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STATYBOS FAKULTETAS

GELŽBETONINIŲ IR MŪRINIŲ KONSTRUKCIJŲ KATEDRA

Jevgenija Riašenceva

**LENKIAMŲ GELŽBETONINIŲ ELEMENTŲ STANDUMO NORMINIS
PATIKIMUMAS
STANDARDIZED STIFFNESS RELIABILITY OF FLEXURAL REINFORCED
CONCRETE ELEMENTS**

Baigiamasis magistro darbas

Statinių konstrukcijos, valstybinis kodas 62402T107

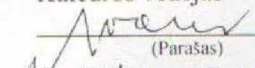
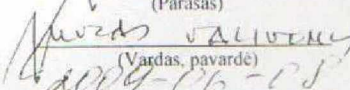
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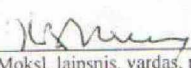
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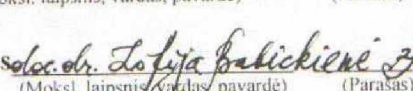
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Vadovas  Benediktas Uzdilinskas, dr., doc.
(Moksl. laipsnis, vardas, pavardė) (Parašas) (Data)

Konsultantas  doc. dr. Lofija Patickienė
(Moksl. laipsnis, vardas, pavardė) (Parašas) (Data) 2009 06 16

Konsultantas _____
(Moksl. laipsnis, vardas, pavardė) (Parašas) (Data)

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Baigiamojo darbo rengimo konsultantai:

(Moksl. laipsnis, vardas, pavardė)

Vadovas

[Signature]
(Paraša)

doc. dr. Benediktas Uspolovicius
(Moksl. laipsnis, vardas, pavardė)

Užduotį gavau

[Signature]
(Parašas)
Jurgenia Rišėnava
(Vardas, pavardė)
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Autorius **Jevgenija Riašenceva**
Vadovas **doc. dr. Benediktas Užpolevičius**

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Anotacija

Baigiamajame magistro darbe yra pateikiamas lenkiamųjų gelžbetoninių elementų standumo patikimumo, normuoto įvairių šalių ir tarptautinėse Skaičiuotinių reikšmių normose (STR 2.05.05, EN 1992-1, ACI 318, SP 52-101), nustatymo apibendrinimas. Apibendrinimas yra grindžiamas lenkiamųjų gelžbetoninių elementų trumpalaikio ir ilgalaikio standumo deterministinių modelių, taikomų įvairiose Skaičiuotinių reikšmių projektavimo normose, sugretinimu ir standumo norminio patikimumo tikimybinio skaičiavimu.

Darbą sudaro 5 dalys: įvadas, literatūros analizė, norminio ribinio įlinkio skaičiavimas, taikant įvairių projektavimo normų deterministinius modelius, lenkiamųjų gelžbetoninių elementų norminio standumo patikimumo valdymas, išvados. Darbo apimtis – 74 p. teksto, 10 iliustracijų, 6 lentelės, 23 bibliografiniai šaltiniai.

Prasminiai žodžiai

Apkrovų tikimybinis modeliavimas, gelžbetoninės sijos, norminis patikimumas, patikimumo indeksas, patikimumo valdymas, ribinis įlinkis, skaičiuotinių reikšmių metodas, tiesioginis-tikimybinis metodas.

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Master Degree Studies Building Structures study programme Master's Thesis		
Title	Standardized stiffness reliability of flexural reinforced concrete elements	
Author	Jevgenija Riašenceva	
Academic supervisor	Assoc Prof Dr Benediktas Užpolevičius	
Thesis language: English		
Annotation The determination of reliability of stiffness of flexural reinforced concrete elements, standardized in different countries and international Design value codes (STR 2.05.05, EN 1992-1, ACI 318, SP 52-101), is generalized in master's degree thesis. The generalization is based on unification of deterministic models of short-term and long-term stiffness of flexural reinforced concrete elements, applied in different Design value codes, and probabilistic calculation of standardized reliability of stiffness. The work contains 5 parts: introduction, literature analysis, calculation of ultimate deflection according to deterministic models of design codes, management of standardized reliability of stiffness of flexural reinforced concrete elements, conclusions. Thesis consists of: 74 p. of text, 10 pictures, 6 tables, 23 bibliographical entries.		
Keywords Probabilistic modeling of loads, reinforced concrete beams, ultimate deflection, standardized reliability, reliability index, reliability management, method of design values, direct probability method.		

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INTRODUCTION

An acceptable, i.e. standardized in codes reliability is managed *non-directly* using partial safety factors (PSF) method and applying current Design Values (DV) codes of design and evaluation of existing elements.

In Direct Probabilistic-Statistic (DPS) codes nor partial safety factors, nor design values are used. The reliability is managed *directly* applying models of probability theory and mathematical statistics. A design and existing elements evaluation criteria in DPS codes is chosen standardized in DV codes reliability of resistance, for example, strength, stiffness.

Standardized in DV codes reliability of *safety* is investigated in details, what cannot be stated about reliability of *stiffness*. It is not so widely studied.

The aim of the master's degree thesis is to generalize the determination of stiffness reliability standardized in different countries DV codes and international DV codes of flexural reinforced concrete elements.

The generalization is based on unification and probabilistic calculation of standardized stiffness reliability of short-term and long-term stiffness deterministic models, applied in different DV design codes of flexural reinforced concrete elements.

1 THE CRITERIA OF STIFFNESS DESIGN OF REINFORCED CONCRETE ELEMENTS

1.1 THE SAFETY AND SERVICEABILITY STANDARDIZED RELIABILITY DEPENDENCE ON DAMAGE IN CASE WHEN LIMIT STATE TAKES PLACE

The standardized reliability is chosen such that the sum of the initial investments for the element manufacture and erection and the losses due to limit state occurrence will be minimal taking into account different time period of these expenditures. These expenditures sum is expressed [1]:

$$C = C_{0,A} + C_{\alpha} \cdot (\alpha_A + \alpha_z) = C_{0,A} + C_{\alpha} \cdot \alpha; \quad (1.1.1)$$

here

$C_{0,A}$ – initial investments needed for the element reliability assurance and which depend on material consumption, for example on element cross-sectional area A ;

C_{α} – losses due to limit state occurrence, for example due to structure or equipment destruction or concession interruption (economic losses), hazard to people (social losses) which may be expressed in money, i.e. as economic losses, taking into account insurance payments;

α_A – probability of limit state occurrence due to insufficient cross-sectional area factor A ;

α_z – probability of limit state occurrence due to human mistakes in design, construction and use of structure, $\alpha = \alpha_A + \alpha_z$.

Table 1.1.1 The average human failure rate

Nr.	A form of activity, where the mistake occurred	Rate in Europe, %	Rate in Lithuania, %
1	Design	25	7
2	Structure manufacturing and construction	45	63
3	Exploitation	8	7
4	External influence (differential and large foundation setting and slip, elements overloading)	15	23
5	Other mistakes	7	

The average human failure rate is determined based on different European countries authors received data, also Lithuanian Inspection of Construction data received during the forty-year period.

The expenses corresponding to summands from Table 1.1.1 take place at different time intervals, so they are calculated at the beginning or at the end of structure working life, taking into account the construction borrowing rate. As the example, in case of 10 % annual interest rate and 50 years of structure life cycle, calculating the expenses at the end of structure working life the $C_{0,A}$ investments are multiplied by the coefficient 117.

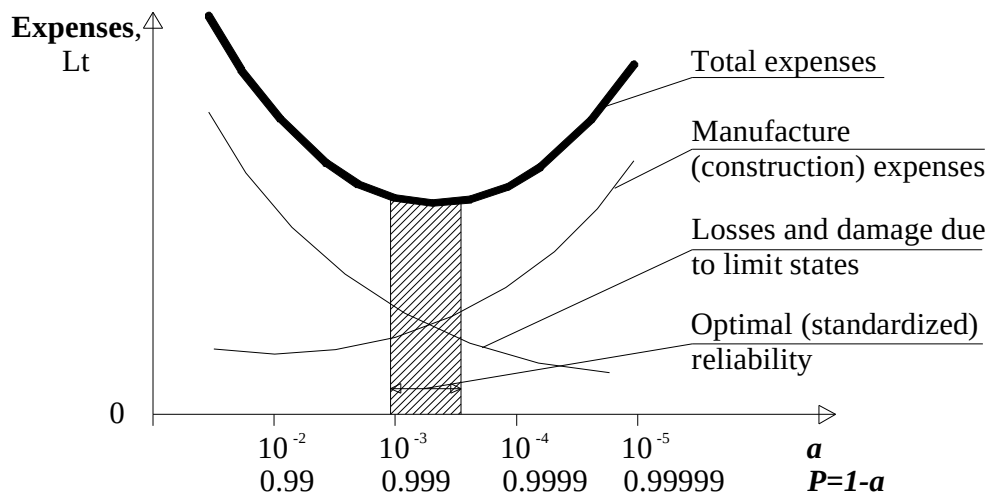


Fig. 1.1.1 Optimal codified reliability of structural elements margin of strength

It is possible to determine optimal values of α and $P = 1 - \alpha$, i.e., standardized reliability by minimizing the expenses and requiring the following condition to be fulfilled [1]:

$$C_{0,A} + C_{\alpha} \alpha \rightarrow \min, \quad (1.1.2)$$

This procedure called economical-social reliability optimization is shown in Fig.1.1.1.

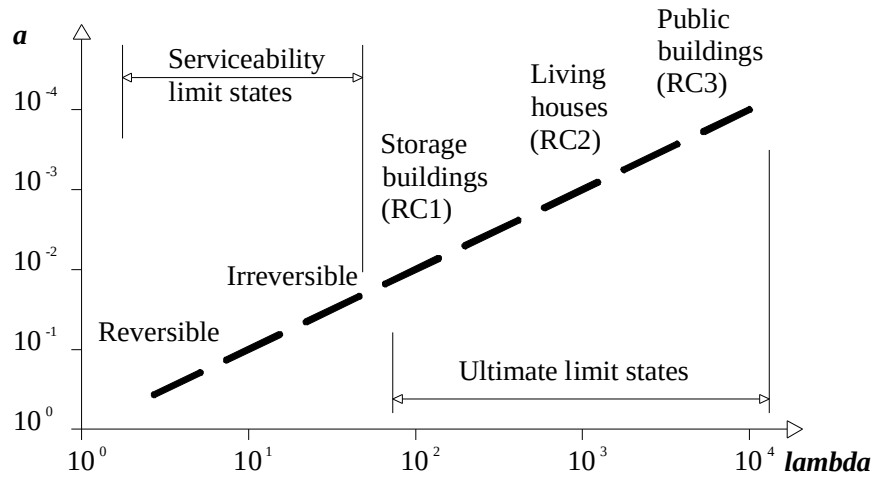


Fig. 1.1.2 The α limit state probability dependence on $\lambda = C_\alpha \alpha / C_{0,A}$ relative damage due to limit state occurrence; C_α , α , $C_{0,A}$ – values according to (1.1.1); RC1, RC2, RC3 – element reliability classes

The standardized α values dependence on λ relative damage due to limit state occurrence for simple elements, for example beams, columns is shown in Fig. 1.1.2. The reliability differentiation from design values design codes is shown there, which is performed applying different partial safety factors for loads and for materials. Often these coefficients for serviceability limit state group are equal to unit and for ultimate limit state group coefficients are larger than unit, they are within the interval $1.15 \div 1.5$. So, using these factors the average serviceability and safety reliability differentiation is done. The reliability difference for RC1, RC2, RC3 elements reliability classes, which depend on element importance, is obtained applying additionally 0.9, 1.0, 1.1 K_f coefficient.

It is determined that $\alpha_z \approx 10\alpha_A$, see Fig. 1.1.3 (a). Moreover, when increasing the element factor A , for example cross-sectional area, by 10%, the α_A for ordinary elements decreases approximately by 30%. And when α_z value is not changed (because the α_z does not depend on α_A , but depends on human mistakes control and application of preventive measures), the α sum of limit state probability decreases only about 3% according to (1.1.1), i.e. changes of α value are negligibly small (see Fig. 1.1.3, (b)). So, the wide spread opinion, that it is possible to decrease the limit state probability by increasing the element cross-sectional area, is wrong. Another method of decreasing the α limit state probability sum is more effective – it is the decreasing of α_z value. It

is determined that if the funds, allocated for 10 % increment of factor A , are addressed for decrease of α_z , for example by strengthening human mistakes control and preventive measures at all structure construction and exploitation stages, the value of α_z will decrease about 40 %. As the result, the α sum of limit state probability will decrease about 30 %, i.e. it will decrease 10 times more than in alternative case of decrease of α_A factor (see Fig. 1.1.3, (b) and (c)).

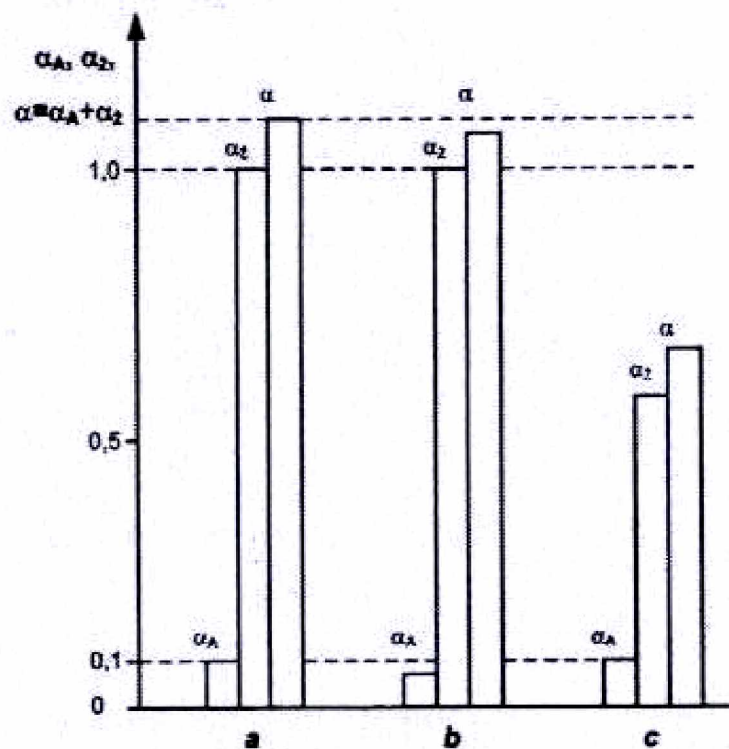


Fig. 1.1.3 The component parts of limit state risk: α_A or α_z – depending on structural material consumption, for example on cross-sectional area A and on applying human errors management and preventive measures; α – the sum according to (1.1.1) $\alpha = \alpha_A + \alpha_z$; *a* – standard environmental conditions, *b* – when the funds are directed to increase the cross-sectional area A , *c* – when the same funds are directed to make the human errors management and preventive measures stricter

Table 1.1.2 The data of annual fatalities calculated for 3.4 mln citizens

№	Type of fatality	In West countries	In Lithuania (for 2005 year)
1	Road accidents	918	892
2	Fallings	340	592
3	Fire and explosions	126	207
4	Accidents in water	98	389
5	Lightning stroke	0.3	
6	Poisonous envenomations	0.2	
7	Structure collapse during exploitation	0.7	0.6
8	Diseases (human of 15-45 age)	2470	3840
9	Murder		314
10	Suicide		1317

It is assumed, that the variable load partial safety factor should be increased up to 1.5 instead of the value of 1.3 restricted in STR code. But only developed EU countries are able to afford this. The example could be Sweden, but in Sweden design codes for all variable loads the partial safety factor is 1.3 and this is explained so, that to increase this coefficient it will cost for Sweden several hundred millions euro. Taking into account the small influence of component α_A of (1.1.1) expression decrement to the total element reliability and a relatively small number of human fatalities in structures in comparison with fatalities from other accidents (see Table 1.1.2), there was selected the same coefficient 1.3 in STR as in Sweden codes. It was considered to pay more attention to decrease of the component α_z in (1.1.1) expression, i.e. to strengthen human mistakes control and preventive measures.

1.2 THE ECONOMICAL OPTIMIZATION OF SAFETY AND SERVICEABILITY RELIABILITY ACCORDING TO R. RACKWITZ

It is proposed [2] to use optimization for assessing targets with special reference to earthquake resistant design. In particular, a distinction was made whether failure would occur upon construction or never and at “large random disturbances” only. A second distinction was made with respect to the reconstruction policy. In the extremes there are just two: no reconstruction after failure and systematic reconstruction or repair after failure, respectively. Whereas it is true that both types of failure should generally be considered depending on the type of loading on the structure the matter of reconstruction policy was apparently overlooked in the past. In fact, for almost all civil engineering structures systematic rebuilding after failure, be it caused by extreme loading, bad construction, fatigue, other deterioration, loss of serviceability, or by demolition after obsolesce, is mandatory, at least ideally because buildings serve the user and society. Optimization for one “mission” is thinkable for certain construction operations only.

A suitable optimization scheme will be proposed. It will be demonstrated that failure rates rather than time-dependent failure probabilities are the basis for setting up safety targets. Important conclusions will be drawn with respect to code making, in particular when assessing safety targets and when deriving partial safety factors for practical use.

According to *Rosenblueth and Hasofer's treatment* it is assumed that the objective function of a structural component is

$$Z(p) = B(p) - C(p) - D(p), \quad (1.2.1)$$

$B(p)$ – the benefit from the existence of the structure,

$C(p)$ – the construction cost,

$D(p)$ – the expected damage cost,

p – a design parameter vector.

Without loss of generality all quantities will be measured in monetary units. Statistical decision theory dictates that the expected values for $B(p)$, $C(p)$ and $D(p)$ have to be taken. $B(p)$, in general, will be unaffected or slightly decrease with each component of p but this will be neglected

without substantial error so that $B = B(p)$. $C(p)$ increases with each component of p under normal circumstances. Frequently, it can be approximated by $C(p) \approx C_0 + \sum c_i p_i$. C_0 are those costs which do not depend on p . In general, there is $C_0 \gg \sum c_i p_i$. $D(p)$ decreases with p in some fashion. For each involved party, i.e. the builder, the user and the society, $Z(p)$ should be positive. Otherwise one should not undertake the realization of the structure. This is illustrated in Fig. 1.2.1. Benefits, cost and damages are not necessarily the same for all involved parties. Therefore, the intersection of the domains where $Z(p)$ is positive is the domain of p , which makes sense for all parties.

In general, the cost components have to be taken into account in an overall economical consideration such as violations of the serviceability conditions and fatigue and other deterioration usually implying immediate repair but also obsolesce. Also, many structures are inspected more or less regularly. For simplicity the objective function now is written as

$$Z(p) = B^* - C(p) - I(p) - U(p) - M(p) - A(p) - D(p), \quad (1.2.2)$$

where

$U(p)$ – is associated with serviceability failures,

$M(p)$ – with fatigue and other aging failure,

$A(p)$ – with obsolesce,

$D(p)$ – the expected cost for ultimate limit state failure. It is assumed that different failure types do not interact,

$I(p)$ – all inspection and resulting maintenance cost.

First of all, the optimal solution for building facilities with or without a systematic rebuilding policy is based on failure intensities and not on time-dependent failure probabilities. It is neither necessary to define arbitrary reference times of intended use nor is it necessary to undertake the complicated task to compute first passage time distributions. No table of recommended reference times of usage of structures is needed. The same targets, in terms of failure rates, can be set for temporary structures and monumental buildings, given the same marginal cost for reliability and failure consequences. Nevertheless, it is necessary to define a time

unit. For civil engineering facilities this is no doubt one year in consideration of the length of their life cycles. Other choices are possible provided that the failure intensities are small and much smaller than the interest rate. Also, the optimum design parameters are independent of assumed, highly variable lifetimes. This does not mean that lifetime aspects, especially in case of fatigue and other deterioration, are ignored. Here, design must be directly for mean failure times which are sufficient to derive the corresponding asymptotic renewal densities to be checked against target failure rates.

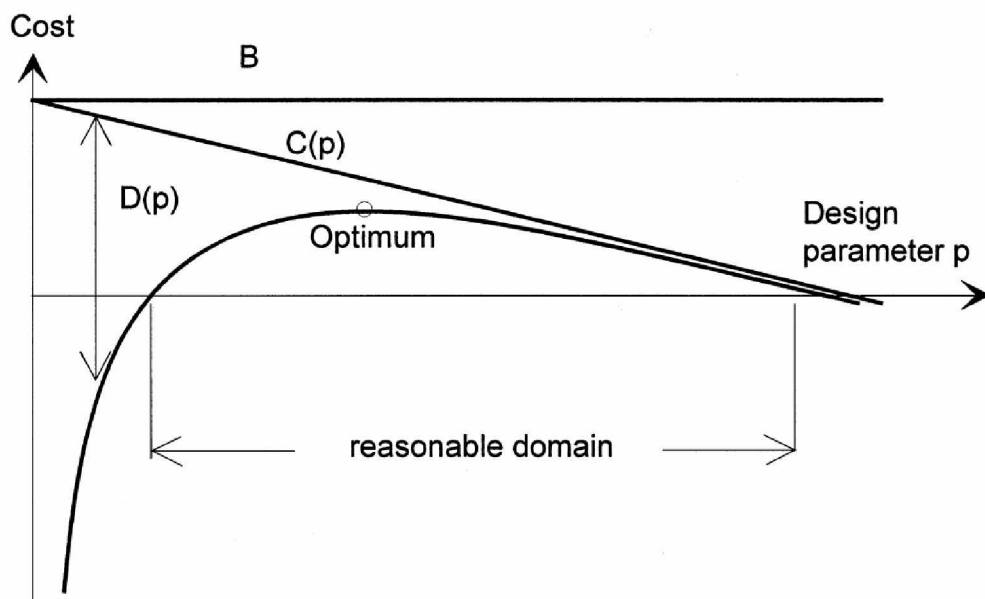


Fig. 1.2.1 Cost and benefit over design parameter p

On the basis of optimization studies an attempt is made to design a table of target values to be used in practice as an alternative to similar tables in codes.

The proposed values agree with values proposed in the industry with high damage potential and elsewhere for accidental situations (for example, the so-called safe shut down earthquake has a return period of 10 000 years and the resulting return period of failure will be yet smaller). The “no safe shut down situation” would require a still smaller failure rate. For normal office, housing and industrial buildings the marginal cost for reliability are very small. One therefore could argue to lower the failure rate down to 10^{-5} . The targets for serviceability limit states appear rather conservative. Present practice appears to accept larger risks. The values are also compatible with

the yearly risk for life and limb set in ISO 2394 as $P < 10^{-6} / P(D|F)$ where $P(D|F)$ is the probability of a person being killed given that the structure collapses. The values in the table have been set for structural components (failure modes). For structural systems they are meant for the structural component dominating structural failure. If there are many components of the same importance or many equally likely failure modes with comparable failure consequences the values in the table have to be suitably lowered.

Table 1.2.1 Proposed target failure rates

	Safety limit state failure		
	Expected failure consequences		
Relative effort to achieve reliability	Insignificant	Normal	Large
High	2.3 (10^{-2})	3.1 (10^{-3})	3.7 (10^{-4})
Normal	3.1 (10^{-3})	3.7 (10^{-4})	4.3 (10^{-5})
Low	3.7 (10^{-4})	4.3 (10^{-5})	4.7 (10^{-6})
Serviceability limit state failure			
High	1.3 (10^{-1})		
Normal	1.7 ($5 \cdot 10^{-2}$)		
Low	2.3 (10^{-2})		

Others than failures by “rare disturbances” include serviceability failures, fatigue failures and ultimate limit state failures but also obsolesce. It is proposed that for almost all civil engineering facilities the only reasonable reconstruction policy is systematic rebuilding or repair. It is found that the failure rate (failure intensity, renewal density) is the decisive criterion for setting up safety/reliability targets for high reliability structures. This is approximately true also for “one mission” structures. Structures should be optimal. A general objective function is proposed. Tools for its maximization in a first order sense are presented. The principles of reliability verification but also of reliability-based design are illustrated. Given codified admissible failure rates, for example, related to one year, it is possible to derive partial safety factors so that a full probabilistic analysis is in complete correspondence with a deterministic partial safety factor design. An exception is fatigue and other deterioration where a more direct approach is necessary.

1.3 THE PROBABILISTIC MEANING OF CHARACTERISTIC AND DESIGN VALUES OF PERMANENT AND VARIABLE LOADS

To insure the needed marginal resistance of structural elements, the smallest design values of material or soil strength and the largest design values of loads are used.

The variable load is represented not only by characteristic, but also by other *representative* values in designing and evaluating of safety and serviceability limit states. The graphical interpretation of these values of the loading is shown in Fig. 1.3.1. The F_k characteristic value is the main representative value of the load. The other representative values are determined by multiplying F_k by ψ_0, ψ_1, ψ_2 , which values are less than unit. The *combinative* value represents the maximum of action effect occurrence of combination of two and more variable loads of the same probability as one load. The *frequentative* value represents such value of variable load, which the structure would be acted during the greatest part of T exploitation period, and the value larger than the frequentative value would act small part of T period or its exceeding frequency would be limited up to determined value. The *quasi-permanent* load represents such variable load, which is acting the largest part of T exploitation design period. The mentioned representative values are determined, i.e. the coefficients ψ_0, ψ_1, ψ_2 are selected in codes, by the probabilistic analysis of E action effect from one variable load and from combination of several variable loads.

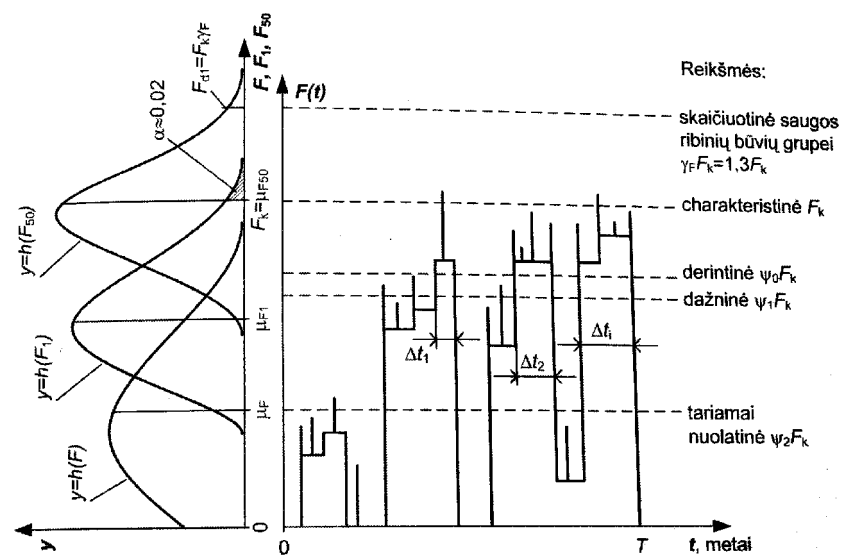


Fig. 1.3.1 The graphical interpretation of loading $F(t)$ values

2 CALCULATION OF ULTIMATE DEFLECTIONS OF REINFORCED CONCRETE ELEMENTS USING DETERMINISTIC MODELS OF DIFFERENT DESIGN CODES

2.1 INTRODUCTORY REMARKS

The aim of this chapter is to calculate deflection of bending reinforced concrete beam under the action of distributed load according to STR 2.05.05:2005 [3], EN1992-1:2004 [4], ACI318-2005 [5], SP 52-101-2003 [6] design codes. The obtained results will further be used for determination of flexural reinforced concrete element reliability of stiffness. The following iterative scheme will be applied:

Step 1

The permanent and variable load values are selected. The necessary condition is that the ratio of permanent to variable load is $g_k / q_k = 1$. Then according to the certain code the normal section strength condition is checked and the reinforcement minimal cross-sectional area is determined. Because of the system and random errors of calculations offered in design codes of different countries, the required reinforcement area calculated using two different design codes can differ even in case when the loading values are the same. The structural scheme of element under investigation is shown in Fig. 2.1.1.

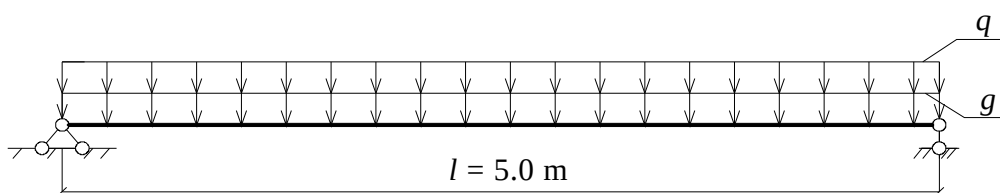


Fig. 2.1.1 The beam structural scheme when $g_k / q_k = 1$

Step 2

Because the element is quite short, it can be assumed another way of how the failure may develop. It is possible that the reinforced concrete beam under the bending action of distributed load will crack before the ultimate deflection will be reached. The cracking moment needs to be

determined and the condition of cracks occurrence has to be checked. However, the section will probably be cracked.

Step 3

The previous step results will determine the further calculations. If the section is uncracked, then there will be one way of deflection determination, and if the section is cracked, then the deflection calculation procedure becomes more difficult. The deflection is calculated according to the selected procedure. The value of deflection has to be such, that the difference between the ultimate deflection w_{ult} and calculated deflection w will be negligibly small ($\pm 2\%$). Otherwise the procedure continues in *Step 1* and using iteration the values of loads g_k and q_k are chosen.

2.2 DESIGN ACCORDING TO STR 2.05.05:2005 CODE

Initial data

The 5 meter-long beam is simply supported. The cross-section is: $h = 360 \text{ mm}$, $b = 200 \text{ mm}$. Concrete class is C20/25, reinforcement class is S400. The material properties are selected according to the requirements of [3] Code, for the further calculations will be also used [7] Code. The structural scheme of the element is shown in Fig. 2.1.1.

Characteristics of concrete C20/25 ([3], Chapter VII, subchapter I, Table 5):

- $E_{cm} = 30 \text{ GPa}$ – modulus of elasticity of concrete;
- $f_{ck} = 20 \text{ MPa}$ – characteristic value of compressive strength of concrete;
- $f_{cd} = 12 \text{ MPa}$ – design value of compressive strength of concrete.

Characteristics of reinforcement S400 ([3], Chapter VII, subchapter II, Table 8):

- $E_s = 200 \text{ GPa}$ – modulus of elasticity of steel;
- $f_{yk} = 400 \text{ MPa}$ – characteristic value of tensile strength of steel;
- $f_{yd} = 365 \text{ MPa}$ – design value of tensile strength of steel.

Effective beam depth is $d = 333 \text{ mm}$. Characteristic value of permanent load is $g_k = 5 \text{ kN / m'}$, characteristic value of variable load is $q_k = 5 \text{ kN / m'}$.

Design values of loadings ([7], Appendix 10, Table 2):

- $g_d = g_k \cdot \gamma_G = 5 \cdot 1.35 = 6.75 \text{ kN / m'}$;
- $q_d = q_k \cdot \gamma_Q = 5 \cdot 1.3 = 6.5 \text{ kN / m'}$.

- γ_G – partial safety factor for permanent load;
- γ_Q – partial safety factor for variable load.

2.2.1 CHECKING THE STRENGTH CONDITION

The selection of longitudinal reinforcement in rectangular section is performed using the following algorithm. When the longitudinal reinforcement is in tensile zone, then $A_{s2} = 0$.

The design concrete strength ([3], Chapter VII, subchapter I, expression 7.4):

$$f_{cd} = \alpha \cdot \alpha_{cc} \cdot f_{ck} / \gamma_c = 0.9 \cdot 1 \cdot \frac{20}{1.5} = 12 \text{ MPa};$$

α_{cc} and α – coefficients:

$$\alpha_{cc} = 1.0,$$

$\alpha = 0.9$ – when in the structure the rectangular shape stress distribution diagram is evaluated and characteristic strength of the concrete is ≤ 50 MPa,

f_{ck} – characteristic strength of the concrete,

γ_c – reliability coefficient. For reinforced concrete structures $\gamma_c = 1.5$.

Determination of ξ_{lim} . For this calculate the characteristics of the concrete compressed zone ω (Chapter VIII, subchapter II, Paragraph 72):

$$\omega = a - 0.008 f_{cd} = 0.85 - 0.008 \cdot 12 = 0.754;$$

a – coefficient evaluating the type of the concrete. For heavy concrete $a = 0.85$,

f_{cd} – design strength of the concrete.

Determination of relative height ξ_{lim} of the element compressed zone:

$$\xi_{lim} = \frac{\omega}{1 + \frac{\sigma_{s,lim}}{\sigma_{sc,lim}} \left(1 - \frac{\omega}{1.1} \right)} = \frac{0.754}{1 + \frac{365}{400} \left(1 - \frac{0.754}{1.1} \right)} = 0.586;$$

$\sigma_{s,lim}$ – stresses in the reinforcement taking into account yielding strength of the reinforcement,

$\sigma_{sc,lim}$ – limit stresses of the reinforcement in the compressed zone,

use $\sigma_{sc,lim} = 500 \text{ MPa}$.

The strength condition has to be satisfied (Chapter VIII, subchapter III):

$$M_{Rd} = f_{cd} \cdot b \cdot d^2 \cdot \mu_{Ed} \geq M_{Ed}.$$

The design value of the bending moment:

$$M_{Ed} = \frac{(g_d + q_d) \cdot l^2}{8} = \frac{(6.75 + 6.5) \cdot 5.0^2}{8} = 41.4 \text{ kNm} = 41.4 \cdot 10^{-3} \text{ MNm}.$$

The coefficient evaluating the compression zone of the cross-section:

$$\mu_{Ed} = \frac{M_{Ed}}{f_{cd} b d^2} = \frac{41.4 \cdot 10^{-3}}{12 \cdot 0.200 \cdot 0.333^2} = 0.156.$$

Determination of relative height ξ of the element compressed zone:

$$\xi_{eff} = \frac{x_{eff}}{d} = 1 - \sqrt{1 - 2\mu_{Ed}} = 1 - \sqrt{1 - 2 \cdot 0.156} = 0.170.$$

Because the condition $\xi_{eff} \leq \xi_{lim}$ is satisfied, then

$$f_{yd} A_{s1} = f_{cd} b d \xi_{eff};$$

$$A_{s1} = \frac{f_{cd} b d \xi_{eff}}{f_{yd}} = \frac{12 \cdot 0.200 \cdot 0.333 \cdot 0.170}{365} = 3.72 \cdot 10^{-4} \text{ m}^2.$$

I take 3Ø14 ($A_{s1} = 462 \text{ mm}^2 = 4.62 \cdot 10^{-4} \text{ m}^2$).

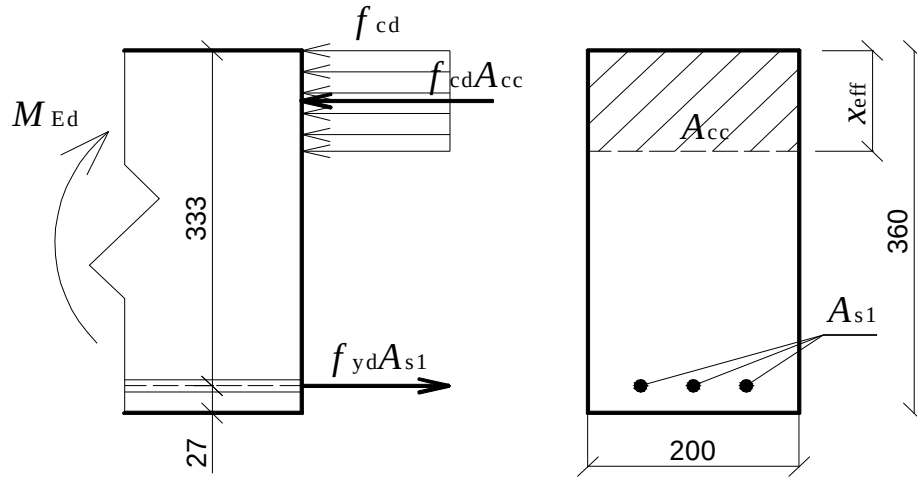


Fig. 2.2.1 The structural scheme of the normal section strength of bending element of the rectangular cross-section

The strength of the element is enough, when the condition $M_{Ed} \leq M_{Rd}$ is satisfied. It is taken that the element is not under-reinforced, if the condition $\xi_{eff} \leq \xi_{lim}$ is satisfied, then its strength is calculated so:

$$M_{Ed} \leq f_{cd} \cdot b \cdot x_{eff} (d - 0.5x_{eff}).$$

The depth of the compressed zone x_{eff} is calculated so:

$$f_{yd} A_{s1} = f_{cd} b \cdot x_{eff};$$

$$x_{eff} = \xi_{eff} d = \frac{f_{yd} \cdot A_{s1}}{f_{cd} \cdot b} = \frac{365 \cdot 462 \cdot 10^{-6}}{12 \cdot 0.200} = 0.0703 \text{ m};$$

$$f_{cd} \cdot b \cdot x_{eff} (d - 0.5x_{eff}) = 12 \cdot 0.200 \cdot 0.0703 \cdot (0.333 - 0.5 \cdot 0.0703) = 0.0503 \text{ MN}.$$

50.3 kN > 41.4 kN it means $M_{Rd} > M_{Ed}$.

Because the resistance of the section is larger than the load action, the strength is enough.

2.2.2 CHECKING THE CRACKING CONDITION

The condition $M > M_{cr}$ is checked according to [3], Chapter XIV, Subchapter I. If it is satisfied, it means the element will crack.

M – the design value of the bending moment acting in the section,

M_{cr} – the cracking moment.

Geometrical characteristics of the effective cross-section:

$$A_{eff} = A_c + \alpha_e A_{s1} = 0.200 \cdot 0.360 + 6.67 \cdot 4.62 \cdot 10^{-4} = 0.0751 \text{ m}^2;$$

$$\alpha_e = \frac{E_s}{E_{cm}} = \frac{200 \cdot 10^3}{30 \cdot 10^3} = 6.67;$$

$$S_{eff} = 0.200 \cdot 0.360 \cdot 0.180 + 6.67 \cdot 4.62 \cdot 10^{-4} \cdot 0.027 = 0.013 \text{ m}^3.$$

The distance from the bottom to centre of gravity:

$$y_{sc} = \frac{S_{eff}}{A_{eff}} = \frac{0.013}{0.0751} = 0.174 \text{ m}.$$

The distance from the reinforcement to centre of gravity:

$$y_{s1} = 0.174 - 0.027 = 0.147 \text{ m},$$

$$I_{eff} = I_c + \alpha_e \cdot A_{s1} \cdot y_{s1}^2 = \frac{0.200 \cdot 0.360^3}{12} + 6.67 \cdot 4.62 \cdot 10^{-4} \cdot 0.147^2 = 8.44 \cdot 10^{-4} \text{ m}^4.$$

Cracking moment

$$M_{cr} = W_{pl} f_{ctk},$$

$$W_{pl} = \gamma \cdot W_{eff} = 1.75 \cdot \frac{I_{eff}}{y_{sc}} = 1.75 \cdot \frac{8.44 \cdot 10^{-4}}{0.174} = 8.49 \cdot 10^{-3} \text{ m}^3.$$

$$\text{Then } M_{cr} = 8.49 \cdot 10^{-3} \cdot 1.5 = 0.0127 \text{ MNm} = 12.7 \text{ kNm}.$$

$41.4 \text{ kN} > 12.7 \text{ kNm}$ it means $M_r > M_{cr}$. This means that the cracks will appear in the tensile zone.

2.2.3 DETERMINATION OF LOAD CORRESPONDING TO ULTIMATE DEFLECTION

w_{ult}

The curvature of the bending element, in case when in the tensile zone there are cracks ([3], Chapter XIV, Subchapter IV):

$$\frac{1}{r} = \frac{M}{d z} \left[\frac{\psi_s}{E_s A_{s1}} + \frac{\psi_c}{(\phi_f + \xi) b d E_{c,m} \nu} \right] - \frac{N_{tot}}{d} \cdot \frac{\psi_s}{E_s A_{s1}},$$

M – the bending moment from the loads about the centre of tensile reinforcement;

$$M = \pm M_1 \pm P_d \cdot e_{sp}.$$

z – the lever arm of the forces;

ψ_s – the coefficient which takes into account the work of the compressed concrete in the cracked zone;

ψ_c – the coefficient which takes into account the compressed concrete layer deformations and it is equal 1.0 for all concrete classes;

ξ – relative height ξ_{lim} of the element compressed zone;

ν – the factor, which evaluate compressed concrete elasto-plastic deformations, $\nu = 0.45$ ([3], Chapter XIV, Subchapter IV, Table 26);

N_{tot} – axial forces;

$$\frac{N_{tot}}{d} \cdot \frac{\psi_s}{E_s A_{s1}} = 0 \text{ – for flexural elements.}$$

The total curvature of the element when the tensile zone is cracked:

$$\frac{1}{r} = \left(\frac{1}{r} \right)_1 - \left(\frac{1}{r} \right)_2 + \left(\frac{1}{r} \right)_3,$$

here

$\left(\frac{1}{r} \right)_1$ – the curvature from the action of short term loadings;

$\left(\frac{1}{r}\right)_2$ – the curvature from the action of short term permanent and quasi-permanent loadings;

$\left(\frac{1}{r}\right)_3$ – the curvature from the action of long term permanent and quasi-permanent loadings.

The curvature from the action of short term loadings

$$\begin{aligned}\left(\frac{1}{r}\right)_1 &= \frac{M}{d \cdot z} \left[\frac{\psi_s}{E_s \cdot A_{p1}} + \frac{\psi_c}{(\varphi_f + \xi) b_w \cdot d \cdot E_{cm} \cdot v} \right] = \\ &= \frac{0.414 \cdot 10^8}{333 \cdot 299.5} \left[\frac{0.942}{2.00 \cdot 10^5 \cdot 462} + \frac{0.9}{(0 + 0.201) \cdot 200 \cdot 333 \cdot 3.0 \cdot 10^4 \cdot 0.45} \right] = \\ &= 415.1 \cdot [1.0195 \cdot 10^{-8} + 4.980 \cdot 10^{-9}] = 6.3 \cdot 10^{-6} \frac{1}{\text{mm}},\end{aligned}$$

$\psi_c = 0.9$ – the coefficient which takes into account the compressed concrete layer deformations ([3], Chapter XIV, Subchapter IV, Paragraph 184);

$v = 0.45$ – the factor, which evaluate compressed concrete elasto-plastic deformations.

The design value of bending moment:

$$M_{Ed} = \frac{(g_d + q_d) \cdot l^2}{8} = \frac{(6.75 + 6.5) \cdot 5.0^2}{8} = 41.4 \text{ kNm} = 41.4 \cdot 10^{-3} \text{ MNm}.$$

Determination of relative height ξ_{lim} of the element compressed zone:

$$\xi = \frac{1}{\beta + \frac{1+5(\delta+\lambda)}{10 \cdot \rho \cdot \alpha_e}} = \frac{1}{1.8 + \frac{1+5 \cdot (0.0933+0)}{10 \cdot 0.00694 \cdot 6.67}} = 0.201,$$

where

$\beta = 1.8$ ([3], Chapter XIV, Subchapter IV, Paragraph 185).

The reinforcement factor of the section:

$$\rho = \frac{A_{s1}}{(b \cdot d)} = \frac{4.62 \cdot 10^{-4}}{(0.200 \cdot 0.333)} = 0.00694.$$

The coefficient δ :

$$\delta = \frac{M}{b_w \cdot d^2 \cdot f_{ck}} = \frac{0.0414}{0.200 \cdot 0.333^2 \cdot 20} = 0.0933.$$

The coefficients φ_f and λ :

$$\varphi_f = \frac{\left((b_{eff} - b_w) h_f + \frac{\alpha_e}{2\nu} A_{s2} \right)}{b_w \cdot d} = \frac{\left((b_{eff} - b_w) \cdot 0 + \frac{\alpha_e}{2\nu} \cdot 0 \right)}{b_w \cdot d} = 0,$$

$$\lambda = \varphi_f \left(1 - \frac{h_f}{2d} \right) = 0 \cdot \left(1 - \frac{0}{333} \right) = 0.$$

The lever arm of the forces:

$$z = d \left[1 - \frac{\frac{h_f}{d} \cdot \varphi_f + \xi^2}{2(\varphi_f + \xi)} \right] = 333 \cdot \left[1 - \frac{\frac{0}{333} \cdot 0 + 0.201^2}{2(0 + 0.201)} \right] = 299.5 \text{ mm}.$$

The factor which takes into account the tensile reinforcement deformations:

$$\psi_s = 1.25 - \varphi_{as} \cdot \varphi_m = 1.25 - 1 \cdot 0.308 = 0.942 < 1.0, \text{ I take } \psi_s = 0.942,$$

where

$$\varphi_{\ell s} = 1.0,$$

$$\varphi_m = \frac{f_{ctk} \cdot W_{pl1}}{M_r - M_{rp}} = \frac{1.5 \cdot 8.49 \cdot 10^{-3}}{0.0414} = 0.308.$$

The curvature from the action of short term permanent and quasi-permanent loadings

$$\begin{aligned}\left(\frac{1}{r}\right)_2 &= \frac{M}{d \cdot z} \left[\frac{\psi_s}{E_s \cdot A_{p1}} + \frac{\psi_c}{(\varphi_f + \xi) b_w \cdot d \cdot E_{cm} \cdot v} \right] = \\ &= \frac{0.333 \cdot 10^8}{333 \cdot 298} \left[\frac{0.868}{2.00 \cdot 10^5 \cdot 462} + \frac{0.9}{(0 + 0.210) \cdot 200 \cdot 333 \cdot 3.00 \cdot 10^4 \cdot 0.45} \right] = \\ &= 335.57 \cdot [9.394 \cdot 10^{-9} + 4.7667 \cdot 10^{-9}] = 4.75 \cdot 10^{-6} \frac{1}{\text{mm}}.\end{aligned}$$

The design value of bending moment from quasi-permanent load:

$$M = M_{Egd} = \frac{(g_d + \psi_2 \cdot q_d) \cdot l^2}{8} = \frac{(6.75 + 0.6 \cdot 6.5) \cdot 5.0^2}{8} = 33.3 \text{ kNm};$$

$d = 333 \text{ mm}$ – effective height of the element cross-section;

$\psi_c = 0.9$ – the coefficient which takes into account the compressed concrete layer deformations;

$v = 0.45$ – the factor, which evaluate compressed concrete elasto-plastic deformations.

Determination of relative height ξ_{lim} of the element compressed zone:

$$\xi = \frac{1}{\beta + \frac{1+5(\delta + \lambda)}{10 \cdot \rho \cdot \alpha_e}} = \frac{1}{1.8 + \frac{1+5 \cdot (0.0751+0)}{10 \cdot 0.00694 \cdot 6.67}} = 0.210,$$

$$\beta = 1.8.$$

The reinforcement factor of the section:

$$\rho = \frac{A_{s1}}{(b \cdot d)} = \frac{4.62 \cdot 10^{-4}}{(0.200 \cdot 0.333)} = 0.00694.$$

The coefficient δ :

$$\delta = \frac{M}{b_w \cdot d^2 \cdot f_{ck}} = \frac{0.0333}{0.200 \cdot 0.333^2 \cdot 20} = 0.0751.$$

The coefficients φ_f and λ :

$$\varphi_f = \frac{\left((b_{eff} - b_w) h_f + \frac{\alpha_e}{2\nu} A_{s2} \right)}{b_w \cdot d} = \frac{\left((b_{eff} - b_w) \cdot 0 + \frac{\alpha_e}{2\nu} \cdot 0 \right)}{b_w \cdot d} = 0,$$

$$\lambda = \varphi_f \left(1 - \frac{h_f}{2d} \right) = 0 \cdot \left(1 - \frac{0}{333} \right) = 0.$$

The lever arm of the forces:

$$z = d \left[1 - \frac{\frac{h_f}{d} \cdot \varphi_f + \xi^2}{2(\varphi_f + \xi)} \right] = 333 \cdot \left[1 - \frac{\frac{0}{333} \cdot 0 + 0.210^2}{2(0 + 0.210)} \right] = 298 \text{ mm}.$$

The factor which takes into account the tensile reinforcement deformations:

$$\psi_s = 1.25 - \varphi_{as} \cdot \varphi_m = 1.25 - 1 \cdot 0.382 = 0.868 < 1.0, \text{ I take } \psi_s = 0.868,$$

where

$$\varphi_{\ell s} = 1.0,$$

$$\varphi_m = \frac{f_{ctk} \cdot W_{pl1}}{M_r - M_{rp}} = \frac{1.5 \cdot 8.49 \cdot 10^{-3}}{0.0333} = 0.382.$$

The curvature from the action of long term permanent and quasi-permanent loadings

$$\begin{aligned} \left(\frac{1}{r} \right)_3 &= \frac{M}{d \cdot z} \left[\frac{\psi_s}{E_s \cdot A_{p1}} + \frac{\psi_c}{(\varphi_f + \xi) b_w \cdot d \cdot E_{cm} \cdot \nu} \right] = \\ &= \frac{0.333 \cdot 10^8}{333 \cdot 298} \left[\frac{0.868}{2.00 \cdot 10^5 \cdot 462} + \frac{0.9}{(0 + 0.210) \cdot 200 \cdot 333 \cdot 3.00 \cdot 10^4 \cdot 0.15} \right] = \\ &= 335.57 \cdot [9.394 \cdot 10^{-9} + 1.43 \cdot 10^{-8}] = 7.95 \cdot 10^{-6} \frac{1}{\text{mm}}, \end{aligned}$$

$d = 333 \text{ mm}$ – effective height of the element cross-section;

$\psi_c = 0,9$ – the coefficient which takes into account the compressed concrete layer deformations;

$\nu = 0.15$ – the factor, which evaluate compressed concrete elasto-plastic deformations.

The design value of bending moment from quasi-permanent load:

$$M = M_{Egd} = \frac{(g_d + \psi_2 \cdot q_d) \cdot l^2}{8} = \frac{(6.75 + 0.6 \cdot 6.5) \cdot 5.0^2}{8} = 33.3 \text{ kNm}.$$

Determination of relative height ξ_{lim} of the element compressed zone:

$$\xi = \frac{1}{\beta + \frac{1+5(\delta+\lambda)}{10 \cdot \rho \cdot \alpha_e}} = \frac{1}{1.8 + \frac{1+5 \cdot (0.0751+0)}{10 \cdot 0.00694 \cdot 6.67}} = 0.210,$$

$$\beta = 1.8.$$

The reinforcement factor of the section:

$$\rho = \frac{A_{s1}}{(b \cdot d)} = \frac{4.62 \cdot 10^{-4}}{(0.200 \cdot 0.333)} = 0.00694.$$

The coefficient δ :

$$\delta = \frac{M}{b_w \cdot d^2 \cdot f_{ck}} = \frac{0.0333}{0.200 \cdot 0.333^2 \cdot 20} = 0.0751.$$

The coefficients φ_f and λ :

$$\varphi_f = \frac{\left((b_{eff} - b_w) h_f + \frac{\alpha_e}{2\nu} A_{s2} \right)}{b_w \cdot d} = \frac{\left((b_{eff} - b_w) \cdot 0 + \frac{\alpha_e}{2\nu} \cdot 0 \right)}{b_w \cdot d} = 0;$$

$$\lambda = \varphi_f \left(1 - \frac{h_f}{2d} \right) = 0 \cdot \left(1 - \frac{0}{333} \right) = 0.$$

The lever arm of the forces:

$$z = d \left[1 - \frac{\frac{h_f}{d} \cdot \varphi_f + \xi^2}{2(\varphi_f + \xi)} \right] = 333 \cdot \left[1 - \frac{\frac{0}{333} \cdot 0 + 0.210^2}{2(0 + 0.210)} \right] = 298 \text{ mm.}$$

The factor which takes into account the tensile reinforcement deformations:

$$\psi_s = 1.25 - \varphi_{s_s} \cdot \varphi_m = 1.25 - 1 \cdot 0.382 = 0.868 < 1.0, \text{ I take } \psi_s = 0.868,$$

where

$$\varphi_{\ell s} = 1.0,$$

$$\varphi_m = \frac{f_{ctk} \cdot W_{pl1}}{M_r - M_{rp}} = \frac{1.5 \cdot 8.49 \cdot 10^{-3}}{0.0333} = 0.382.$$

The total curvature of the element

$$\frac{1}{r} = \left(\frac{1}{r} \right)_1 - \left(\frac{1}{r} \right)_2 + \left(\frac{1}{r} \right)_3 = 6.3 \cdot 10^{-6} - 4.75 \cdot 10^{-6} + 7.95 \cdot 10^{-6} = 9.5 \cdot 10^{-6} \frac{1}{\text{mm}}.$$

The deflection of the beam is:

$$w_m = \left(\frac{1}{r} \right) \cdot \frac{5}{48} \cdot l^2 = 9.5 \cdot 10^{-6} \cdot \frac{5}{48} \cdot 5000^2 = 24.7 \text{ mm.}$$

The limited deflection of the beam (beam length $l = 5000 \text{ mm}$):

$$w_{ult} = \frac{l}{200} = \frac{5000}{200} = 25 \text{ mm.}$$

The deflection of the beam due to bending moment w_m is less than the limited deflection w_{ult} . The serviceability limit state condition is fulfilled.

2.3 DESIGN ACCORDING TO EN 1992-1:2004 CODE

Initial data

The 5 meter-long beam is simply supported. The cross-section is: $h = 360$ mm, $b = 200$ mm. Concrete class is C20/25, reinforcement class is S400. The material properties are selected according to the requirements of [4] Code, for the further calculations will be also used [8, 9]. The structural scheme of the element is shown in Fig. 2.1.1.

Characteristics of concrete C20/25 ([4], Section 3, Table 3.1):

- $E_{cm} = 29 \text{ GPa}$ – modulus of elasticity of concrete;
- $f_{ck} = 20 \text{ MPa}$ – characteristic value of compressive strength of concrete;
- $f_{cd} = 13.3 \text{ MPa}$ – design value of compressive strength of concrete.

Characteristics of reinforcement S400 ([4], Section 3.2):

- $E_s = 200 \text{ GPa}$ – modulus of elasticity of steel;
- $f_{yk} = 400 \text{ MPa}$ – characteristic value of tensile strength of steel;
- $f_{yd} = 365 \text{ MPa}$ – design value of tensile strength of steel.

Effective beam depth is $d = 333$ mm. Characteristic value of permanent load is $g_k = 5 \text{ kN / m'}$, characteristic value of variable load is $q_k = 5 \text{ kN / m'}$.

Design values of loadings ([9], Section 4; [16], Annex A, Table A1.2):

- $g_d = g_k \cdot \gamma_G = 5 \cdot 1.35 = 6.75 \text{ kN / m'}$;
- $q_d = q_k \cdot \gamma_Q = 5 \cdot 1.5 = 7.5 \text{ kN / m'}$.

- γ_G – partial safety factor for permanent load ;
- γ_Q – partial safety factor for variable load.

2.3.1 CHECKING THE STRENGTH CONDITION

The selection of longitudinal reinforcement in rectangular section is performed using the following algorithm. When the longitudinal reinforcement is in tensile zone, then $A_{s2} = 0$.

The design concrete strength ([4], Section 3, Paragraph 3.1.6):

$$f_{cd} = \alpha_{cc} \cdot f_{ck} / \gamma_c = 1 \cdot \frac{20}{1.5} = 13.3 \text{ MPa}.$$

Coefficient $\alpha_{cc} = 1.0$,

f_{ck} – characteristic strength of the concrete, MPa,

γ_c – reliability coefficient. For reinforced concrete structures $\gamma_c = 1.5$ ([4], Section 2, Table 2.1N).

Determination of relative height ξ_{lim} of the element compressed zone (see [9], example):

$$\xi_{lim} = \frac{0.0035}{0.0035 + \frac{f_{yd}}{E_s}} = \frac{0.0035}{0.0035 + \frac{365}{200 \cdot 10^3}} = 0.657;$$

f_{yd} – design value of reinforcement yield strength;

E_s – modulus of elasticity of reinforcement.

The strength condition have to be satisfied ([4], Section 6, Part 6.1):

$$M_{Rds} = f_{cd} \cdot b \cdot d^2 \cdot \mu_{Eds} \geq M_{Eds}.$$

The design value of the bending moment:

$$M_{Eds} = \frac{(g_d + q_d) \cdot l^2}{8} = \frac{(6.75 + 7.5) \cdot 5.0^2}{8} = 44.5 \text{ kNm} = 44.5 \cdot 10^{-3} \text{ MNm}.$$

The coefficient evaluating the compression zone of the cross-section

$$\mu_{\text{Eds}} = \frac{M_{\text{Eds}}}{f_{\text{cd}} b d^2} = \frac{44.5 \cdot 10^{-3}}{13.3 \cdot 0.200 \cdot 0.333^2} = 0.150.$$

Determination of relative height ξ of the element compressed zone:

$$\xi = \frac{1}{\lambda} \left(1 - \sqrt{1 - \frac{2\mu_{\text{Eds}}}{\eta}} \right) = \frac{1}{0.8} \left(1 - \sqrt{1 - \frac{2 \cdot 0.150}{1}} \right) = 0.204.$$

λ – reduction coefficient of compression zone of the cross-section. When the characteristic compression strength of the concrete $f_{\text{ck}} \leq 50 \text{ MPa}$ then $\lambda = 0.8$;

μ_{Eds} – the coefficient evaluating the compression zone of the cross-section;

η – reduction coefficient of compression stresses acting in the cross-section. When the characteristic compression strength of the concrete $f_{\text{ck}} \leq 50 \text{ MPa}$ then $\eta = 1.0$.

Because the condition $\xi_{\text{eff}} \leq \xi_{\text{lim}}$ is satisfied, then the following equilibrium takes place:

$$f_{\text{yd}} A_{\text{s1}} = f_{\text{cd}} b d v_{\text{cd}}.$$

The coefficient describing the compression zone of the element:

$$v_{\text{cd}} = \lambda \cdot \eta \cdot \xi = 0.8 \cdot 1.0 \cdot 0.204 = 0.163.$$

The required area of the reinforcement in the tensile zone:

$$A_{\text{s1}} = \frac{f_{\text{cd}} b d v_{\text{cd}}}{f_{\text{yd}}} = \frac{13.3 \cdot 0.200 \cdot 0.333 \cdot 0.163}{365} = 3.96 \cdot 10^{-4} \text{ m}^2.$$

I take 2Ø18 ($A_{\text{s1}} = 509 \text{ mm}^2 = 5.09 \cdot 10^{-4} \text{ m}^2$).

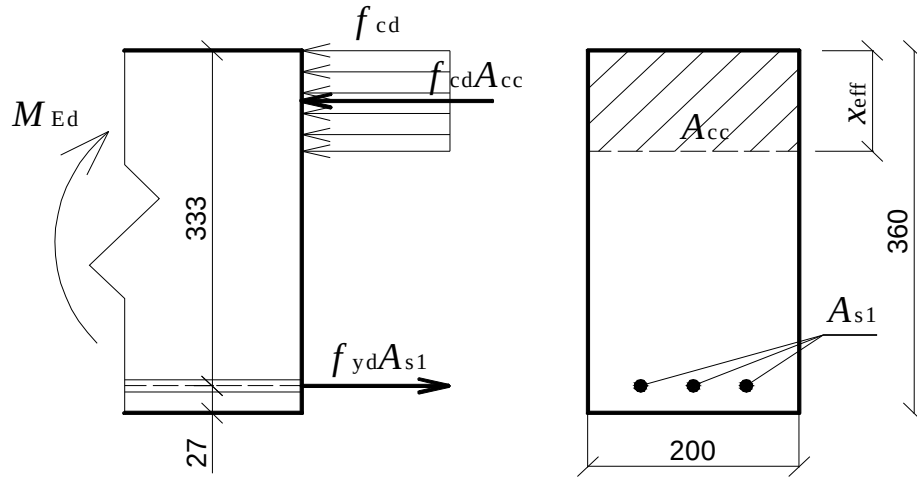


Fig. 2.3.1 The structural scheme of the normal section strength of bending element of the rectangular cross-section

The strength of the element is enough, when the condition $M_{Eds} \leq M_{Rds}$ is satisfied. It is taken that the element is not under-reinforced, if the condition $\xi \leq \xi_{lim}$ is satisfied, then its strength is calculated so:

$$M_{Rds} = f_{cd} \cdot b \cdot d^2 \cdot \left(1 - \frac{0.5 \cdot v_{cd}}{\eta}\right) \cdot v_{cd} = 13.3 \cdot 0.200 \cdot 0.333^2 \cdot \left(1 - \frac{0.5 \cdot 0.210}{1.0}\right) \cdot 0.210 = 0.0554 \text{ MNm};$$

where

$$v_{cd} = \frac{f_{yd} A_{s1}}{f_{cd} b d} = \frac{365 \cdot 5.09 \cdot 10^{-4}}{13.3 \cdot 0.200 \cdot 0.333} = 0.210;$$

$$55.4 \text{ kN} > 44.5 \text{ kN} \text{ it means } M_{Rds} > M_{Eds}.$$

Because the resistance of the section is larger than the load action, the strength is enough.

2.3.2 CHECKING THE CRACKING CONDITION

The condition $M > M_{cr}$ is checked. If it is satisfied, it means the element will crack ([4], Section 7).

M – the design value of the bending moment acting in the section,

M_{cr} – the cracking moment.

Geometrical characteristics of the effective cross-section:

$$A_{eff} = A_c + \alpha_e A_{s1} = 0.200 \cdot 0.360 + 6.90 \cdot 5.09 \cdot 10^{-4} = 0.0755 \text{ m}^2;$$

$$\alpha_e = \frac{E_s}{E_{cm}} = \frac{200 \cdot 10^3}{29 \cdot 10^3} = 6.90;$$

$$S_{eff} = 0.200 \cdot 0.360 \cdot 0.180 + 6.90 \cdot 5.09 \cdot 10^{-4} \cdot 0.027 = 0.0131 \text{ m}^3.$$

The distance from the bottom to centre of gravity:

$$y_{sc} = \frac{S_{eff}}{A_{eff}} = \frac{0.0131}{0.0755} = 0.173 \text{ m}.$$

The distance from the reinforcement to centre of gravity:

$$y_{s1} = 0.173 - 0.027 = 0.146 \text{ m},$$

$$I_{eff} = I_c + \alpha_e \cdot A_{s1} \cdot y_{s1}^2 = \frac{0.200 \cdot 0.360^3}{12} + 6.90 \cdot 5.09 \cdot 10^{-4} \cdot 0.146^2 = 8.52 \cdot 10^{-4} \text{ m}^4.$$

Cracking moment

$$M_{cr} = W_{pl} f_{ctk},$$

$$W_{pl} = \gamma \cdot W_{eff} = 1.75 \cdot \frac{I_{eff}}{y_{sc}} = 1.75 \cdot \frac{8.52 \cdot 10^{-4}}{0.173} = 8.62 \cdot 10^{-3} \text{ m}^3.$$

$$\text{Then } M_{cr} = 8.80 \cdot 10^{-3} \cdot 1.5 = 0.0129 \text{ MNm} = 12.9 \text{ kNm}.$$

$44.5 \text{ kN} > 12.9 \text{ kNm}$ it means $M_r > M_{cr}$. This means that the cracks will appear in the tensile zone.

2.3.3 DETERMINATION OF LOAD CORRESPONDING TO ULTIMATE DEFLECTION

W_{ult}

Calculation of curvature

Curvature under the action of the quasi-permanent load should be calculated based on both the cracked and uncracked section. An estimate of an “average” value of curvature can be obtained using ([4], Section 7, Part 7.4.3):

$$\frac{1}{r} = \zeta \cdot \left(\frac{1}{r} \right)_{cr} + (1 - \zeta) \cdot \left(\frac{1}{r} \right)_{uc},$$

where

$\left(\frac{1}{r} \right)_{cr}$ – values of curvature calculated for the cracked case;

$\left(\frac{1}{r} \right)_{uc}$ – values of curvature calculated for the uncracked case;

ζ – is a distribution coefficient.

A distribution coefficient:

$$\zeta = 1 - \beta_1 \cdot \beta_2 \cdot \left(\frac{M_{cr}}{M} \right)^2;$$

β_1 – bond coefficient: $\beta_1 = 1$ for high bond and $\beta_1 = 0.5$ for plain bars;

β_2 – load duration factor: $\beta_2 = 1$ for a single short term loading and $\beta_2 = 0.5$ for sustained loads;

M – acting bending moment in the tension reinforcement calculated on the basis of a cracked section;

M_{cr} – cracking moment in the tension reinforcement calculated on the basis of a cracked section under the loading conditions causing first cracking.

Calculation of curvature for the uncracked condition

The curvature from bending moment action:

$$\left(\frac{1}{r}\right)_{uc} = \frac{12 \cdot M_{Ed}}{E_{c,eff} \cdot b \cdot h^3} = \frac{12 \cdot 44.5 \cdot 10^6}{10741 \cdot 200 \cdot 360^3} = 5.33 \cdot 10^{-6} \frac{1}{mm}.$$

The effective modulus of elasticity taking into account the creep of concrete:

$$E_{c,eff} = \frac{E_{cm}}{1 + \varphi_{(\infty,t0)}} = \frac{290000}{1 + 1.7} = 10741 \text{ N / mm}^2,$$

$\varphi_{(\infty,t0)}$ – the final creep coefficient of concrete.

$$\text{For 365 age at loading and } \frac{2A_c}{u} = \frac{2 \cdot 360 \cdot 200}{2 \cdot (360 + 200)} = 129, \varphi_{(\infty,t0)} = 1.7.$$

Calculation of curvature for the fully cracked condition

To calculate the curvature of the cracked section the I value of the transformed concrete section must be calculated.

Calculate the neutral axis position. Taking area moments about the neutral axis:

$$b \cdot x \cdot \frac{x}{2} = \alpha_e \cdot A_s \cdot (d - x);$$

$$200 \cdot \frac{x^2}{2} = \frac{200}{10.741} \cdot 509 \cdot (333 - x);$$

$$x^2 + 94.8x - 31561 = 0;$$

$$x = 231 \text{ mm}.$$

Calculate the second moment of area of the cracked section:

$$I_{cr} = b \cdot \frac{x^3}{3} + \alpha_e \cdot A_s \cdot (d - x)^2 = 200 \cdot \frac{231^3}{3} + \frac{200}{10.741} \cdot 509 \cdot (333 - 231)^2 = 9.2 \cdot 10^{-4} \text{ m}^4.$$

Calculate the curvature of the cracked section:

$$\left(\frac{1}{r}\right)_{cr} = \frac{M_{Ed}}{E_{c,eff} \cdot I_{cr}} = \frac{44.5 \cdot 10^6}{10741 \cdot 9.2 \cdot 10^8} = 4.5 \cdot 10^{-6} \frac{1}{mm}.$$

Calculate the average of the cracked and uncracked curvature:

$$\frac{1}{r} = \zeta \cdot \left(\frac{1}{r}\right)_{cr} + (1 - \zeta) \cdot \left(\frac{1}{r}\right)_{uc}.$$

The cracking strength of the concrete $f_{ctm} = 2.2 \text{ MPa}$. Hence from elastic bending theory and considering the uncracked concrete section, the moment that will just cause cracking of the section is:

$$M_{cr} = \frac{f_{ctm} \cdot b \cdot h^2}{6} = \frac{2.2 \cdot 0.2 \cdot 0.360^2}{6} = 9.5 \text{ kNm};$$

$$\zeta = 1 - \beta_1 \cdot \beta_2 \cdot \left(\frac{M_{cr}}{M}\right)^2 = 1 - 1.0 \cdot 0.5 \cdot \left(\frac{9.5}{44.5}\right)^2 = 0.977.$$

Calculate the average curvature:

$$\frac{1}{r} = \zeta \cdot \left(\frac{1}{r}\right)_{cr} + (1 - \zeta) \cdot \left(\frac{1}{r}\right)_{uc} = 0.977 \cdot 4.5 \cdot 10^{-6} + (1 - 0.977) \cdot 5.33 \cdot 10^{-6} = 4.52 \cdot 10^{-6} \frac{1}{mm}.$$

Calculation of shrinkage curvature

for the fully cracked condition

$$\left(\frac{1}{r}\right)_{cs} = \epsilon_{cs} \cdot \alpha_e \cdot \frac{S}{I_{cr}} = \frac{330 \cdot 10^{-6} \cdot 200 \cdot 5.19 \cdot 10^4}{10.741 \cdot 9.2 \cdot 10^8} = 3.47 \cdot 10^{-6} \frac{1}{mm};$$

$\epsilon_{cs,\infty}$ – the final shrinkage deformation, it is assumed $\epsilon_{cs,\infty} = -0.33 \text{ } ^\circ / _{00}$;

$$S = A_s \cdot (d - x) = 509 \cdot (333 - 231) = 5.19 \cdot 10^5 \text{ mm}^3.$$

for the uncracked condition

$$\left(\frac{1}{r}\right)_{cs} = \epsilon_{cs} \cdot \alpha_e \cdot \frac{S}{I_{uc}} = \frac{12 \cdot 330 \cdot 10^{-6} \cdot 200 \cdot 7.79 \cdot 10^4}{10.741 \cdot 200 \cdot 360^3} = 0.62 \cdot 10^{-6} \frac{1}{mm};$$

$$S = A_s \cdot (d - x) = 509 \cdot (333 - 180) = 7.79 \cdot 10^5 \text{ mm}^3.$$

Calculate the average shrinkage curvature:

$$\left(\frac{1}{r}\right)_{cs} = \zeta \cdot \left(\frac{1}{r}\right)_{cr} + (1 - \zeta) \cdot \left(\frac{1}{r}\right)_{uc} = 0.977 \cdot 3.47 \cdot 10^{-6} + (1 - 0.977) \cdot 0.62 \cdot 10^{-6} = 3.41 \cdot 10^{-6} \frac{1}{mm}.$$

Calculate the deflection:

$$\text{Curvature due to loading} = 4.5 \cdot 10^{-6} \frac{1}{mm};$$

$$\text{Curvature due to shrinkage} = 3.4 \cdot 10^{-6} \frac{1}{mm};$$

$$\text{Total curvature} = 7.9 \cdot 10^{-6} \frac{1}{mm}.$$

For a simply supported span subjected to a uniformly distributed load, the maximum midspan deflection is:

$$w_m = k \cdot I_{eff}^2 \left(\frac{1}{r}\right)_m = \frac{5}{48} \cdot 5000^2 \cdot 7.9 \cdot 10^{-6} = 20.5 \text{ mm}.$$

The limited deflection of the beam is:

$$w_{ult} = \frac{l}{250} = \frac{5000}{250} = 20 \text{ mm}.$$

The deflection of the beam due to bending moment w_m is larger than the limited deflection w_{ult} . However, $\frac{20.5 - 20}{20.5} \cdot 100 \% = 2.4 \%$. It may be stated that the serviceability limit state condition is fulfilled.

2.4 DESIGN ACCORDING TO ACI 318-89 CODE

Initial data

The 5 meter-long beam is simply supported. The cross-section is: $h = 360$ mm, $b = 200$ mm. Concrete class is C20/25, reinforcement class is S400. The material properties are selected according to the requirements of [5] Code, for the further calculations will be also used [10, 11]. The structural scheme of the element is shown in Fig. 2.1.1.

Characteristics of concrete C20/25:

$E_{cm} = 30 \text{ GPa}$ – modulus of elasticity of concrete;

$f_{ck} = 20 \text{ MPa}$ – characteristic value of compressive strength of concrete;

Characteristics of reinforcement S400:

$E_s = 200 \text{ GPa}$ – modulus of elasticity of steel;

$f_{yk} = 400 \text{ MPa}$ – characteristic value of tensile strength of steel;

Effective beam depth is $d = 333$ mm. Characteristic value of permanent load is $g_k = 5 \text{ kN/m'}$, characteristic value of variable load is $q_k = 5 \text{ kN/m'}$.

Design values of loadings ([5], Chapter 9, Part 9.2):

$g_d = g_k \cdot \gamma_G = 5 \cdot 1.2 = 6.0 \text{ kN/m'}$;

$q_d = q_k \cdot \gamma_Q = 5 \cdot 1.6 = 8.0 \text{ kN/m'}$.

γ_G – partial safety factor for permanent load;

γ_Q – partial safety factor for variable load.

2.4.1 CHECKING THE STRENGTH CONDITION

The selection of longitudinal reinforcement in rectangular section is performed using the following algorithm. When the longitudinal reinforcement is in tensile zone, then $A_{s2} = 0$.

Use 2Ø20 ($A_{s1} = 628 \text{ mm}^2 = 6.28 \cdot 10^{-4} \text{ m}^2$).

Assume a minimum thickness from ACI deflection table ([10], Section 5.12.1, Example 5.11):

$$\frac{l}{16} = \frac{5.0}{16} = 0.3125 \text{ m} = 312.5 \text{ mm}.$$

Required factored moment:

$$M_u = \frac{(g_d + q_d) \cdot l^2}{8} = \frac{(6.0 + 8.0) \cdot 5.0^2}{8} = 43.75 \text{ kNm} = 43.75 \cdot 10^{-3} \text{ MNm}.$$

Required resisting moment:

$$M_n = \frac{M_u}{\phi} = \frac{43.75}{0.65} = 67.3 \text{ kNm} = 67.3 \cdot 10^{-3} \text{ MNm};$$

ϕ – a factor, for compression-controlled state $\phi = 0.65$ ([5], Section 9, Figure R9.3.2).

When the net tensile strain in the extreme tension fibers, ϵ , is small, such as in short flexural members, being equal or less than a *compression-controlled* strain limit, a brittle mode of failure is expected, with little warning of such an impending. For compression-controlled state:

$$\frac{c}{d} = \frac{600}{600 + f_y} = \frac{600}{600 + 400} = 0.6, \text{ hence, compression-controlled ([5], Section 9, Figure R9.3.2).}$$

The depth of the compression zone ([5], Section 9, Part 9.4; [10], Section 5.12.1, Example 5.11):

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{6.28 \cdot 10^{-4} \cdot 400}{0.85 \cdot 20 \cdot 0.200} = 0.074 \text{ m}.$$

The compression stress:

$$C = 0.85 \cdot f'_c \cdot b \cdot a = 0.85 \cdot 20 \cdot 0.200 \cdot 0.074 = 0.252 \text{ MN} = 252 \text{ KN} .$$

The balanced neutral axis depth

$$c = \frac{a}{\beta_1} = \frac{0.074}{0.85} = 0.087 \text{ m} = 87 \text{ mm} ,$$

β_1 – a stress block depth factor, for strengths below 30 MPa $\beta_1 = 0.85$ ([5], Section 9, Figure R9.3.2).

Checking the strain limit. If the section corresponds to the compression–controlled or tension–controlled state:

$$\varepsilon = 0.003 \cdot \left(\frac{d - c}{c} \right) = 0.003 \cdot \left(\frac{333 - 87}{87} \right) = 0.00085, \text{ hence, compression–controlled.}$$

The available nominal moment strength is :

$$M_n = A_s \cdot f_y \cdot \left(d - \frac{a}{2} \right) = 6.28 \cdot 10^{-4} \cdot 400 \cdot \left(0.333 - \frac{0.074}{2} \right) = 74.4 \text{ kNm} > \text{required } M_n = 67.3 \text{ kNm}.$$

2.4.2 CHECKING THE CRACKING CONDITION

At a low load, a small bending moment develops, and the stress at the extreme tension fibers will be less than the modulus of rupture of concrete, $f_r = 0.7\sqrt{f'_c}$ ([5], Section 9, Paragraph 9.5.2.3). If the load is increased until the tensile stress reaches an average stress of the modulus of rupture, f_r , cracks will develop. If the tensile stress is higher than f_r , the section will crack, and a cracked section case will develop.

When the tensile stress, f_r , is equal to the modulus of rupture, $f_r = 0.7\sqrt{f'_c}$, a crack may start to develop, and the moment causes this stress is called the cracking moment ([11], Section 8).

$$M_{cr} = f_r \frac{I_g}{c},$$

f_r – modulus of rupture of concrete;

I_g – gross moment of inertia;

c – the distance from the neutral axis to the extreme tension fibers.

$$f_r = 0.7\sqrt{f'_c} = 0.7\sqrt{20} = 3.13 \text{ MPa};$$

$$I_g = \frac{bh^3}{12} = \frac{0.200 \cdot 0.360^3}{12} = 7.776 \cdot 10^{-4} \text{ m}^4;$$

$$c = \frac{h}{2} = \frac{0.360}{2} = 0.180 \text{ m};$$

$$M_{cr} = 3.13 \frac{7.776 \cdot 10^{-4}}{0.180} = 0.0135 \text{ MNm} = 13.5 \text{ kNm}.$$

$43.75 \text{ kN} > 13.5 \text{ kNm}$ it means $M_u > M_{cr}$. This means that the cracks will appear in the tensile zone.

2.4.3 DETERMINATION OF LOAD CORRESPONDING TO ULTIMATE DEFLECTION

W_{ult}

Assume 60% live load sustained for 24 months ([11], Section 8, Example 8.13).

$$E_c = w_c^{1.5} \cdot 0.043 \cdot \sqrt{f'_c} = 2400^{1.5} \cdot 0.043 \cdot \sqrt{20} = 22610 \text{ MPa};$$

w_c – for standard normal weight concrete $w_c = 2400 \text{ kg/m}^3$ ([5], Section 8, Part 8.5).

Modular ratio:

$$n = \frac{E_s}{E_c} = \frac{200000}{22610} = 8.85.$$

Depth of neutral axis c :

$$\frac{200c^2}{2} = nA_s(d - c);$$

$$100 \cdot c^2 = 8.85 \cdot 628 \cdot (333 - c);$$

$$c^2 + 55.6 \cdot c - 18507 = 0;$$

$$c = 167 \text{ mm}.$$

Cracking moment of inertia ([5], Section 9):

$$I_{cr} = \frac{bc^3}{3} + nA_s(d - c)^2 = \frac{200 \cdot 167^3}{3} + 8.85 \cdot 628 \cdot (333 - 167)^2 = 4.64 \cdot 10^{-4} \text{ m}^4.$$

Dead load:

$$M_a = \frac{g_d \cdot l^2}{8} = \frac{6.0 \cdot 5.0^2}{8} = 18.75 \text{ kNm} = 18.75 \cdot 10^{-3} \text{ MNm};$$

$$\left(\frac{M_{cr}}{M_a} \right)^3 = \left(\frac{13.5}{18.75} \right)^3 = (0.72)^3 = 0.373.$$

Dead load + 60% live load:

$$\left(\frac{M_{cr}}{M_a}\right)^3 = \left(\frac{13.5}{18.75 + 0.6 \cdot 25.0}\right)^3 = (0.4)^3 = 0.064.$$

Dead load + live load:

$$\left(\frac{M_{cr}}{M_a}\right)^3 = \left(\frac{13.5}{18.75 + 25.0}\right)^3 = (0.309)^3 = 0.0294.$$

Effective moment of inertia:

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 \cdot I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] \cdot I_{cr}.$$

Dead load:

$$I_e = 0.373 \cdot 7.776 \cdot 10^{-4} + 0.627 \cdot 4.64 \cdot 10^{-4} = 5.810 \cdot 10^{-4} \text{ m}^4.$$

Dead load + 60% live load:

$$I_e = 0.064 \cdot 7.776 \cdot 10^{-4} + 0.936 \cdot 4.64 \cdot 10^{-4} = 4.841 \cdot 10^{-4} \text{ m}^4.$$

Dead load + live load:

$$I_e = 0.0294 \cdot 7.776 \cdot 10^{-4} + 0.9706 \cdot 4.64 \cdot 10^{-4} = 4.732 \cdot 10^{-4} \text{ m}^4.$$

Short-term deflection ([5], Section 10):

$$\Delta = \frac{5 \cdot M \cdot l^2}{48 \cdot E \cdot I} = \frac{5 \cdot M \cdot 5000^2}{48 \cdot 22610 \cdot I} = 115 \cdot \frac{M}{I} \text{ mm}.$$

Initial live-load deflection:

$$\Delta_L = \frac{115 \cdot (18.75 + 25.0) \cdot 10^6}{4.732 \cdot 10^8} - \frac{115 \cdot 18.75 \cdot 10^6}{5.810 \cdot 10^8} = 6.9 \text{ mm}.$$

Initial dead-load deflection:

$$\Delta_D = \frac{115 \cdot 18.75 \cdot 10^6}{5.810 \cdot 10^8} = 3.7 \text{ mm}.$$

Initial 60% sustained live-load deflection:

$$\Delta_{LS} = \frac{115 \cdot (18.75 + 0.6 \cdot 25.0) \cdot 10^6}{4.841 \cdot 10^8} - \frac{115 \cdot 18.75 \cdot 10^6}{5.810 \cdot 10^8} = 4.3 \text{ mm}.$$

Long-term deflection ([5], Section 9, Paragraph 9.5.2.5):

$$\Delta_{LT} = \Delta_L + \lambda_{\infty} \Delta_D + \lambda_t \Delta_{LS} = 6.9 + 2.0 \cdot 3.7 + 1.65 \cdot 4.3 = 21.4 \text{ mm};$$

λ_{∞} – long-term deflection multiplier, for 5 years or more $\lambda_{\infty} = 2.0$;

λ_t – long-term deflection multiplier, for 24 months $\lambda_t = 1.65$.

Deflection requirements ([5], Section 9, Table 9.5(a, b)):

$$w_{ult} = \frac{l}{240} = \frac{5000}{240} = 20.8 \text{ mm},$$

$$w_m = 21.4 \text{ mm}.$$

The deflection of the beam due to bending moment w_m is larger than the limited deflection w_{ult} . However, $\frac{21.4 - 20.8}{20.8} \cdot 100 \% = 2.9 \%$. It may be stated that the serviceability limit state condition is fulfilled.

2.5 DESIGN ACCORDING TO SP 52-101-2003 CODE

Initial data

The 5 meter-long beam is simply supported. The beam cross-section is: $h = 360$ mm, $b = 200$ mm. Concrete class is B25, reinforcement class is A400. The material properties are selected according to the requirements of [6] Code, for the further calculations will be also used [12] Code. The structural scheme of the element is shown in Fig. 2.1.1.

Characteristics of concrete B25, compatible with C20/25 ([6], Section 5, Tables 5.1, 5.2):

- $E_{cm} = 30 \text{ GPa}$ – modulus of elasticity of concrete;
- $f_{ck} = 18.5 \text{ MPa}$ – characteristic value of compressive strength of concrete;
- $f_{cd} = 14.5 \text{ MPa}$ – design value of compressive strength of concrete.
- $f_{ctk} = 1.55 \text{ MPa}$ – characteristic value of compressive strength of concrete;

Characteristics of reinforcement A400, compatible with S400 ([6], Section 5.2, Table 5.7):

- $E_s = 200 \text{ GPa}$ – modulus of elasticity of steel;
- $f_{yk} = 400 \text{ MPa}$ – characteristic value of tensile strength of steel;
- $f_{yd} = 355 \text{ MPa}$ – design value of tensile strength of steel.

Effective beam depth is $d = 333$ mm. Characteristic value of permanent load is $g_k = 5 \text{ kN / m'}$, characteristic value of variable load is $q_k = 5 \text{ kN / m'}$.

Design values of loadings:

$$g_d = g_k \cdot \gamma_G = 5 \cdot 1.3 = 6.75 \text{ kN / m' ;}$$

$$q_d = q_k \cdot \gamma_Q = 5 \cdot 1.2 = 6.5 \text{ kN / m' .}$$

γ_G – partial safety factor for permanent load ([12], Section 2, Table 1);

γ_Q – partial safety factor for variable load ([12], Section 3, Table 2).

2.5.1 CHECKING THE STRENGTH CONDITION

The selection of longitudinal reinforcement in rectangular section is performed using the following algorithm. When the longitudinal reinforcement is in tensile zone, then $A_{s2} = 0$. It will be selected 3Ø14 ($A_{s1} = 462 \text{ mm}^2 = 4.62 \cdot 10^{-4} \text{ m}^2$) for the compressed zone. The concrete class C20/25 is the same as B25 in SP 52-102-2004 [6].

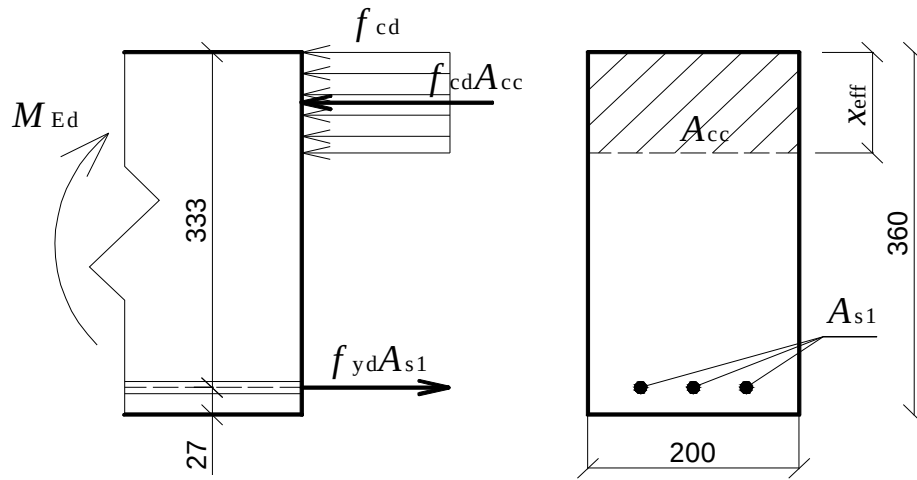


Fig. 2.5.1 The structural scheme of the normal section strength of bending element of the rectangular cross-section

The design concrete strength ([6], Section 5):

$$f_{cd} = \frac{f_{ck}}{\gamma_c} = \frac{18.5}{1.3} = 14.5 \text{ MPa}.$$

f_{ck} – characteristic strength of the concrete,

γ_c – reliability coefficient. For reinforced concrete structures $\gamma_c = 1.3$.

Determination of relative height ξ_{lim} of the element compressed zone ([6], Section 6.2.7):

$$\xi_{lim} = \frac{0.8}{1 + \frac{\epsilon_{s,el}}{\epsilon_{b,ult}}} = \frac{0.8}{1 + \frac{1.775 \cdot 10^{-3}}{0.0035}} = 0.531;$$

$\varepsilon_{s,el}$ – relative deformations of the reinforcement of tensile zone under the action of loading,

$\varepsilon_{b,ult}$ – relative deformations of compressed concrete when the stresses are equal to the strength of the concrete,

$$\varepsilon_{s,el} = \frac{f_{yd}}{E_s} = \frac{355}{200 \cdot 10^3} = 1.775 \cdot 10^{-3}.$$

The strength condition for bending elements ([6], Section 6.2):

$$M < M_{ult},$$

$$M_{ult} = f_{cd} \cdot b \cdot x \cdot (d - 0.5x) = 14.5 \cdot 0.200 \cdot 0.0566 \cdot (0.333 - 0.5 \cdot 0.0566) = 0.0500 \text{ MNm}.$$

The height of the compressed zone x is determined so:

$$x = \frac{f_{yd} \cdot A_s}{f_{cd} \cdot b} = \frac{355 \cdot 462 \cdot 10^{-6}}{14.5 \cdot 0.200} = 0.0566 \text{ m}.$$

The design value of the bending moment:

$$M_{Ed} = \frac{(g_d + q_d) \cdot l^2}{8} = \frac{(6.75 + 6.5) \cdot 5.0^2}{8} = 41.4 \text{ kNm} = 41.4 \cdot 10^{-3} \text{ MNm}.$$

The condition $M < M_{ult}$ is satisfied.

$$\xi = \frac{x}{d} = \frac{0.0566}{0.333} = 0.170.$$

The condition $\xi \leq \xi_{lim}$ is satisfied.

2.5.2 CHECKING THE CRACKING CONDITION

The condition $M > M_{cr}$ ([6], Section 7.2) is checked. If it is satisfied, it means the element will crack.

M – the design value of the bending moment acting in the section,

M_{cr} – the cracking moment.

Geometrical characteristics of the effective cross-section. The area A_{red} of the reduced section:

$$A_{eff} = A_c + \alpha_e A_{s1} = 0.200 \cdot 0.360 + 6.67 \cdot 4.62 \cdot 10^{-4} = 0.0751 \text{ m}^2;$$

$$\alpha_e = \frac{E_s}{E_{cm}} = \frac{200 \cdot 10^3}{30 \cdot 10^3} = 6.67;$$

$$S_{eff} = 0.200 \cdot 0.360 \cdot 0.180 + 6.67 \cdot 4.62 \cdot 10^{-4} \cdot 0.027 = 0.013 \text{ m}^3.$$

The distance from the bottom to centre of gravity:

$$y_{sc} = \frac{S_{eff}}{A_{eff}} = \frac{0.013}{0.0751} = 0.174 \text{ m}.$$

The distance from the reinforcement to centre of gravity:

$$y_{s1} = 0.174 - 0.027 = 0.147 \text{ m},$$

$$I_{eff} = I_c + \alpha_e \cdot A_{s1} \cdot y_{s1}^2 = \frac{0.200 \cdot 0.360^3}{12} + 6.67 \cdot 4.62 \cdot 10^{-4} \cdot 0.147^2 = 8.44 \cdot 10^{-4} \text{ m}^4.$$

Cracking moment $M_{cr} = W_{pl} f_{ctk}$ ([6], Section 7.2.9),

$$W_{pl} = \gamma \cdot W_{eff} = 1.75 \cdot \frac{I_{eff}}{y_{sc}} = 1.75 \cdot \frac{8.44 \cdot 10^{-4}}{0.174} = 8.49 \cdot 10^{-3} \text{ m}^3.$$

$$\text{Then } M_{cr} = 8.49 \cdot 10^{-3} \cdot 1.5 = 0.0127 \text{ MNm} = 12.7 \text{ kNm}.$$

$41.4 \text{ kN} > 12.7 \text{ kNm}$ it means $M_r > M_{cr}$. This means that the cracks will appear in the tensile zone.

2.5.3 DETERMINATION OF LOAD CORRESPONDING TO ULTIMATE DEFLECTION

w_{ult}

The total curvature of the section with cracks in the tensile zone ([6], Section 7.3.8):

$$\frac{1}{r} = \left(\frac{1}{r}\right)_1 - \left(\frac{1}{r}\right)_2 + \left(\frac{1}{r}\right)_3,$$

where

$\left(\frac{1}{r}\right)_1$ – curvature from characteristic action of short-term loadings;

$\left(\frac{1}{r}\right)_2$ – curvature from characteristic action of short-term permanent and quasi-permanent loadings;

$\left(\frac{1}{r}\right)_3$ – curvature from characteristic action of long-term permanent and quasi-permanent loadings.

The curvature from the action of short term loadings

$$\left(\frac{1}{r}\right)_1 = \frac{M_{Ed}}{D} = \frac{0.0414}{4.91 \cdot 10^3} = 8.43 \cdot 10^{-6} \frac{1}{mm}.$$

The design value of bending moment:

$$M_{Ed} = \frac{(g_d + q_d) \cdot l^2}{8} = \frac{(6.75 + 6.5) \cdot 5.0^2}{8} = 41.4 \text{ kNm} = 41.4 \cdot 10^{-3} \text{ MNm}.$$

The bending stiffness of the reduced section ([6], Section 7.3.13):

$$D = E_{s,red} \cdot A_s \cdot z \cdot (h_0 - x_m) = 200 \cdot 10^3 \cdot 4.62 \cdot 10^{-4} \cdot 0.2843 \cdot (0.333 - 0.146) = 4.91.$$

The reduced modulus of elasticity of steel:

$$E_s = E_{s,red} = 200 \text{ GPa}.$$

The distance from the centre of tensile reinforcement to the point of applied force in the compressed zone ([6], Section 7.3.13):

$$z = h_0 - \frac{1}{3} x_m = 333 - \frac{1}{3} \cdot 146 = 284.3 \text{ mm}.$$

The depth of compressed zone ([6], Section 7.3.12):

$$\begin{aligned} x_m &= h_0 \left(\sqrt{(\mu_s \cdot \alpha_{s2})^2 + 2\mu_s \cdot \alpha_{s2}} - \mu_s \cdot \alpha_{s2} \right) = \\ &= 0.333 \cdot \left(\sqrt{(6.94 \cdot 10^{-3} \cdot 24.8)^2 + 2 \cdot 6.94 \cdot 10^{-3} \cdot 24.8} - 6.94 \cdot 10^{-3} \cdot 24.8 \right) = 0.146 \text{ m}; \end{aligned}$$

$$\mu_s = \frac{A_s}{b \cdot h_0} = \frac{462}{200 \cdot 333} = 6.94 \cdot 10^{-3}.$$

The reduced modulus of elasticity of concrete:

$$E_{b,1} = E_{b,red} = \frac{R_b}{\epsilon_{b1,red}} = \frac{14.5}{0.0018} = 8.056 \cdot 10^3 \text{ MPa}.$$

The ratio of steel-to-concrete elastic modules:

$$\alpha_e = \frac{E_s}{E_{b,red}} = \frac{200 \cdot 10^3}{8.056 \cdot 10^3} = 24.8.$$

The curvature from the action of short term permanent and quasi-permanent loadings

$$\left(\frac{1}{r} \right)_2 = \frac{M_{Egd}}{D} = \frac{0.0333}{4.91 \cdot 10^3} = 6.78 \cdot 10^{-6} \frac{1}{\text{mm}}.$$

The design value of bending moment from quasi-permanent load:

$$M = M_{Egd} = \frac{(g_d + \psi_2 \cdot q_d) \cdot l^2}{8} = \frac{(6.75 + 0.6 \cdot 6.5) \cdot 5.0^2}{8} = 33.3 \text{ kNm}.$$

The bending stiffness of the reduced section ([6], Section 7.3.13):

$$D = E_{s,red} \cdot A_s \cdot z \cdot (h_0 - x_m) = 200 \cdot 10^3 \cdot 4.62 \cdot 10^{-4} \cdot 0.2843 \cdot (0.333 - 0.146) = 4.91.$$

The reduced modulus of elasticity of steel:

$$E_s = E_{s,red} = 200 \text{ GPa}.$$

The distance from the centre of tensile reinforcement to the point of applied force in the compressed zone ([6], Section 7.3.13):

$$z = h_0 - \frac{1}{3} x_m = 333 - \frac{1}{3} \cdot 146 = 284.3 \text{ mm}.$$

The depth of compressed zone ([6], Section 7.3.12):

$$\begin{aligned} x_m &= h_0 \left(\sqrt{(\mu_s \cdot \alpha_{s2})^2 + 2\mu_s \cdot \alpha_{s2}} - \mu_s \cdot \alpha_{s2} \right) = \\ &= 0.333 \cdot \left(\sqrt{(6.94 \cdot 10^{-3} \cdot 24.8)^2 + 2 \cdot 6.94 \cdot 10^{-3} \cdot 24.8} - 6.94 \cdot 10^{-3} \cdot 24.8 \right) = 0.146 \text{ m}; \end{aligned}$$

$$\mu_s = \frac{A_s}{b \cdot h_0} = \frac{462}{200 \cdot 333} = 6.94 \cdot 10^{-3}.$$

The reduced modulus of elasticity of concrete:

$$E_{b,1} = E_{b,red} = \frac{R_b}{\epsilon_{b1,red}} = \frac{14.5}{0.0018} = 8.056 \cdot 10^3 \text{ MPa}.$$

The ratio of steel-to-concrete elastic modules:

$$\alpha_e = \frac{E_s}{E_{b,red}} = \frac{200 \cdot 10^3}{8.056 \cdot 10^3} = 24.8.$$

The curvature from the action of long term permanent and quasi-permanent loadings

$$\left(\frac{1}{r}\right)_3 = \frac{M_{Egd}}{D} = \frac{0.0333}{3.01 \cdot 10^3} = 11.06 \cdot 10^{-6} \frac{1}{mm}.$$

The design value of bending moment from quasi-permanent load:

$$M = M_{Egd} = \frac{(g_d + \psi_2 \cdot q_d) \cdot l^2}{8} = \frac{(6.75 + 0.6 \cdot 6.5) \cdot 5.0^2}{8} = 33.3 \text{ kNm}.$$

The bending stiffness of the reduced section ([6], Section 7.3.13):

$$D = E_{s,red} \cdot A_s \cdot z \cdot (h_0 - x_m) = 200 \cdot 10^3 \cdot 4.62 \cdot 10^{-4} \cdot 0.263 \cdot (0.333 - 0.209) = 3.01.$$

The reduced modulus of elasticity of steel:

$$E_s = E_{s,red} = 200 \text{ GPa}.$$

The distance from the centre of tensile reinforcement to the point of applied force in the compressed zone ([6], Section 7.3.13):

$$z = h_0 - \frac{1}{3} x_m = 0.333 - \frac{1}{3} \cdot 0.209 = 0.263 \text{ m}.$$

The depth of compressed zone ([6], Section 7.3.12):

$$\begin{aligned} x_m &= h_0 \left(\sqrt{(\mu_s \cdot \alpha_{s2})^2 + 2\mu_s \cdot \alpha_{s2} - \mu_s \cdot \alpha_{s2}} \right) = \\ &= 0.333 \cdot \left(\sqrt{(6.94 \cdot 10^{-3} \cdot 38.3)^2 + 2 \cdot 6.94 \cdot 10^{-3} \cdot 38.3 - 6.94 \cdot 10^{-3} \cdot 38.3} \right) = 0.209 \text{ m}; \end{aligned}$$

$$\mu_s = \frac{A_s}{b \cdot h_0} = \frac{462}{200 \cdot 333} = 6.94 \cdot 10^{-3}.$$

The reduced modulus of elasticity of concrete:

$$E_{b,1} = E_{b,red} = \frac{R_b}{\varepsilon_{b1,red}} = \frac{14.5 \cdot 0.9}{2.5 \cdot 10^{-3}} = 5.22 \cdot 10^3 \text{ MPa} .$$

The ratio of steel-to-concrete elastic modules:

$$\alpha_e = \frac{E_s}{E_{b,red}} = \frac{200 \cdot 10^3}{5.22 \cdot 10^3} = 38.3 .$$

The deflection of the beam is ([6], Section 7.3.6):

$$w_m = \left(\frac{1}{r} \right) \cdot \frac{5}{48} \cdot l^2 = (8.4 - 6.78 + 11.06) \cdot 10^{-6} \cdot \frac{5}{48} \cdot 5000^2 = 33.02 \text{ mm} .$$

The limited deflection of the beam, when the beam length is $l = 5000 \text{ mm}$ ([6], Section 7.3.3):

$$w_{ult} = \frac{l}{150} = \frac{5000}{150} = 33 \text{ mm} .$$

The deflection of the beam due to bending moment w_m is less than the limited deflection w_{ult} . The serviceability limit state condition is fulfilled.

2.6 GENERALIZATION OF DESIGN APPLYING DETERMINISTIC MODELS OF DIFFERENT DESIGN CODES

In the performed calculations the flexural concrete members were designed for serviceability using different design codes [3-6]. The members are considered to be safe if they are designed according to equations and limitations defined by certain design code. Consequently, for defined in advance geometrical properties of the members the reinforcement required to maintain an internal moment capacity equal or greater than that of the external moment is determined. Once the final dimensions were determined, the beam was checked for serviceability: cracks and deflection.

In this paper, there are discussed the design peculiarities, differences in the design calculations as well as obtained results according to mentioned design codes.

The strength condition defines the reinforcement areas and the acting loads. The most conservative are the Eurocode and ACI norms, which require $A_{s1} = 5.09 \times 10^{-4} \text{ m}^2$ and $A_{s1} = 6.28 \times 10^{-4} \text{ m}^2$ correspondingly for $g_k = q_k = 5 \text{ kN/m'}$. The STR and SP norms require the same reinforced area, it is $A_{s1} = 4.62 \times 10^{-4} \text{ m}^2$ for $g_k = q_k = 5 \text{ kN/m'}$. Such not equal results are needed to satisfy different serviceability requirements of the design codes. And the main reason of such not equal required reinforcement area is different values of partial safety factors for permanent and variable loads. STR uses values 1.35 and 1.3, EN uses 1.35 and 1.5, ACI – 1.2 and 1.6, SP – 1.3 and 1.2 for permanent and variable loads correspondingly.

The largest deflection is allowable by SP norms, it is $l/150 = 33 \text{ mm}$. Approximately the same deflections are given by Eurocode and ACI norms, they are $l/250 = 20 \text{ mm}$ and $l/240 = 20.8 \text{ mm}$. The STR code allowable deflection is $l/200 = 25 \text{ mm}$.

Also the deflection calculations according to different design codes evaluate different load actions or differently evaluate the same load action. However, deflection according to every design code contains such parts as deflections from long-term and short-term load actions.

There are also differences in cracking moment calculations. When the applied external moment exceeds the cracking moment, M_{cr} , a cracked section case is developed, and the concrete in the tension zone is neglected. A transformed cracked section is used to calculate the cracking

moment of inertia, I_{cr} , using the concrete area in compression and the transformed steel area nA_s . Such definition uses all design codes, however the ACI code allows to use simplified expression, where the section is said to be uncracked and instead of cracking moment of inertia the gross moment of inertia is used. In fact, the cracking moment values according to different design codes do not differ much.

The calculated deflection values will be used further for calculation of stiffness reliability of flexural reinforced concrete elements.

An additional calculation was performed in order to compare economical aspects of different design codes for required reinforcement area. The value of load was taken $g_k = q_k = 5 \text{ kN/m'}$. The ultimate deflection was assumed $l/200 = 25 \cdot 10^{-3} \text{ m}$. The same procedure was carried out according to requirements of every design code for the value of ultimate deflection $l/200 = 25 \cdot 10^{-3} \text{ m}$. By iterating required reinforcement area, the obtained deflection value was maximally approached to the ultimate deflection value. Only SP codes was not included in such comparison, because the calculated deflection was very close to ultimate deflection $l/150 = 33 \cdot 10^{-3} \text{ m}$, and by decreasing the ultimate deflection up to $l/200 = 25 \cdot 10^{-3} \text{ m}$, the reinforcement area also has to be decreased, but in such case the strength condition won't be satisfied. The calculation results for STR, Eurocode and ACI codes are shown in table 2.6.1.

Table 2.6.1 Comparing of required reinforcement area according to STR, EN and ACI codes for accepted deflection value $l/200 = 25 \cdot 10^{-3} \text{ m}$ for load $g_k = q_k = 5 \text{ kN/m'}$

Code	Load $g_k = q_k$, kN/m'	Required reinforcement area A_s for w_{ult} according to STR, EN, ACI			Required reinforcement area A_s for $l/200 = \text{const}$,	
		w_{ult}	A_s		$\times 10^{-4} \text{ m}^2$	%
			$\times 10^{-4} \text{ m}^2$	%		
STR [3]	5	$l/200$	4.62	100	4.62	100
EN [4]		$l/250$	5.09	110	4.52	98
ACI [5]		$l/240$	6.28	136	5.56	120

3 MANAGEMENT OF STANDARDIZED STIFFNESS RELIABILITY OF FLEXURAL REINFORCED CONCRETE ELEMENTS

3.1 INTRODUCTORY REMARKS

In current design codes of Design Values (DV) based on method of Partial Safety Factors the standardized reliability is chosen as criteria of resistance, for example strength, stiffness of structural and foundation elements. To determine the standardized reliability it is necessary to perform the analysis of marginal resistance management from DV codes. The standardized methods of management of marginal reliability in DV codes are discussed in this paper. The methods are proved by summation performed in [18] reference of deterministic models of stiffness of flexural reinforced concrete elements, which are applied in international DV design codes and DV design codes of different countries [3-7, 15-17]. Based on analysis of standardized in DV design codes marginal stiffness management the following method of determination and use of standardized reliability of stiffness factor in Direct Probabilistic-Statistical (DPS) codes is discussed.

3.2 PROBABILISTIC MODELING OF LOADS

As shown in [18], using different DV design codes the total deflection w commonly is determined as the sum of accumulated long-term deflection w_l during the whole exploitation period. The total deflection is composed from the deflection under the quasi-permanent load p_l action and short-term deflection w_{sh} under the short-term load action:

$$w = w_l + w_{sh}. \quad (3.2.1)$$

The deflection w_l accumulated at the end of exploitation period composes the major part from the total deflection w_l . To use the standardized reliability of stiffness in *direct probabilistic-statistical* codes [21] it is necessary to choose the upper limit of standardized reliability of stiffness, i.e. the maximum standardized value. Therefore, the standardized reliability of stiffness is analyzed and it is determined for the end of exploitation period applying the *randomly timed*

probability distribution of short-term loading. So, the upper limit of reliability of stiffness is analyzed for the end of exploitation period.

For this reason the total loading p is considered as the sum of p_l and p_{sh} . ([16] 6.5.3 paragraph and [7] 90 paragraph):

$$p = p_l + p_{sh} = g + \psi_2 q + (1 - \psi_2) q. \quad (3.2.2)$$

where $p_l = g + \psi_2 q$ – permanent loading part composed from self-weight of structure g and q variable load quasi-permanent part $q = \psi_2 q$. The characteristic value of this load is determined from $p_{l,k} = g_k + \psi_2 q_k$; here g_k and q_k – the characteristic values of loads, ψ_2 – the coefficient describing the quasi-permanent part of q_k characteristic value and calculated depending on the premise purpose (example according to [16] table A1.1), i.e. the random process probabilistic properties of load q changing time t .

One of the methods how to manage the standardized reliability of stiffness in different DV design codes is to choose different $p_{sh,k,j} = q_{sh,k,j}$, $j = 1, 2, 3$ characteristic values of short-term part q_{sh} of loading q , which for serviceability limit state corresponds the design values:

1) if according to (3.2.1) w standardized value is limited due to *esthetical* requirements, then characteristic value $p_{sh,k,1} = q_{sh,k,1} = 0$ is chosen,

2) if w deflection is reversible, then $p_{sh,k,2} = q_{sh,k,2} = (\psi_1 - \psi_2) q_k$ characteristic value is chosen,

3) if w deflection is irreversible, then $p_{sh,k,3} = q_{sh,k,3} = (1 - \psi_2) q_k$ characteristic value is chosen (see Fig 3.2.1).

The coefficients ψ_1 and ψ_2 are chosen from the [16] table A1.1. The reliability of stiffness depends on $q_{sh,k,j}$, $j = 1, 2, 3$ values fraction compatibility, i.e. depends on difference between $q_{sh,k,j}$, $j = 1, 2, 3$ and mean value $\mu_{q,sh} - q_{sh,k,j}$, $j = 1, 2, 3$. The values of $q_{sh,k,j}$, $j = 1, 2, 3$ and its fraction compatibility is shown graphically in Fig.3.2.1 based on [1] Fig.2.13.

When the upper limit of reliability of stiffness is determined, the load p_l according to (3.2.2) distribution mean value $\mu_{p,l}$ is determined applying $\mu_{l,k} = g_k + \psi_2 q_k$. This equation is

based on data from statistical analysis, which defined that the distribution mean value μ_g of the load g taking into account the weight of walls and fixed-site equipment is $\mu_g \approx g_k$ (see example, [21]). According to the description of the quasi-permanent part q_l of the load q , its distribution mean value corresponds to characteristic value $\mu_{q,l} = q_{l,k} = \psi_2 q_k$, i.e. the fraction compatibility of characteristic value is $\alpha_{q,l,k} = P(q_l > q_{l,k}) = 0.5$ (see Fig.3.2.1).

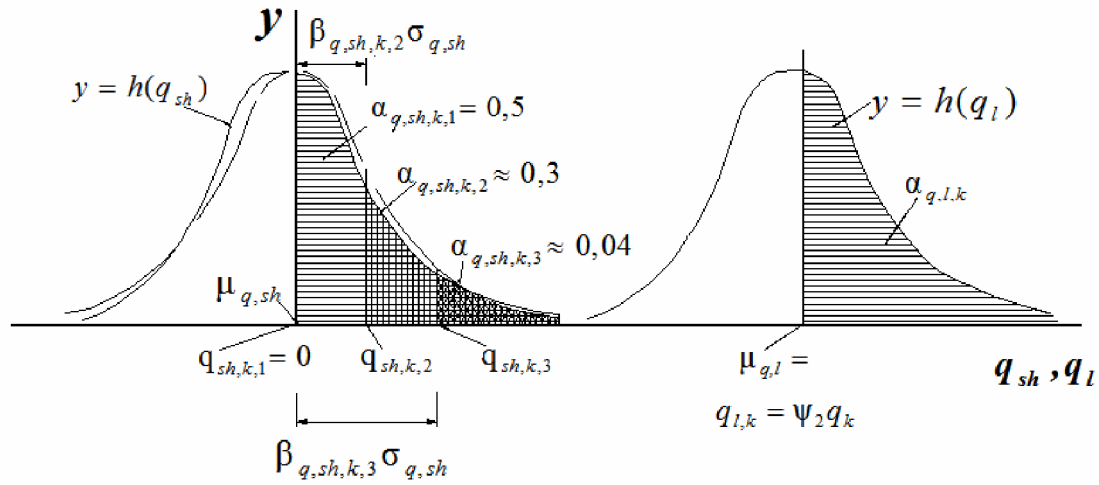


Fig. 3.2.1 The modeling of variable load $q = q_l + q_{sh}$ quasi-permanent part q_l and short-term part according to (3.2.2) equation using the probability distribution $h(q_l)$ and $h(q_{sh})$, which mean value $\mu_{q,l} = q_{l,k} = \psi_2 q_k$ and $\mu_{q,sh} = 0$; $q_{l,k}$ and $q_{sh,k,j}$, $j=1,2,3$ are the characteristic values of loads q_l and q_{sh} , used to satisfy esthetical requirements for determination of reversible and irreversible deflections

The part of the load $p_{sh} = q_{sh}$ mean value $\mu_{q,sh} = 0$, and the standard deviation is determined from the condition that the fraction compatibility of the characteristic value $q_{sh,k,3}$ of the load q_{sh} is $\alpha_{q,sh,k,3} = P(q_{sh} > q_{sh,k,3}) = 0.04$, i.e. the value of $\beta_{q,sh,k,3}$ reliability factor $\beta_{q,sh,k,3} = 1.7$ corresponding to $q_{sh,k,3}$. The values of $\alpha_{q,sh,k,3}$ and $\beta_{q,sh,k,3}$ are determined from the condition that the q load $q = q_l + q_{sh}$ fraction compatibility of the characteristic value q_k is $\alpha_{q,k} = P(q > q_k) \approx 0.02$, i.e. this compatibility corresponds to load q_l maximum fraction

compatibility of the q_k characteristic value $\alpha_{q_l,k} = P(q_l > q_k) = 0.02$ [15, 16, 1]. So, assuming that the q_l and q_{sh} parts of loads are independent, $\alpha_{q_{sh},k,3} = \alpha_{q,k} / \alpha_{q_l,k} = 0.02/0.5 = 0.04$, here $\alpha_{q_l,k} = P(q_l > q_{l,k}) = 0.5$ (see Fig. 3.2.1).

The standard deviation $\sigma_{q,sh}$ of the load q_{sh} is determined from the condition: $\sigma_{q,sh} = q_{sh,k,3} / \beta_{q_{sh},k,3} = (1 - \psi_2) q_k / 1.7 \approx 0.6(1 - \psi_2) q_k = 0.24 q_k$; here $\beta_{q_{sh},k,3} = 1.7$ corresponds to fraction compatibility $\alpha_{q_{sh},k,3} = 0.04$ and also it was selected that $\sigma_{q,l} = \sigma_{q,sh} = 0.24 q_k$.

In cases when the characteristic value $q_{sh,k,2} = (\psi_1 - \psi_2) q_k$ is chosen (see Fig.3.2.1) the reliability factor is determined from the condition:

$$\beta_{q_{sh},k,2} = \beta_{q_{sh},k,3} \frac{\psi_1 - \psi_2}{1 - \psi_2}. \quad (3.2.3)$$

Here ψ_1 and ψ_2 are the discussed coefficients. Because the ratio $\frac{\psi_1 - \psi_2}{1 - \psi_2}$ for the most variable loads is equal to 0.3, from the (3.2.3) follows $\beta_{q_{sh},k,2} = 1.7 \cdot 0.3 = 0.5$, i.e. the characteristic value $q_{sh,k,2}$ fraction compatibility is $\alpha_{q_{sh},k,2} = P(q_{sh} > q_{sh,k,2}) = 0.3$ (see Fig. 3.2.1).

In case when the characteristic value $q_{sh,k,1} = \mu_{q,sh} = 0$ is chosen (see Fig. 3.2.1), then $\beta_{q_{sh},k,1} = 0$ and $\alpha_{q_{sh},k,1} = 0.5$.

3.3 THE MODEL OF STANDARDIZED STIFFNESS RELIABILITY MANAGEMENT

Summarizing different [3-7, 15-17] DV codes it was determined [18], that standardized marginal stiffness is managed in two ways $u=1,2$ in cases of non-esthetic, reversible and irreversible, i.e. $j=1,2,3$ deflection limitation. By first $u=1$ way, which was discussed in the previous chapter the different standardized marginal stiffness is managed by choosing the different $q_{sh,k,j}$, $j=1,2,3$ characteristic values of the short-term part q_{sh} of the variable load q . By second

$u = 2$ way the different standardized marginal stiffness is managed by choosing the different $w_{ult,u,j}$, $j = 1, 2, 3$ standardized in DV codes ultimate values of deflection. So in general the marginal stiffness $Z_{u,j} = w_{ult,u,j} - w_{u,j}$ taking into account (3.2.1) is determined so

$$\begin{aligned} Z_{u,j} &= w_{ult,u,j} - w_{u,j} = w_{ult,u,j} - w_{l,u,j} - w_{sh,u,j} = \\ &= w_{ult,u,j} - w_{l,u}(E_c, E_s, d, g, f_c, q_l, \theta_l) - w_{sh,u}(E_c, E_s, d, g, f_c, q_l, \theta_{sh}) = \\ &= w_{ult,ref,u} + \Delta w_{ult,u,j} - w_{l,u}(X_1, X_2, \dots, X_7) - w_{sh,u}(X_1, X_2, \dots, X_7), \end{aligned} \quad (3.3.1)$$

$u = 1, 2$, $j = 1, 2, 3$.

Here $w_{l,u,j}$ and $w_{sh,u,j}$ correspond to w_l and w_{sh} values from the (3.2.1) equation, $w_{ult,ref,u}$, $u = 1, 2$, $j = 1, 2, 3$ is the standardized *reference* value of limited deflection, $\Delta w_{ult,u,j}$ is the change of limited deflection, $w_{l,u}(\cdot)$ and $w_{sh,u}(\cdot)$ are the standardized deterministic models of long-term and short-term deflections according to (3.2.1), $X_1 = E_c$ and $X_2 = E_s$ are the modules of elasticity of concrete and reinforcement, $X_3 = d$ – is the effective depth of the cross-section, $X_4 = g$ – the load from the self-weight of the structure, $X_5 = f_{c,t}$ is the tensile strength of the concrete. In model $w_{l,u}(\cdot)$ the components $X_6 = q_l$ and $X_7 = \theta_{w,l}$ are applied and in model $w_{sh,u}(\cdot)$ the components $X_6 = q_{sh}$ and $X_7 = \theta_{w,sh}$ are applied, here q_l and q_{sh} are the discussed loads, $\theta_{w,l}$ and $\theta_{w,sh}$ are the errors of deterministic models $w_{l,u}(\cdot)$ and $w_{sh,u}(\cdot)$ respectively.

3.4 DETERMINATION OF STANDARDIZED STIFFNESS RELIABILITY

The methods of determination of standardized in DV design codes reliability of stiffness

Further it is accepted that coefficients X_i , $i=1,2,...,7$ in (3.3.1) expression are not correlated and their values are distributed by Gaussian law. If these conditions are not satisfied then these values have to be decorrelated and normalized by the known way. The mean value, standard deviation, characteristic value of these random values X_i , $i=1,2,...,7$ not correlated and distributed by Gaussian law are defined as μ_{Xi} , σ_{Xi} , x_{ik} respectively. Moreover, the values of μ_{Xi} and x_{ik} , $i=1,2,...,7$ hardly differ and most of them are equal. So using (3.3.1) expression the standardized in DV design codes factor of stiffness reliability $\beta_{DV,u,j}$, $u=1,2$, $j=1,2,3$ may be determined applying expressions (3.4.1) and (3.4.2):

$$\beta_{DV,u,j} = \frac{\mu_{Z,u,j} - z_{k,u,j}}{\sigma_Z} = \left\{ w_{ult,ref} + \Delta w_{ult,2,j} - \mu_{\theta,w,l} \frac{\partial w_l}{\partial f_{ct}} (\mu_{f_{ct}} - f_{ct,k}) - \right. \\ \left. \mu_{\theta,w,sh} \left[\frac{\partial w_{sh}}{\partial f_{ct}} (\mu_{f_{ct}} - f_{ct,k}) - \frac{\partial w_{sh}}{\partial q_{sh}} (\mu_{q_{sh}} - q_{sh,k,1,j}) \right] \right\} / \sigma_Z, \quad (3.4.1)$$

$$u=1, 2 \quad j=1, 2, 3,$$

$$\sigma_Z = \left[\mu_{\theta,w,l}^2 \sum_{i=1}^7 \left(\frac{\partial w_l}{\partial X_i} \sigma_{Xi} \right)^2 + \mu_{\theta,w,sh}^2 \sum_{i=1}^7 \left(\frac{\partial w_{sh}}{\partial X_i} \sigma_{Xi} \right)^2 + 2\rho_{w,l,w,sh} \right. \\ \left. \mu_{\theta,w,l} \sum_{i=1}^7 \left(\frac{\partial w_l}{\partial X_i} \sigma_{Xi} \right) \mu_{\theta,w,sh} \sum_{i=1}^7 \left(\frac{\partial w_{sh}}{\partial X_i} \sigma_{Xi} \right) \right]^{0,5}. \quad (3.4.2)$$

Here $\mu_{Z,u,j}$, $z_{k,u,j} = 0$, $u=1, 2$, $j=1, 2, 3$ and σ_Z are the mean value, characteristic value and standard deviation of the $Z_{u,j}$; $\rho_{w,l,w,sh}$ is the correlation factor of the deflections $w_{l,u}$ and $w_{sh,u}$ calculated by the (3.2.1) expression.

In expressions (3.4.1) and (3.4.2) the standardized in different DV codes stiffness models and standardized reliability of stiffness management parameter values are applied. They are generalized by 2 types $u = 1, 2$ and by 3 cases $j = 1, 2, 3$.

For values X_i , $i = 1, 2, \dots, 7$ not correlated and distributed by Gaussian law, and for small differences $\mu_{f,ct} - f_{ct,k}$ and $\mu_{q,sh} - q_{sh,k,1,j}$, $j = 1, 2, 3$ the probability distribution of random value $Z_{u,j}$, $u = 1, 2$, $j = 1, 2, 3$ is also approximated as Gaussian distribution. This assertion is valid when the function (3.3.1) is not discontinued $\mu_{f,ct} - f_{ct,k}$ and $\mu_{q,sh} - q_{sh,k,1,j}$, $j = 1, 2, 3$ in two-dimensional system, for example when there are no cracks. So it is necessary to check the condition when the system is uncracked using expressions (3.4.1) and (3.4.2).

The reliability of stiffness was determined for the differently reinforced (0,5 %, 1,0 %, 1,5 %) 5 m long reinforced concrete beam with effective beam depth $d = 0.333$ m and width $b = 0.200$ m. The beam concrete class is C25/30, reinforcement class is S400 according to EN 1992 [4]. The beam is loaded by distributed permanent load g and distributed variable load q , which characteristic value ratio is $g_k / q_k = 1$. In the analyzed case the loads represent offices premises, for which the characteristic value of quasi-permanent load is

$p_{l,k} = g_k + \psi_2 q_k = g_k + \psi_2 g_k = (1 + \psi_2) g_k = 1,3 g_k$ and the $q_{sh,k,j}$, $j = 1, 2, 3$ characteristic values of q_{sh} short-term part of q variable load when $q_k = g_k$:

$$q_{sh,k,1} = 0,$$

$$q_{sh,k,2} = (\psi_1 - \psi_2) q_k = 0,2 g_k,$$

$$q_{sh,k,3} = (1 - \psi_2) q_k = 0,7 g_k,$$

here $\psi_1 = 0,5$ and $\psi_2 = 0,3$ are coefficients taken from [16] table A1.1 and corresponding to short-term load permanent and quasi-permanent part.

Factors and models generalization features

The deflections $w_{ult,ref,u}$, $w_{ult,u,j}$ and deterministic models and probability factors of their parts of the expression (3.3.1) were generalized.

The generalizing of $\Theta_{w,l,u}$ and $\Theta_{w,sh,u}$, $u = 1, 2$ errors in different DV codes are evaluated according to experimentally determined „obs“ and applying the model of calculated „cal“ values ratios. The errors are characterized by system errors $\mu_{\Theta,w,l} = \mu_{w,l,obs} / \mu_{w,l,cal}$, $\mu_{\Theta,w,sh} = \mu_{w,sh,obs} / \mu_{w,sh,cal}$ and by the variation coefficient of random errors $\delta_{\Theta,w,l}$, $\delta_{\Theta,w,sh}$.

The $w_{l,u}(\cdot)$ generalized errors of *long-term* deflection models were determined for the EN [4] ir ACI [5] codes using the [22]. The errors $\mu_{\Theta,w,l,EN} = 1,02$, $\mu_{\Theta,w,l,ACI} = 0,88$ and $\delta_{\Theta,w,l,EN} = 0,19$, $\delta_{\Theta,w,l,ACI} = 0,28$ were determined based on analysis of deflection errors of different authors [22]. There are about two hundred cases of flexural reinforced concrete elements of different span, cross-section and reinforcement-to-concrete ratios. By generalizing the errors for the EN and ACI codes, it is determined that $\mu_{\Theta,w,l} = 0,95$ and $\delta_{\Theta,w,l} = 0,23$. Such values were taken for SNIP, STR, SP [18, 7, 10] codes as well. It is necessary to take into account in the calculation of $\beta_{SR,u,j}$ values (see table 4.4.1), that the system error of SP code model is $\mu_{\Theta,w,l,SP} = 0,70$ and it differs from $\mu_{\Theta,w,l} = 0,95$ chosen value. This error difference was determined not directly, performing the correlation analysis of calculated deflection values according to selected codes models.

The generalized errors of *short-term* deflections applying EN, ACI, SNIP, STR codes models were determined according to data in [23] work. The data was obtained by 4 authors work on analysis of deflection errors of 80 flexural reinforced concrete elements of different span, cross-section and reinforcement-to-concrete ratios. The bending moment is $M = 0,8 M_{fr}$, where M_{fr} – rupture moment. According to [23] table 4.5: $\mu_{\Theta,w,sh,EN} = 1,01$, $\mu_{\Theta,w,sh,ACI} = 0,99$, $\mu_{\Theta,w,sh,SNIP} = 1,0$, $\delta_{\Theta,w,sh,EN} = 0,24$, $\delta_{\Theta,w,sh,ACI} = 0,14$, $\delta_{\Theta,w,sh,SNIP} = 0,13$, i. e. for EN, ACI, SNIP codes the generalized errors values are $\mu_{\Theta,w,sh} = 1,0$, $\delta_{\Theta,w,sh} = 0,17$. Such generalized values of errors were selected for STR and SP codes as well.

It is necessary to mention that in expressions (3.4.1) and (3.4.2) partial derivatives $\partial w_l / \partial X_i$, $\partial w_{sh} / \partial X_i$, $i = 1, 2, \dots, 7$ correspond to generalization of the models $w_{l,u}(\cdot)$ ir $w_{sh,u}(\cdot)$

according to (3.3.1) expression of two-type $u = 1, 2$ three-case $j = 1, 2, 3$ for flexural reinforced concrete elements with different reinforcement-concrete ratios, i. e. 0,5 %, 1,0 %, 1,5 % .

It was also mentioned that for management of the standardized marginal stiffness by $u = 1$ type in $j = 1, 2, 3$ cases for all DV codes the generalized $q_{sh,k,1,j} = q_{sh,k,j}$ $u = 1, j = 1, 2, 3$ characteristic values were selected. The $q_{sh,k,j}$, $j = 1, 2, 3$ values are determined for management of marginal stiffness in the beginning of Chapter 3.4. The mentioned values were chosen for the case of generalized *serviceability limit state*, when the ultimate deflection is $w_{ult} = l / 400 = 0.0125 \text{ m}$. It is determined by accepting that the ratio $w_l / w_{sh} = 2$ according to the (3.2.1) expression $w_{ult} = w_l + w_{sh}$. So, taking onto account the previously mentioned, it was determined:

$$\sigma_{\theta,w,l} = \frac{2}{3} w_{ult} \cdot \delta_{\theta,w,l} = 0.67 \cdot 0.0125 \cdot 0.23 = 1.9261 \cdot 10^{-3} \text{ m and}$$

$$\sigma_{\theta,w,sh} = 0.33 \cdot w_{ult} \cdot \delta_{\theta,w,l} = 0.33 \cdot 0.0125 \cdot 0.17 = 0.701 \cdot 10^{-3} \text{ m}.$$

The generalized values of parameters X_i , $i=1, 2, \dots, 7$ from (3.3.1) expression and generalized values of parameters from (3.4.1) and (3.4.2) expressions are shown in table 3.4.1.

Table 3.4.1 The unified values by areas of office premices ($\psi_1 = 0,5$, $\psi_2 = 0,3$ according to [17] table A1.1) of $\partial w_l / \partial X_i$, $\partial w_{sh} / \partial X_i$, $i = 1, 2, \dots, 7$ partial derivatives of the X_i , $i = 1, 2, \dots, 7$ coefficients in (3.3.1) expression μ_{X_i} mean value, σ_{X_i} standard deviation, x_{ik} characteristic value and $w_{l,u}(\cdot)$ and $w_{sh,u}(\cdot)$ expressions for the flexural reinforced concrete beams when $q_k / g_k = 1$

Parameters	$X_1 = E_c$	$X_2 = E_s$	$X_3 = d$	$X_4 = g$	$X_5 = f_{ct}$	models of the (3.3.1) expression			
						$w_{l,u}(\cdot) = w_l(\cdot)$		$w_{sh,u}(\cdot) = w_{sh}(\cdot)$	
						$X_6 = q_l$	$X_7 = \Theta_{w,l}$	$X_6 = q_{sh}$	$X_7 = \Theta_{w,sh}$
μ_{X_i}	3,15	2,00	0,333	$\mu_g = g_k$	2,56	$\psi_2 g_k$	0,95 (0,70)	0	1,00
σ_{X_i}	0,630	0,220	0,010	$0,1g_k$	0,513	$0,24g_k$	0,23	$0,24g_k$	0,17
x_{id}	3,15	2,00	0,333	g_k	0,765	$\psi_2 g_k$	0	$q_{sh,k,j}, j=1,2,3^{1)}$	0
Measurement units	10^4 MPa	10^5 MPa	m	kN/m	MPa	kN/m	–	kN/m	–
$\partial w_l / \partial X_i$	–2,09	–4,34	–1,40	1,78	–1,84	0,50	1,0	0,50	1,0
$\partial w_{sh} / \partial X_i$	–0,47	–2,56	–0,44	–0,11	–0,07	0,79	1,0	0,79	1,0
Measurement units	10^{-7} m/MPa	10^{-8} m/MPa	10^{-1} m/m	10^{-3} m ² /kN	10^{-3} m/MPa	10^{-3} m ² /kN	–	10^{-3} m ² /kN	–
¹⁾ $q_{sh,k,1} = 0$, $q_{sh,k,2} = (\psi_1 - \psi_2)q_k = 0,1g_k$, $q_{sh,k,3} = (1 - \psi_2)q_k = 0,7g_k$.									

Determination of standardized reliability index

Standardized according to different DV codes $\beta_{SR,j}$ reliability index values due to expressions (3.4.1) and (3.4.2) are listed in table 3.4.2. The calculation of $\beta_{SR,j}$ values was performed by the following steps. The load g_k was determined by iteration and it corresponds to ultimate deflection w_{ult} . When the value g_k is determined, the $\beta_{SR,j}$, $j=1, 2, 3$ values are calculated according to (3.4.1) and (3.4.2) expressions, depending on which $q_{sh,k,j}$, $j=1, 2, 3$ case is used to reach limit state.

Table 3.4.2 Reliability index values $\beta_{SR,j}$, $j=1, 2, 3$ according to (3.4.1)-(3.4.2) expressions and the corresponding to it $\alpha_{SR,j}$ probability, which were determined from different [3-7, 15-17] DV codes generalizing the $j=1, 2, 3$ three cases of marginal stiffness management

$\sigma_Z^{1)}$ m	$f_{ct,k} - \mu_{fc,t}$ MPa	j	$q_{sh,k,j} - \mu_{q,sh}$ kN/m	$\beta_{SR,j}$	$\alpha_{SR,j}$
0.00425	−0.76	1	0	0.3	0.38
		2	1.0	0.5	0.31
		3	3.5	1.0	0.16
¹⁾ Calculating σ_Z according to (3.4.2) the $\rho_{w,l,w,sh} = 0,7$ value was used, which was calculated applying experiment					

Peculiarities of standardized in DV codes reliability of stiffness use in direct probabilistic-statistic (DPS) codes

Applying DPS codes it is better to use as criteria standardized in DV codes reliability. The values of the $\beta_{SR,saf,T}$ reliability index corresponding the group of ultimate limit state, for example strength, represent reliability during the whole T exploitation period. And the values of the $\beta_{SR,u,j}$ reliability of stiffness according to (3.4.1)-(3.4.2) represent reliability at the end of T exploitation at the freely selected moment of time, i.e. represent the reliability of stiffness upper limit during the whole T exploitation period. As it was discussed, this upper limit is chosen because of peculiarities of determination of standardized in DV codes marginal stiffness deterministic models.

Different probabilistic meaning of $\beta_{SR,saf,T}$ and $\beta_{SR,u,j}$ reliability indices is due to different damage degree in case when the safety or serviceability limit state occurs, because in case of the ultimate limit state the exploitation of element often is stopped. However, when the serviceability limit state occurs, the structure is still being exploited.

The important advantage of $\beta_{SR,u,j}$ reliability of stiffness parameters determination and use in DPS codes is that the reliability of stiffness is determined much accurately than in case if the reliability of stiffness parameter would be determined during the whole T exploitation period. Such T period corresponding $\beta_{SR,ser,T} = \mu_{Z,ser,T} / \sigma_{Z,ser,T}$ values of reliability index are determined with large relative error, because the mean values $\mu_{Z,ser,T}$ are very small for the whole exploitation period. Because of this reason the relative error of determination $\beta_{SR,ser,T}$ values several times is larger than the relative errors of determination $\beta_{SR,1,j}$ values according to (3.4.1)-(3.4.2).

4. CONCLUSIONS

1. Generalizing different DV design codes it may be stated that the deflection of flexural reinforced concrete elements is managed by choosing different $q_{sh,k,j}$, $j=1,2,3$ characteristic values of the short-term part q_{sh} of the variable load q . These values, first of all, are selected depending on damage in case when the limit state occurs and correspond to esthetic requirements, reversible, irreversible limit deflections.

So, the stiffness reliability standardized management may be compared with the safety reliability standardized management, which performance requires division into three reliability classes RC1, RC2, RC3. These classes correspond to different damage in case of limit state occurrence.

2. The determined values of $\beta_{SR,l,j}$, $j=1,2,3$ standardized reliability index are 0.3, 0.5, 1.0 (see table 4.4.2), i.e. the minimum and the maximum values of this index differs about three times, when the control of the reliability is performed by the first method. The more information about standardized reliability of stiffness control is available in EN 1990 code [16].

3. The determined interval of index $\beta_{SR,j} = 0.3 \dots 1.0$ is similar to the serviceability limit state reliability index $\beta_{SR} = 0 \dots 1.5$ intervals in ISO 2394 [16] table E 4.2 and EN 1990 [17].

4. The determined values of $\beta_{SR,l,j}$, $j=1,2,3$ reliability index may be used in direct probabilistic design and evaluation of existing structures of reinforced concrete elements.

5. The carried out calculations show that the least reinforcement area is needed in design of stiffness according to STR code (100 %). In designing according to EN and ACI codes the required reinforcement area is increased 10 % and 36 % comparing with STR code (see table 2.6.1).

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