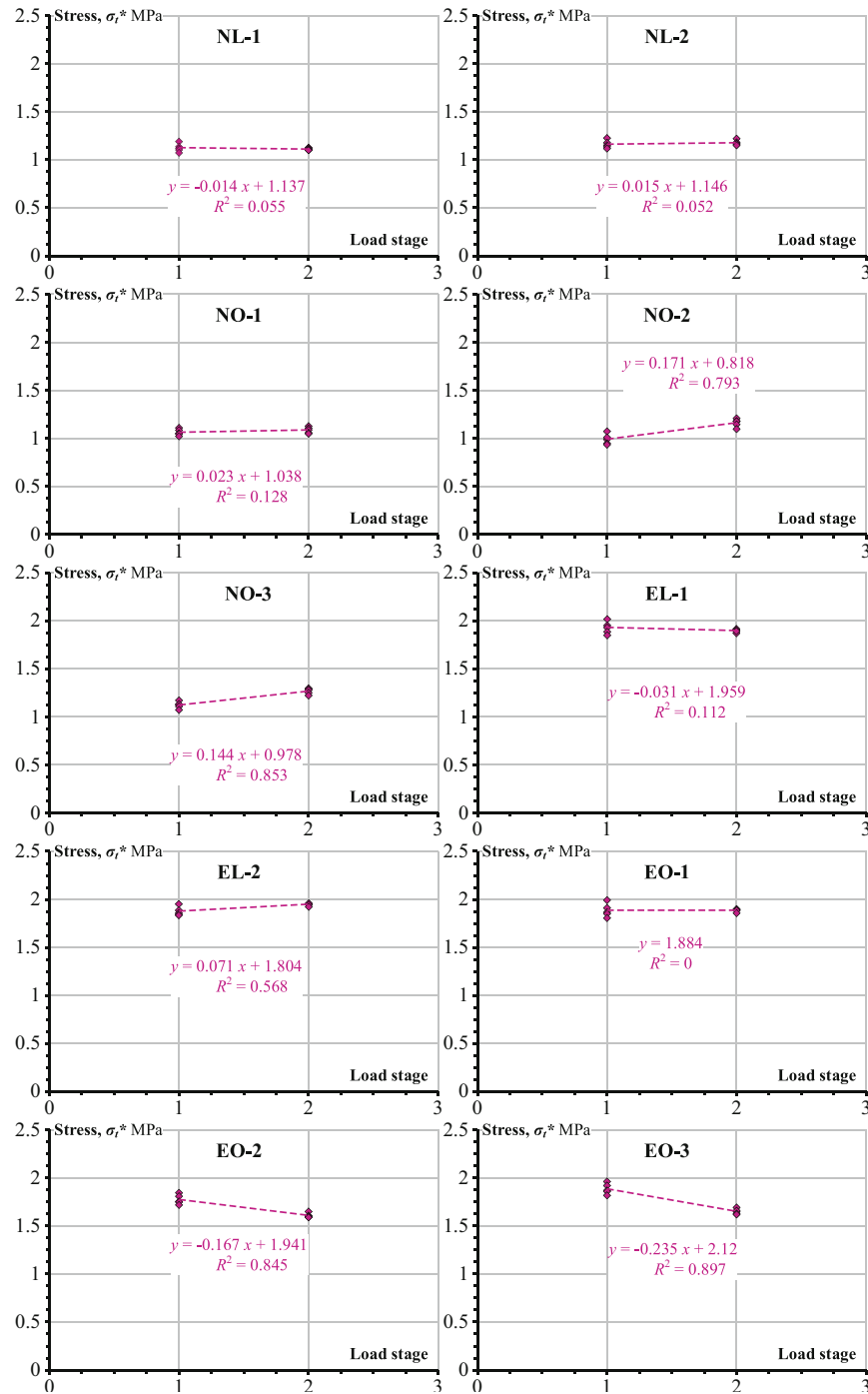


Fig. 5. The equivalent strength trends of the beams from Table 1.

point to avoid the regression skewness resulting from the excessive stiffness of the beam during the first load.

Figure 5 shows the equivalent stresses determined at the service load for all beam samples listed in Table 1, highlighting the regression trends. Remarkably, none of the reference beams (stored in the laboratory conditions) revealed stiffness decay during the test. Contrarily, the test samples with EBR were vulnerable to environmental conditions—the specimens show an average 16% decay in the stiffness (expressed in terms of the slope coefficient of the regression line) after the outdoor exposition. Alternatively, the NSM specimens demonstrated immunity to environmental conditions.

These outcomes determine the first attempt to identify the stiffness decay under outdoor conditions, representing a part of the ongoing research project; the limited size of the dataset and the test duration reduces the reliability of the identified trends. Therefore, upcoming tests will verify the hypothesized EBR vulnerability to environmental impacts. However, the developed analysis procedure demonstrated its feasibility and sensitivity in analyzing the mechanical performance of composite reinforcement systems.

4 Conclusions

This manuscript describes a new analysis methodology quantifying the long-term decay of residual stiffness of concrete beams with composite reinforcement. The study employs the partial results of an ongoing research project to illustrate the sensitivity of the proposed analysis procedure. This illustrative study considers 10 beam samples and two composite layouts, i.e., externally bonded reinforcement (EBR) with carbon fiber-reinforced polymer (CFRP) sheets and near-surface mounted (NSM) CFRP strips. The following essential findings resulted from this work:

- The EBR demonstrated a particular vulnerability to outdoor conditions—the average 16% decay was observed after half a year of tests. The beams with NSM reinforcement were less vulnerable to the outdoor environment. The observed degradation in the mechanical performance of EBR was probably due to the extended bonding surface open to the environmental impact.
- The reported outcomes determine the first attempt to identify the stiffness decay under outdoor conditions. Still, the proposed analysis methodology was capable of identifying the stiffness decay. Therefore, the upcoming tests should verify the EBR's vulnerability to environmental impacts.
- The proposed analytical model does not explicitly determine a constitutive model but defines an equivalent measure of the residual stiffness of the composite system.

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