

<table border="1"> <tr><td>Vilnius Gediminas Technical University</td></tr> <tr><td>Antanas Gustaitis' Aviation Institute</td></tr> <tr><td>Department of Aeronautical Engineering</td></tr> </table>		Vilnius Gediminas Technical University	Antanas Gustaitis' Aviation Institute	Department of Aeronautical Engineering	<table border="1"> <tr><td>ISBN</td><td>ISSN</td></tr> <tr><td>Copies No.</td><td></td></tr> <tr><td>Date-.....-.....</td><td></td></tr> </table>		ISBN	ISSN	Copies No.		Date-.....-.....	
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Author		Rakshith Gowda Hoovinahalli Kantharaju										
Academic supervisor		Laurynas Mačiulis										
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Annotation In the final Master's Thesis, the study of pointing errors that occurred due to micro-vibrations on the CubeSat/small satellite caused by the reaction wheel was investigated. Pointing errors due to the reaction wheel jitter were measured by finite element and experimental methods during the work. Structure: Introduction, Literature analysis, research on pointing errors due to reaction wheel jitter and performed finite element and experimental methods, proposed methods to analyze such errors, conclusions and suggestions, references. The thesis consists of 51 p. text without appendixes, 45 pictures, 5 tables, 35 bibliographical entries. (Thesis language : English)												
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Pavadinimas	Reakcijos ratų sukeliamų virpesių eksperimentinis nustatymas CubeSat palydove (Thesis language : English)
Autorius	Rakshith Gowda Hoovinahalli Kantharaju
Vadovas	Laurynas Mačiulis

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Anotacija
Baigiamajame magistro darbe buvo tiriamos dėl reakcijos ratų sukeltų mikrovirpesių atsirandančios "CubeSat" mažojo palydovo nukreipimo paklaidos. Darbo metu baigtinių elementų ir eksperimentiniais metodais buvo išanalizuoti ir išmatuoti poslinkiai, atsirandantys dėl reakcijos ratų sukeltų dinaminių virpesių. Struktūra: Įvadas, literatūros analizė, mokslinių tyrimų ir metodų apžvalga, atlikti baigtinių elementų bei eksperimentiniai tyrimai, išvados ir pasiūlymai, literatūros šaltiniai. Disertaciją sudaro: 51 p. teksto be priedų, 45 paveikslai, 5 lentelės, 35 bibliografiniai įrašai. (Thesis language : English)

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DECLARATION OF AUTHORSHIP IN THE FINAL DEGREE PROJECT

May 23, 2023

I declare that my Final Degree Project entitled „Experimental measurement of Reaction Wheel Jitter on CubeSat Satellite“ is entirely my own work. I have clearly signalled the presence of quoted or paraphrased material and referenced all sources. **(Thesis language : English)**

I have acknowledged appropriately any assistance I have received by the following professionals/advisers:
Doctor Laurynas Mačiulis.

The academic supervisor of my Final Degree Project is Doctor Laurynas Mačiulis.

No contribution of any other person was obtained, nor did I buy my Final Degree Project.

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Rakshith Gowda Hoovinahalli Kantharaju

**EXPERIMENTAL MEASUREMENT OF REACTION WHEEL JITTER ON
CUBESAT SATELLITE**

Master's degree Thesis

Aerospace Engineering study program, state code s 6211EX060

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Vilnius, 2023

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ABBREVIATIONS

RW – Reaction Wheel

FW – Fly Wheel

ATP – Acquisition, tracking, and pointing mechanisms

FOV – Field of View

RF – Radio Frequency

RWA's – Reaction Wheel Assemblies

INTRODUCTION

An artificial satellite is an object intentionally placed into earth's orbit to monitor the earth, transmit information and assist people in improving their lives on a daily basis, which leads to even greater innovation. In 1957 the first successful satellite "Sputnik" was launched by Soviet Union, a quick response to this US also launched their 1st successful satellite "Explorer 1" in 1958.

As compared to satellites which were designed back in 90's differ allot to the one's which are produced now in their size, working techniques, etc., nowadays Nano Satellites are manufactured as they are small and less expensive comparing to the bigger ones. One of the most prevalent uses is for communication. Previously CubeSats relied on radiofrequency (RF). As the future is developing, Laser Communication or free space optical communication is being as a mode for transmission of the data. The most difficult aspect of using this technology to a Nano satellite is precision pointing. Laser beam is relatively narrow, therefore satellite should be extremely robust, and any disruptions should be considered. Micro-vibrations are low-level mechanical vibrations which typically occur at frequencies ranging from a few Hz to 1 kHz. They're frequently caused by internal satellite systems like Reaction Wheel Assemblies (RWAs). The dynamics of the micro vibration sources will interact with those of the satellite structure, making micro vibration effects more difficult to anticipate.

This thesis shall focus on analysis and development of experimental methodology to directly measure pointing errors caused by reaction wheel jitter on a CubeSat satellite. Satellite communications require a high level of stability. The amount of pointing required for space missions is rapidly rising. Internal vibrations in the satellite have a direct impact on pointing capability and core purpose effectiveness. Reaction Wheel is one of the major sources of onboard vibrations, hence research and experimental analysis into micro-vibrations on CubeSat induced by Reaction Wheel is required.

Investigation problem. Experimental measurement of Reaction Wheel Jitter on CubeSat Satellite

Object of project. Analysis and development of an experimental methodology to directly measure pointing errors caused by reaction wheel jitter on a CubeSat satellite.

Tasks to solve the problem.

These actions are performed in order to meet the project's necessary goals:

1. Literature review on experimental methods to validate the results of Jitter
2. Initial results of Mathematical-numerical modelling/ Finite Element Method
3. Experimental analysis of vibrations caused by Reaction Wheel Jitter
4. Analysis of the result and Conclusions

1. ANALYSIS OF THE LITERATURE

1.1.CubeSat

A CubeSat of $(10 \times 10 \times 10)$ cm is termed as CubeSat 1 U (one unit) and extends in one direction only. Similarly, CubeSats 2 U and 3 U have also been built and launched for a specific mission and it weighs between 1 to 1.33 kg. There are several types of CubeSats that have been developed and deployed for specific mission such as research and development satellites, earth remote sensing satellites, and space tethers satellites. Typical examples of earth remote sensing CubeSats are Quakesat, SwissCube, PLUME, and Firefly, and so forth.

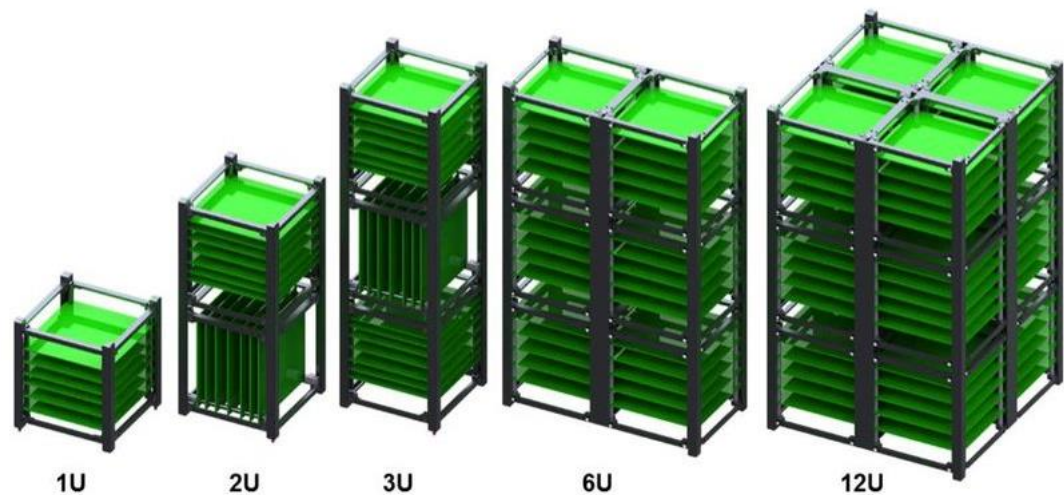


Figure 1. CubeSat sizes (Radius Space)

The various international CubeSat programs have developed in some universities, private companies, and government organizations due to the low-cost of developing the CubeSat satellites (Duijsens et al., n.d.). CubeSats have become an exciting innovation in the world of space research and applications, with objectives for a variety of space aspects such as education, science, and defence, being pursued. Initially, researchers were using CubeSats only in low Earth orbit (LEO) for communications, but recently they have even been planned for deployment around the moon, Mars moons and potentially even Jupiter moons and Saturn moons (Duijsens et al., n.d.). In our solar system, there are more than 200 moons with different shapes and sizes, some of them have atmospheres and oceans. The cold, dusty, desert world with a thin atmosphere is the fourth planet in the solar system with the name of Mars. Jupiter is the giant planet in the solar system, and Saturn is the second largest one (Israr, 2014a). Up until May 31st, 2018, the total number launched CubeSats was 855. Currently, the leading application sector for CubeSats, remote sensing, corresponds to about 45% of all launches. Now, with the development of CubeSats, a total of fifty-eight countries have already been involved (Chen et al., 2019a). While CubeSats are small satellites, with weights significantly less than 300kg (1100 lb), they must still be approved to a specific series of standards

for controlling the factors such as shape, dimensions and mass (Weipeng et al., 2014). As noted earlier, the first launches of CubeSats were in the early 2000s, and they were very simple satellite designs to achieve the demands of student education or to meet some amateur needs. A CubeSat can operate solely or in a group of multiple units (maximum 24 units) as a ‘constellation’.

The expected length of a cube-shaped (1U) pico-/nanosatellite is 100 mm per side, and it may not weigh more than ~1.3 kg for every single 1U CubeSat. So, for double (2U) and triple (3U) designs, they should weigh be less than ~2.6 kg and ~3.9 kg, respectively. The dimension for a double is 227 mm, and 340.5 mm for a triple; the changes should apply on the Z direction only, so the X and Y dimensions are fixed (at 100 mm) (de Lellis et al., 2019). In the launch scenario, the CubeSat must be strong enough to survive launch without damage or dimensional change in the deployer. The cube's edges need to have a minimum radius of 1 mm, to ensure smooth movement within the confines of the deployer. The two crucial structural requirements of the rails are to have flat faces and rounded corners (around 1 mm radius) (Ampatzoglou & Kostopoulos, 2018).

Figure 2. shows an example of a 1U CubeSat. It contains a frame, different panels, power board, command and data handling (CDH), electronics, payload electronics, a camera, and the actual payload. CDH is the system for controlling the other CubeSats subsystems such as communication, ACDS (attitude determination and control subsystem) and payload(Kim et al., 2010).

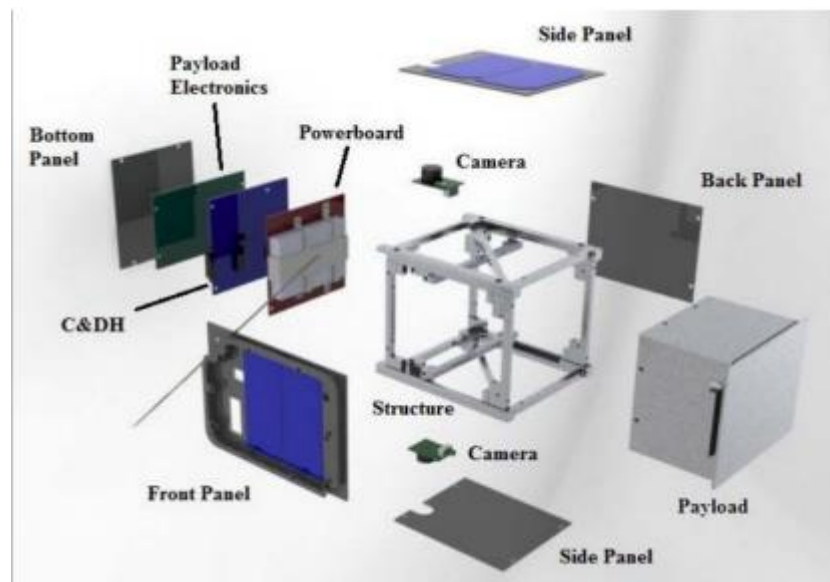


Figure 2. 1U CubeSat (Eric Olson,...2019)

CubeSats need to have some general requirements to allow a regular mission operation. Firstly, they are entirely self-contained from the moment they are charged in the deployer and then transported to the ISS. Any supports or charging of batteries is prepared after ending the integration. Secondly, during the launch, detachable parts, or the creation of any 13 space foreign object debris (FOD) is

forbidden. Thirdly, the fragile materials used for solar cells should meet suitable approval requirements (Inamori et al., 2013a). In terms of the electrical system requirements, during the launch, any electronics must not be active due to avoiding any electrical interference (or interface (I/F)) with the launch vehicle. CubeSats have rechargeable batteries, and during the launch, they must be fully deactivated. Below there are some operational requirements for integration and the related legal obligations for the safety of the CubeSats and associated systems (e.g., the ISS):

- 1- The capability of receiving the transmitter shutdown command for CubeSats with rechargeable batteries.
- 2- The antenna and solar panels may be deployed a minimum of 30 seconds after ejection of the CubeSat from the deployer.
- 3- Preparing an orbital FOD reduction plan by developers and obtaining the approval documentation with NASA space debris mitigation rule recorded in NASA Technical Standard (NASA-STD-8719.144)(de Lellis et al., 2019).

1.2.Acceleration Loads

The below table shows the acceleration loads in all six directions, so the structures should be designed to survive in the accelerations documented and potentially for scenarios when all of them are acting simultaneously.

Table 1. Launch load Factors.

	Nx (g)	Ny (g)	Nz (g)
Launch	+/- 7.0	+/- 4.0	+/- 4.0

All of the safety-critical structures must be identified to determine the requirements for verification. The safety-critical aspects are considered in all CubeSats designs because of the need to avoid failure of the elements and the associated production of space debris; they could potentially impact ISS visiting vehicles (which is known as a catastrophic hazard in terms of ISS strategy) (Inamori et al., 2013b).

1.3.Vibration

- a) The Static Analysis is for estimating the stress, strain, displacements, and forces in the structure. The weight of the satellite system and the gravitational force of (gforce), in this analysis, are applied to the centre of the entire satellite with the fixed bases on the bottom (Israr, 2014b).
- b) In Modal Analysis, the fundamental frequency of the whole structure must be ≥ 30 Hz, according to global stiffness requirements. This frequency will help the entire system to

survive during the launch. In this modal analysis, to obtain the frequencies and mode shapes of structure, the dynamic loading condition is needed (Sonawane & Talmale, 2017).

- c) Harmonic analysis, like the static one, is conducted to obtain the maximum stress on the system.

1.4.Reaction Wheel and its Jitter

On satellites, a response wheel is a key component for attitude control and pointing capability. As shown in Figure 3, a RW consists essentially of a flywheel (spinning mass) supported by ball bearings and operated by a brushless Direct Current (DC) motor.



Figure 3. Reaction wheel structure (Sensel....2012)

RWs can spin in both direction and have a notional zero speed (or several hundred rpm to avoid stiction concerns near zero speed (Mankour et al., 2021)). It can be utilized as a MW to deliver momentum imbalance to the satellite if the speed of rotation can be maintained at 10000 rpm. Some of them, particularly small satellites, can perform both jobs.

The residual error, after displacement and smear are subtracted, is called jitter. Main sources of jitter are,

- Flywheel mass imbalances
- Ball-bearing imperfections and friction
- Torque ripple and cogging torque

1.5. Different types of vibration measurement tools reviewed

Microvibrations induced by a Cantilevered Wheel Assembly with a Soft-Suspension system
by Zhe Zhang and Guglielmo Aglietti. Mechanical disturbances (particularly rotating components like

reaction wheel assemblies and momentum wheel assemblies) onboard a spaceship can substantially compromise the operation of sensitive instruments. Wheel-induced high-frequency (above 100–200 Hz) vibrations are notoriously difficult to regulate, owing to a variety of factors ranging from bearing noise to dynamic amplification due to internal resonances. In this Journal they investigated the dynamic response of a newly developed wheel assembly with a cantilevered flywheel structure assisted by a soft-suspension system. The mathematical model for such wheel assembly is created and then verified by vibration tests. A seismic-mass microvibration measurement system, which is an alternative to traditional microvibration measurement setups, is used to accurately detect wheel-assembly-induced lateral and axial microvibrations. Finally, through a comparison of frequency spectra, the performance of this wheel assembly in terms of microvibration emissions is compared to a standard design (with a stiff suspension), and it is proven that this design produces substantially lesser vibrations at high frequency (Zhang et al., 2012).

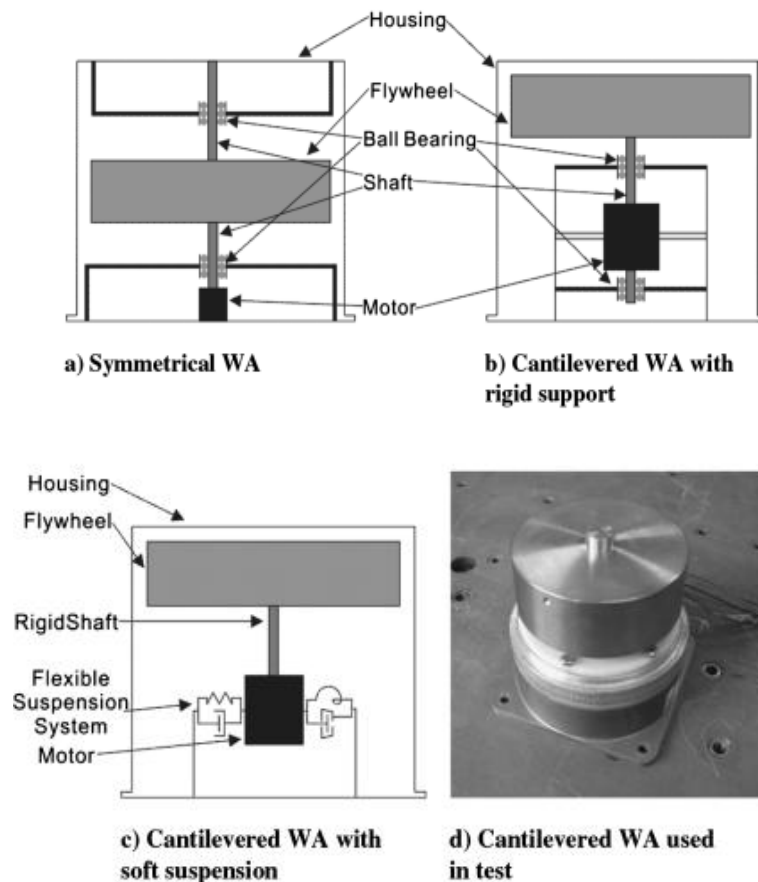


Figure 4. Cantilever configured WA with soft-suspension system (Zhe Zhang.... 2011)

Coupled microvibration analysis of a reaction wheel assembly (WA) by the research group of University of Southampton shows the complete analysis of coupled microvibrations of a WA-structure system. In addition, this article describes a microvibration measurement system to measure WA-induced microvibrations in coupled boundary condition that has been developed to support the

work. Several approaches for reproducing the forces and moments created by a cantilever-configured WA when connected with a supporting structure (in this example, seismic mass) have been constructed and compared to test results. The traditional analysis method, which relies solely on hard-mounted microvibrations as input stresses to the supporting structures, has been shown to miss significant aspects in the projected responses. The classic coupled analysis technique, which employs the accelerances of both the WA (flywheel properties only) and the supporting structure to correct the hard-mounted data and mimic the effects of the dynamic coupling, has been demonstrated to offer better test results predictions. However, because this method does not account for WA gyroscopic effects in the accelerance, this article proposes a procedure for calculating the accelerance that does. The suggested method's results are compared to test results and indicate that it outperforms other available analysis methods in the literature (Zhang et al., 2013). The measurement system developed here is an improved version of the “Seismic Mass Vibration Measurement System (SMVMS)”

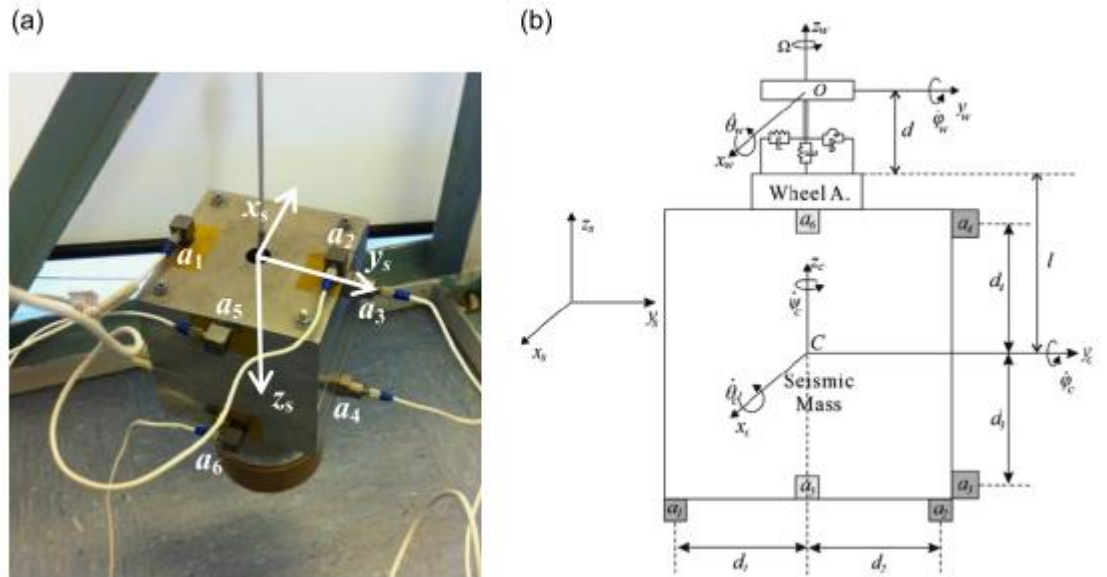


Figure 5. The SMVMS and Its simplified model (Zhe zhang....2013)

Sources of reaction wheel jitter:

Due to static and dynamic imbalances in reaction wheel assemblies, reaction wheels are the most common source of high amplitude jitter disruption in many satellite missions.

Each imbalance can be modeled as shown in Fig. 6 (Hatutori et al., 2004) (Taniwaki et al., 2005)

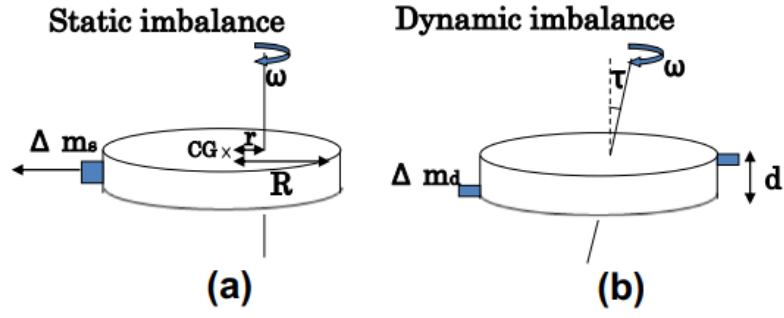


Figure 6. Disturbance model (Hatutori et al., 2004)

Static imbalance is the condition where the principal inertia axis of a rotor is offset from and parallel to the axis of rotation. Static imbalance can theoretically be corrected by the addition of a single correction mass. Static imbalance and the disturbance caused by the static imbalance can be expressed as follows:

$$\begin{aligned} F_s &= M_r \omega^2 \\ &= \Delta m_s r \omega^2 \\ &= u_s \omega^2 \end{aligned} \quad (1.1)$$

where Δm_s , u_s , r , and ω are the unbalance mass that causes static imbalance, the static imbalance, radius of the wheel, and angular velocity of the wheel, respectively. If the reaction wheel is mounted at a distance l from the CG, then the disturbance torque caused by the static imbalance is

$$T_s = u_s \omega^2 l. \quad (1.2)$$

Dynamic imbalance is the condition where the principal inertia axis is not parallel to the axis of rotation, as shown in Fig. 5. The disturbance torque caused by the dynamic imbalance is expressed as follows:

$$T_d = \Delta m_s r d \omega^2 \quad (1.3)$$

$$T_d = u_d \omega^2 \quad (1.4)$$

The imbalance of a rotor induces a centripetal force at the frequency of the rotation. In the research, they assumed that a reaction wheel to have only these two types of disturbances (Inamori et al., 2013a).

In the research paper (Inamori et al., 2013a), the jitter of the reaction wheels is reduced by using accurate sensors, small reaction wheels, and slow rotation speed RW (Reaction wheels). The objective of the reaction wheel, in many satellite missions, is to manage the satellites angular momentum, which increases because of attitude disturbances. If the magnitude of the disturbance is reduced using the other actuators, the magnitude of the angular momentum that the reaction wheels

gain in orbit becomes smaller. Therefore, satellites can stabilize their attitude using only smaller reaction wheels, which cause relatively smaller vibrations. In nano- and micro-satellite missions, the dominant attitude disturbance is the magnetic torque, which can be estimated and cancelled using an extended Kalman filter and magnetic actuators.

Experimental analysis of microvibrations is using Acceleration Sensor-Based Jitter measurements. The current study employs an approach that combines calculations and tests associated with high acceleration sensors to address microvibration impacts on an optical satellite. By inputting flywheel disruption data from a six-component test bench to the optical satellite's finite element model, the displacement and angular displacement of each electro - optic component may be calculated. The pixel offset of the entire optical system is derived using the pixel offset and an optical amplification factor deduced from the linear optical model. To validate the simulation, a new microvibrations measurement experimental device with excellent precision and a large frequency range is built(Chen et al., 2019b).

Testing the acceleration signals of each optical component and determining optical amplification factors can be used to determine the pixel offset of the entire optical system. The results are congruent with optical imaging test results, demonstrating that the experimental scheme is correct and that the simulation is effective. The findings reveal that the influence of microvibrations on a camera can be confirmed using mechanical simulators rather than a full optical camera in the experiment scheme, which has been shown to be an efficient technique of enhancing jitter measurement efficiency (Kim, 2014; Zhang et al., 2014).

The test of flywheel perturbation is performed with a six-component test bench (Kistler table/HR-FP3402) installed in the air bearing a floating platform, as shown in Figure 6. The flywheel is fixed to the test bench through a rigid fixture (GOCE Platform Micro-Vibration Verification By Test And Analysis - NASA/ADS, N.D.; TANIWAKI & OHKAMI, 2003). Under push broom imaging conditions during orbit, the flywheel spins at a constant rotation speed. Therefore, the output data are intercepted under steady state in different rotation speeds. In light of the misalignment between the centroid of the flywheel and the centre of the test bench, the perturbing force/moment of the flywheel can be calculated in the following equation:

$$F_i = F_i^t \quad (1.5)$$

$$M_i = M_i^t - \Sigma(F_i^t \Delta) \quad (1.6)$$

where F_i/M_i is the perturbing force/moment of the flywheel in each direction ($i = 1, 2, 3$); F_i^t/M_i^t is the measured perturbing force/moment in each direction; and Δ represents the distance from the centroid of the flywheel to the center of the test bench in each direction.

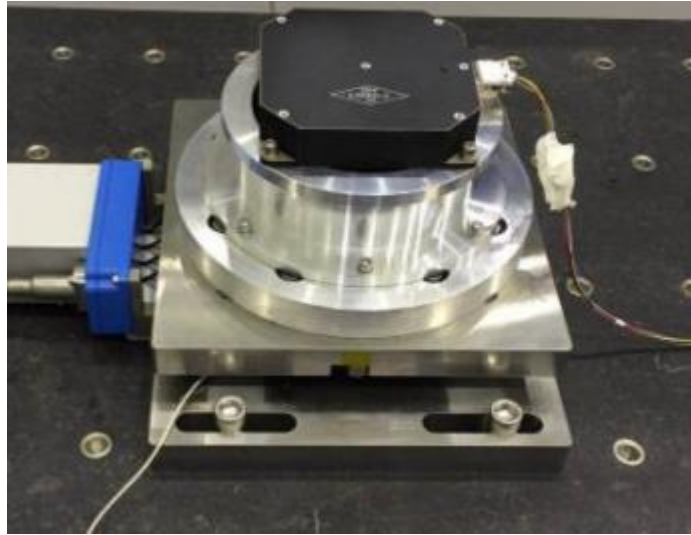


Figure 7. Testing of Flywheel perturbation (Zhou...2012)

The disturbing force/moment of the flywheel is attributed to several reasons, including rotor imbalance, bearing irregularity, flexibility of the flywheel structure, and nonlinear stiffness of bearing (Zhou et al., 2012).

Another measuring methods was also conducted by considering the disturbance of reaction wheel assembly is related with the fixing condition, one is ‘rigid six-axis test bed’ and another is ‘flexible six-axis test bed’. During this method comparison results show that same harmonic numbers are obtained in different measuring conditions, but lower stiffness of fixing condition reduces the natural frequency of RWA. As a result of the dampening effect of flexible fixing conditions, disturbance peaks are reduced (de Lellis et al., 2019).

Considering all these methods we planning to use Accelerometer sensor to validate our results, where the 1U Cubesat is hanged using a string and at the bottom by using weak spring we evaluating the results (Kennedy Space Center et al., 2007, Chen et al., 2019a)

1.6. Acquisition, Tracking, and Pointing Mechanisms

ATP Mechanisms used in free-space optical (FSO) communications systems. It helps to avoid or reduce pointing errors by continuously measuring system-wide performance metrics, that are obtained indicators energy and Strehl ratio, which may be defined because the ratio of the height aberrated picture depth from a factor supply as compared to the most plausible depth the usage of a perfect optical gadget constrained handiest through diffraction over the gadget’s aperture (Kaymak et al., 2018).

Pointing, Acquisition and Tracking

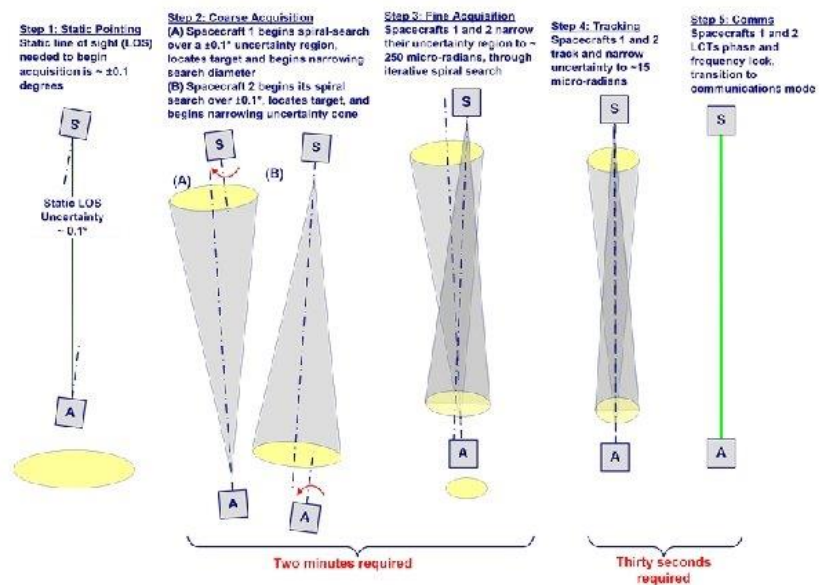


Figure 8. Concept of acquisition link. (Davis H. Hartman)

Pointing is the technique of aligning a transmitter within the field-of-view (FOV) of the receiver. Signal acquisition is the alignment approach of the receiver in the arrival path of the beam. Tracking is the upkeep of every pointing and signal acquisition at a few level within the optical communications amongst speaking terminals. In addition to ATP mechanisms, an appropriate desire of the beam divergence also can moreover allow to mitigate pointing errors. (*Pointing, Acquisition, and Tracking Systems for Free-Space Optical Communication Links*, n.d.)

Mirror-Based ATP Mechanisms:

Mirror-based ATP mechanisms use fast Steering Mirrors (FSMs) to perform beam stabilization, pointing, and tracking. FSMs are one means to compensate for disturbances that could keep the laser beam from hitting its target receiver precisely. (*Fast Steering Mirrors (FSM) | Controller Performance Tuning | Piezo Tip/Tilt | Manufacturer*, n.d.)

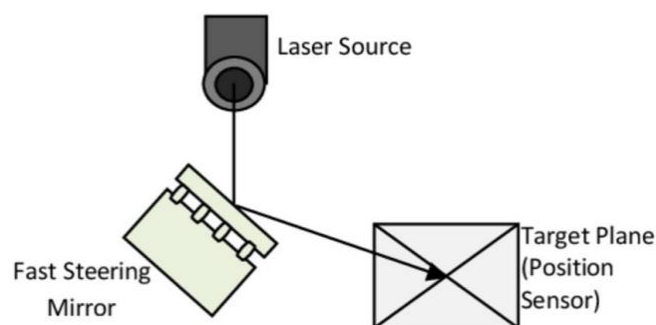


Figure 9. Simple concept of FSM (*Fast-Steering-Mirrors-Improve-Active-Beam-Stabilization*, n.d.)

FSMs are ordinarily desired for first-class-grained and excessive-velocity ATP operations in loose area optical communications, due to the fact they may be mild weight. As a result, FSMs have excessive steerage velocity and first-class pointing resolution (Andrews et al., 2005). One downside of rapid steerage mirrors is their restricted angular variety of movement which could limit their field-of-view for records acquisition or tracking. FSMs can be kind of categorized into 3 classes in step with their actuator types: mechanical, piezoelectric, and microelectromechanical systems (Hemmati, 2009).

1.7. Satellite pointing stability

All satellites are designed to perform a specific mission. Earth observation, sign retransmission, and different superior clinical missions require excessive satellite tv for pc pointing balance. Mechanical disturbances are an issue due to the fact they lessen it. Low pointing balance motive an unfavourable impact at the optical functionality of satellites instruments. In the picture (see Figure 10) low pointing balance because of mechanical disturbances is presented (Song et al., 2017).

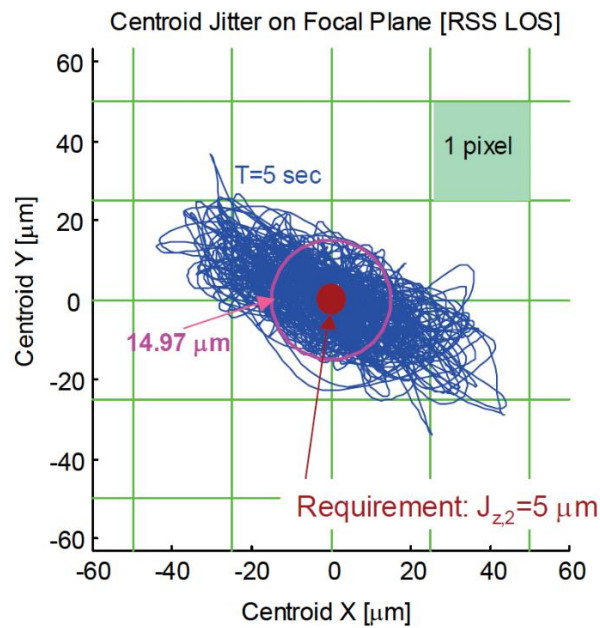


Figure 10. Unstable pointing(Valentin, 2017)

In addition to external disturbances such a gravity gradient, aerodynamic drag, and solar radiation, onboard disturbances also contribute to the mechanical disturbances. The onboard disturbances consist of transient, random disturbances and vibrations (i.e., periodic signals) (i.e. non-periodic signals) (Wang & Jing, 2019).

- The operation of numerous machineries, including reaction wheels, momentum wheels, cryogenic coolers, and solar array driving mechanisms, causes vibration. Because the resulting vibration on the satellite level is typically minimal, in the region of micro-g, the vibrations in these mechanisms are known as micro-vibrations. (TANIWAKI & OHKAMI, 2003).

- Antenna pointing mechanisms, mirror scan mechanisms, latch valves, thrusters, stress releases from thermal expansion/contraction, and foil buckling are some of the components that cause transient and random disturbances.

Note that in some cases the vibrations can be transmitted by some payload elements to the satellite structure, thus in this case the payload is the source and the vibration receiver.

The micro-vibrations caused by reaction wheels at the satellite level can be reduced using a variety of techniques. The first alternative is to use a reaction wheel, which has higher capabilities for producing micro-vibrations. Due to reaction wheel design and manufacturing techniques, it might not be enough to reduce the levels. Micro-vibration is decreased by extending the transmission line from the reaction wheels to the payload. RWs are often placed as far away from the payload as possible in mechanical construction, and particular bracket designs are suggested to reduce micro-vibrations. Another choice for interrupting the transmission line is to use dampers or isolators.

1.8. Jitter

To analyse the stability performance of the satellite, three categories of pointing faults can be established. They are jitter, smear, and displacement. Displacement is the average pointing angle disparity measured throughout an exposure interval at a constant value. Smear refers to the linear fluctuation in attitude during imaging. Smear and displacement should, ideally, not contain noise for 20 brief intervals. The remaining mistake, known as jitter, results from the removal of displacement and smear. The main jitter sources and techniques for reducing them are discussed in this section.

Previously in section 1.5. we already talked about Flywheel mass imbalances. So let's look deeply into other variations which are responsible in causing jitter inside a reaction wheel.

1.8.1. Ball-bearing imperfections and friction

Imperfections related to ball bearings can be found in the inner and outer races, the balls themselves, and even the bearing cage. These flaws include mechanical run-out, uneven surfaces, out-of-roundness, and ball faults. These anomalies have the potential to cause disturbances at the RW rotation rate's subharmonic and higher order harmonics. Although bearing-related disturbances are typically of secondary concern, as RW manufacturers develop their capacity to mass-balance their wheels, bearing-related disturbances may eventually overtake other sources of disturbance torque. The effects of friction in a RW motor can be separated into coulomb friction, which is constant with polarity depending on the direction of rotation of the wheel, and viscous friction, which changes with speed and temperature. Rarely are friction and Stribeck friction effects observed. Any thorough simulation of friction is typically unnecessary.

1.8.2. Torque ripple and cogging torque

The change in motor torque brought on by the type of commutation employed and the form of the back electromotive force is known as torque ripple. When operating close to zero speed, the torque ripple can be large because it is inversely related to wheel speed.

Analysts have a strong justification to rule out this disturbance source from jitter forecasts because resonances are at a minimum at zero-wheel speed. This disturbance, known as cogging torque, can be found in a typical brushless DC motor. This is because as the magnets rotate at a frequency corresponding to the number of poles in the rotor and teeth in the stator, the resistance of the iron stator changes. Thus, it is not dependent on the speed of the wheels, and ironless motor designs do not have cogging torque.

With frequency, the torque ripple's amplitude grows. More so than 0.01 Nm, the low-frequency torque ripple is minimal. A high amplitude (less than 0.01 Nm) high-frequency torque ripple (greater than 500 Hz) is present. However, the spacecraft structure does not adequately convey the vibrations at this high frequency to the cargo. This thesis does not continue to examine the torque ripple for this reason.

1.8.3. Methods for jitter reduction

For high-frequency disturbances, a standard-sized satellite generally uses isolators. An isolator is a device that absorbs waves having oscillatory energy from the reaction wheels. In many space missions, however, they require higher power consumption and mass, which is unacceptable in nano and microsatellites. Instead of isolators, the jitter can be reduced using accurate sensors, small reaction wheels, and slow rotation speed RW. A satellite can reduce its magnetic disturbance with accurate gyro sensors and magnetometers, enabling small reaction wheels with more negligible jitter.

This method's accurate gyro sensors and magnetometers can also be used for accurate attitude control. In addition, smaller reaction wheels reduce a satellite's mass, space, and power consumption. The RW disturbance gets smaller with the smaller rotor mass and its rotation speed. RW disturbances are reduced by smaller reaction wheels or slow rotation speed. In general, the disturbance of a reaction wheel depends on the reaction wheel size and speed, i.e., a more significant reaction wheel and higher rotation speed cause more substantial disturbances.

2. ANALYSIS OF RESEARCH METHODS

In this thesis 1U CubeSat vibration analysis are performed. Experimental test performed with RW attached in 1U CubeSat placed/Hanged to the designed Vibration test bench.

2.1. Natural frequency

Body with herbal frequency is a frame that vibrates unquestionably when disturbed (eigenfrequency). Any type of pressurized frequency that is equal to the herbal frequency of an item can be used. In this example, we are increasing resonance, which causes oscillations on the natural frequency of the object. If this occurs in a small number of cases, the oscillations will continue to rise in strength, ultimately leading to structural failure. This oscillating frequency is the natural frequency, and we measure it in Hz (hertz) and calculating by this formula:

$$f = \omega \div 2\pi \quad (2.1)$$

Here, the ω is the angular frequency, that are measured in radians and calculated:

$$\omega = \sqrt{(k \div m)} \quad (2.2)$$

So eigenfrequency can be calculated:

$$f = \sqrt{(k \div m)} \div 2\pi \quad (2.3)$$

Where, m is the mass of body and k is the body constant.

Typically, eigenfrequency can be described as single most critical property of virtually any system. Excessive vibrations in any system lead to functional or structural failures. The reason for this is the natural frequencies can match with systems resonant frequencies. For example, if you employ a time varying force to a system and select a frequency equivalent to one of the eigenfrequencies, this will result immense amplitude vibration this risk to system failure.

2.2. Modal Analysis

Modal evaluation is the examination of dynamic homes of shape that are subject to Neath vibrational excitation. The widely used evaluation is quasi-static evaluation, in which the weight is carried out at a very slow charge, so the acceleration is essentially zero. Dynamic evaluation is the opposite, and the result of acceleration cannot be ignored. Both types are the selected input (eg, the pressure performed by the machine) and the reaction of the machine (eg, the displacement of the machine due to its load). Compared to quasi-static and dynamic, modal evaluation provides an overview of the limits of a machine's response.

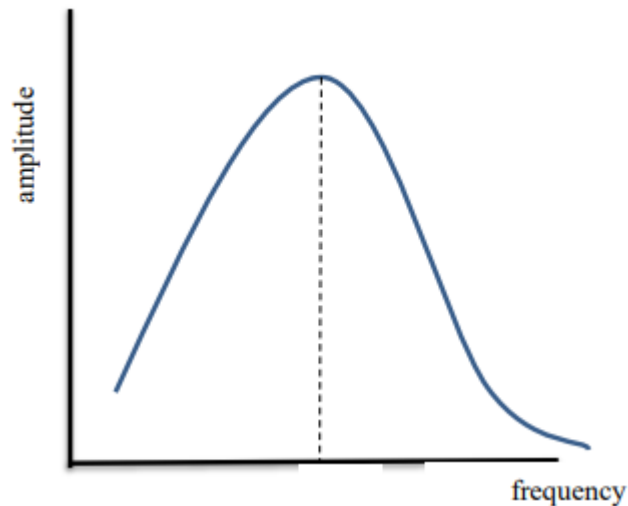


Figure 11. Amplitude of system response

As shown in the Figure 11, every object has an internal frequency (or resonant frequency) at which the object can naturally vibrate. It is about the frequency at which energy can be transferred between forms with the least amount of loss, specifically from vibrational to kinetic energy. As the frequency gets closer to the resonant frequency, the response amplitude increases infinitely.

2.3. Time Domain and Frequency Domain

The time domain is where all the signals are represented. Time-domain signals can be analysed using an oscilloscope. This can help to ensure the signal is valid and intact.

In time domain signals are represented by amplitude on Y axis and time on X axis.

The frequency domain can provide more detail about a time-domain signal. This is useful for deeper analysis. Frequency domain analysis is useful for studying the frequency contents of discrete time domain signals as well as continuous time domain signals. The frequency domain signal can be analysed using a spectrum analyser. This allows for detailed analysis of the signal's frequency content.

In frequency domain signals are represented by power(amplitude) on Y axis and frequency on X axis.

Time domain signal can be converted to Frequency domain signal with the use of Discrete Fourier Transform or Fast Fourier Transform(FFT).

Figure-12 depicts representation of various waveforms in time domain and frequency domain. The waveforms include sinewave, triangle, sawtooth, rectangle, pulse etc. As shown sine wave will have single peak at frequency of the sine wave. As we know frequency is the inverse of time period of the sinewave.

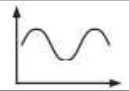
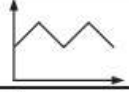
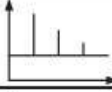
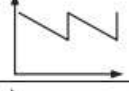
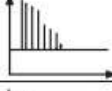
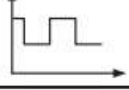
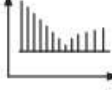
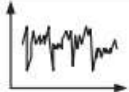
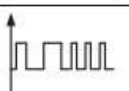
Waveform	Time domain	Frequency domain
Sinewave		
Triangle		
Sawtooth		
Rectangle		
Pulse		
Random noise		
Bandlimited noise		
Random binary sequence		

Figure 12. Representation of various waveforms(Difference between Time Domain and Frequency Domain, n.d.)

- **Fast Fourier Transform**

The Fourier transform is a mathematical operation that converts waveform data in the time domain into the frequency domain. This is useful for analysing and understanding the patterns and characteristics of waveforms. The Fourier transform achieves this by breaking down the original time-based waveform into a series of sinusoidal terms, each with a unique magnitude, frequency, and phase. This process effectively converts a time domain waveform that is difficult to describe mathematically into a more manageable series of sine functions that, when added together, reproduce the original waveform exactly. The amplitude of each sinusoidal term in a waveform change with its frequency, creating a power spectrum. Figure 13 illustrates this time to frequency domain conversion concept (Ronald N,...1989)

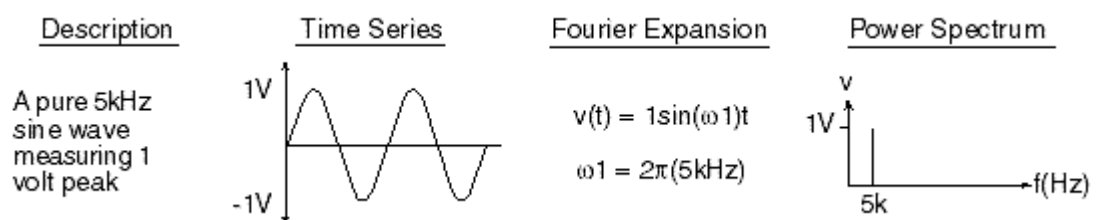


Figure 13. Illustration of Time to Frequency domain (Ronald N,...1989)

2.4. Time Harmonic Loads

An imbalanced wheel, for instance, can cause your car's structure to vibrate at the same frequency as the wheel's rotation by creating sinusoidally time-varying stresses. All of these partial differential equations share the same basic structure (Walter Frei, 2016).

$$Mu_{tt} + Cu_t + \nabla * (-K\nabla u) = F \text{ on } \Omega \quad (2.4)$$

$$n * (K\nabla u) + Au = f \text{ on } \Gamma_1 \quad (2.5)$$

$$u = g \text{ on } \Gamma_2 \quad (2.6)$$

Where $u = (x,t)$ is the space and time-varying field of interest (displacement). The terms M , K , and C represent different material properties and are usually termed the mass, stiffness, and damping terms, respectively. The term $F = (x,t)$ represents the domain's scattered loads. Additionally, a set of boundary conditions that represent the loads is present at all times $f = (x,t)$, and constraints, $g = g(x,t)$, on the system.

Response of the displacement of this system over time may now be calculated if we additionally have a set of initial circumstances, such as $u = u(x,t = 0)$ (see Figure 14). The mass f 0 frequency amplitude 30 oscillates back and forth under these circumstances, starting at zero. The displacement becomes periodic after a period of initial start-up time; it repeats sinusoidally with a constant angular frequency and will be independent of the original setting.

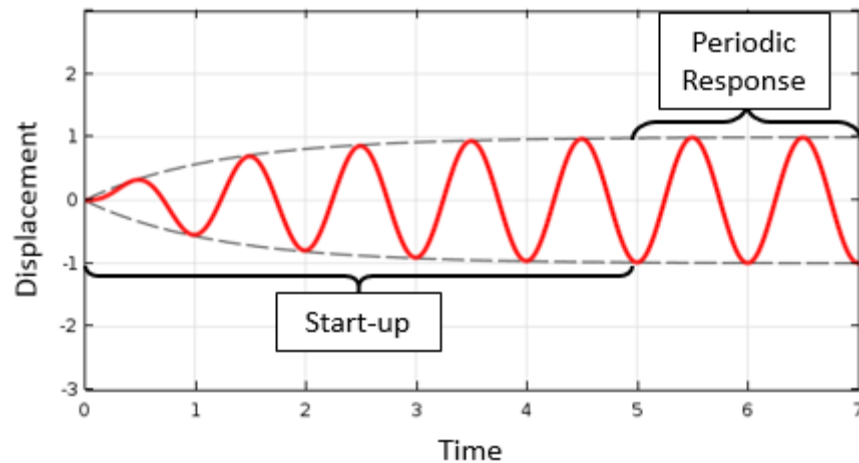


Figure 14. Simple Harmonic oscillator (Walter Frei, 2016)

2.5. Displacement amplitude

In physics, amplitude refers to the greatest displacement or distance that a point on a vibrating body or wave can move relative to its equilibrium location. It is equivalent to the vibration path's half-length. The bob travels halfway across the pendulum's amplitude when it moves from one to another. Sources that vibrate produce waves, and the amplitude of the waves is inversely proportional to the source's amplitude (Walter Frei, 2016).

2.6. Harmonic response analysis of RW attached to a cantilever beam.

As an initial date we planned to take measurements using a Reaction Wheel attached to a cantilever beam placed on a Vibration Isolation table to have a stable environment to opt out the results. Throughout this experiment we use a camera to record the images and a mirror to adjust pointing and stabilize beam of Laser. Refer to Figure 15

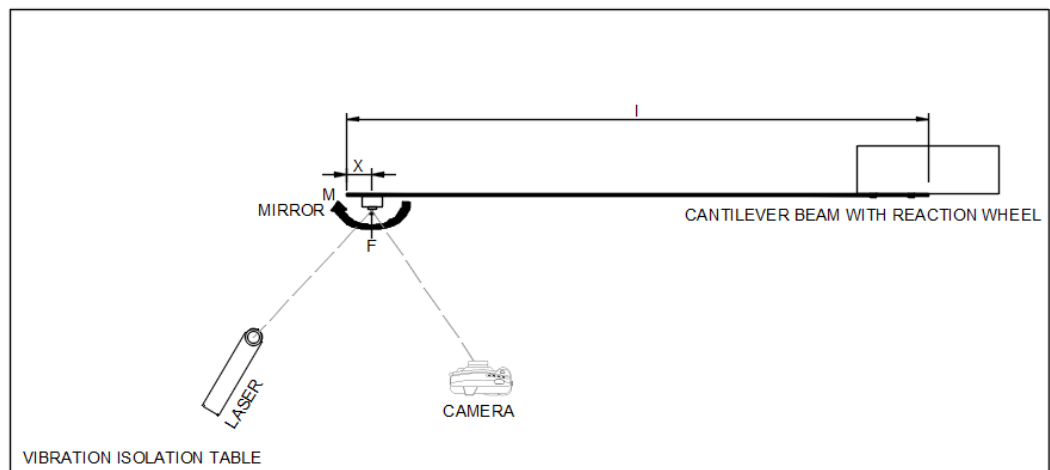


Figure 15. Schematic representation of experimental design

From the Literature analysis we know that the measured imbalance of the reaction wheel is about 59,8g-mm while the manufacturer's claim of imbalance was <50 g-mm (Laurynas Šišovas MT, 2021). By keeping this value as reference we analysed Harmonic response of beam from Ansys software. And also we are representing the same conditions to have experimental values.

Specifications of the reaction wheel used in the project of Laurynas Šišovas are presented in the table below,

Table 2. RW Specifications

Dimensions	43.5 mm x 43.5 mm x 24 mm
Temperature range	-40 °C to +85 °C
Mass	140 g
Maximum speed	6500 RPM
Maximum torque	3.2 mNm
Maximum momentum storage	20 mNms

Finite element analysis results will be compared with experimental test results.

Table 3. RW vibration modes.

MODE 1	689,07 Hz
MODE 2	703,69 Hz
MODE 3	751,24 Hz
MODE 4	808,96 Hz

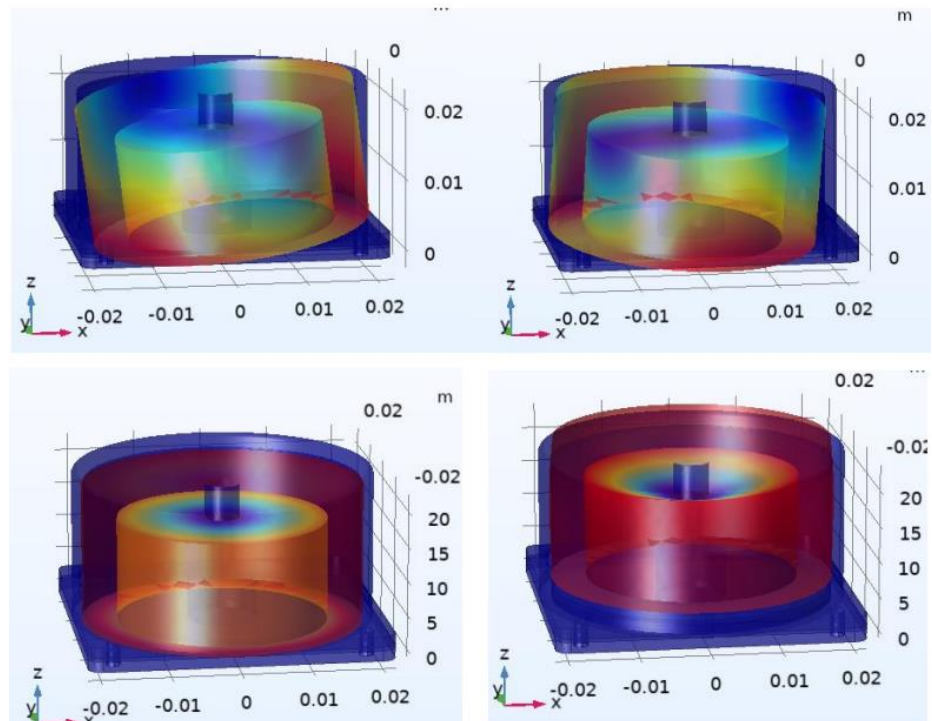


Figure 16. Reaction wheel deformations at different modal frequencies.(Laurynas_Šišovas_MT, 2021)

2.6.1. Finite element analysis of cantilever beam

Detailed finite element analysis (Vibration analysis) is presented in this section. Analysis is performed by Ansys software, using boundary conditions and results as mentioned before in section 2.6 and in the table 1 and 2.

In this section we are analysis Harmonic response of cantilever beam attached with reaction wheel. Conditions applied during this analysis are.

- Reaction wheel attached directly to the beam
- Reaction wheel attached with an angle of 90°

Analysis of Natural frequency of the beam used:

The natural frequency of a beam refers to the frequency at which it tends to vibrate freely when subjected to an external force. It is an essential characteristic of the beam's dynamic behaviour.

The formula to calculate the natural frequency of a beam depends on several factors, including the beam's material properties, geometry, and boundary conditions. The most commonly used formula for calculating the natural frequency of a supported beam with a uniform cross-section is given:

$$f = \frac{1}{2\pi} \sqrt{\frac{EI}{mL^3}} \quad (2.7)$$

Where:

f is the beam's natural frequency,

E is the material's modulus of elasticity,

I is the cross-sectional moment of inertia,

m is the beam's mass in units of length

L is the measurement of the beam.

It is important to note that this formula assumes that the beam is vibrating in its fundamental mode, which means it vibrates with the lowest possible frequency. For higher modes of vibration, additional factors and equations are involved.

Also, it is worth mentioning that the natural frequency can be affected by various factors such as boundary conditions (e.g., clamped, supported, free), distributed loads, and other complexities in the beam's geometry. In real-world scenarios, finite element analysis or experimental testing is often used to determine the precise natural frequencies of complex beam structures.

We have chosen beam length of 1000 mm, width of 50mm and thickness of 2 mm. Material used for this analysis is structural steel.

Table 4. Frequencies from modal analysis

Mode	Frequency [Hz]
1.	1.6445
2.	10.306
3.	28.866
4.	40.748
5.	56.597
6.	63.217

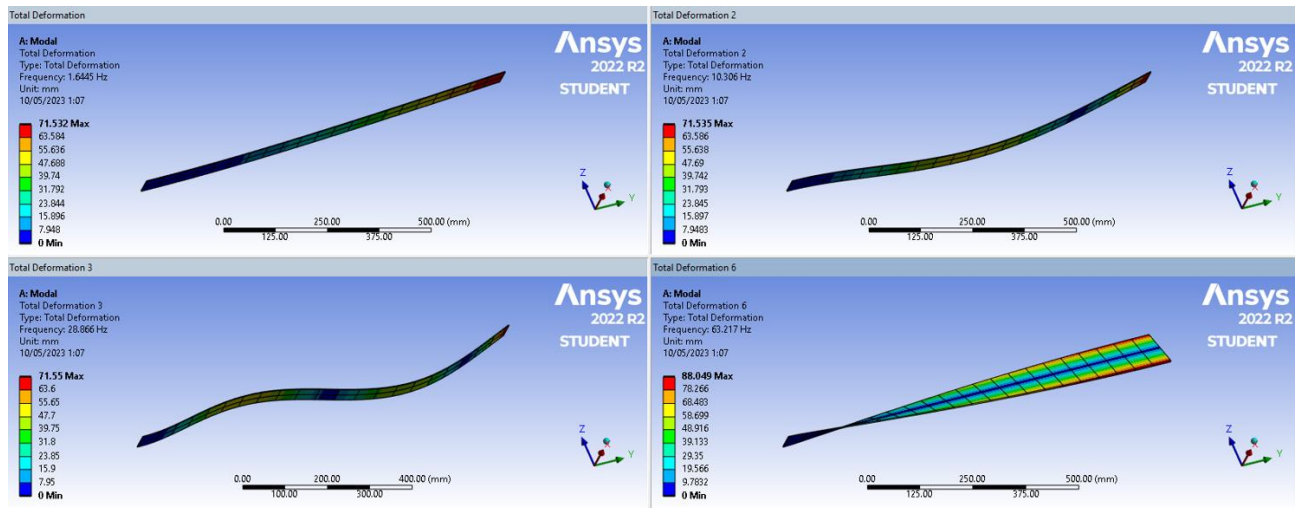


Figure 17. Modal analysis of Cantilever beam.

We have used minimum frequency of 1 Hz to maximum frequency was 786 Hz during Harmonic response analysis.

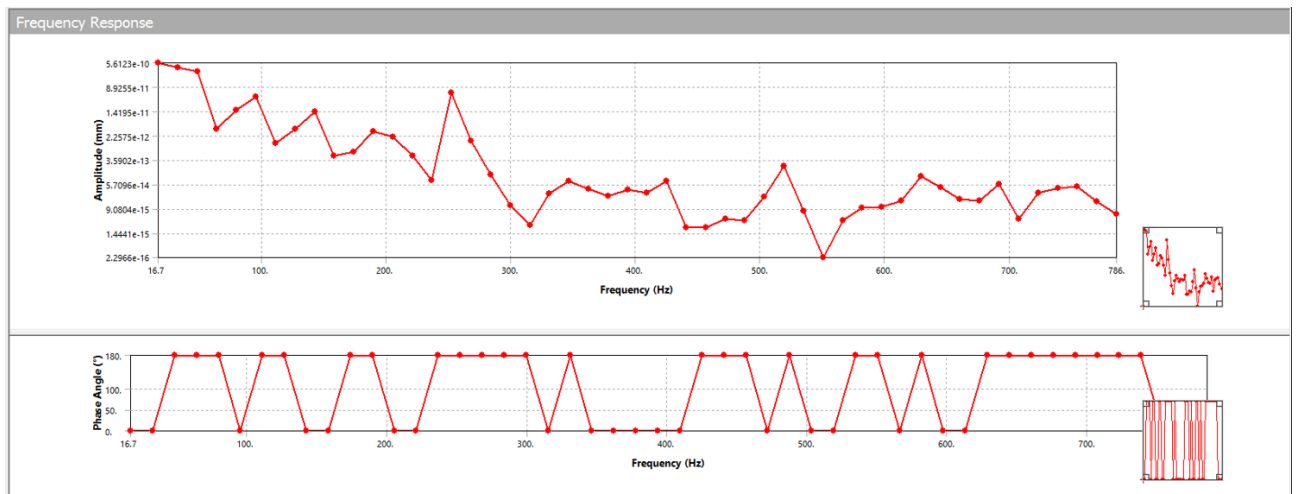


Figure 18. Frequency response and Phase angle of Cantilever beam

Analysis of Reaction wheel attached to cantilever beam:

When the beam of length 1m is fixed to the brackets and reaction wheel is attached 100mm away from the edge of the beam in the opposite side we receive the way more less deformation and it is better to place the reaction wheel in that position so that we can easily point out the Jitter. The total deformed angle should be below 0.11 degrees.

The Rotating force or measured imbalance of Reaction wheel i.e. 59,8 g-mm is used for the calculation. Frequency is calculated by the software automatically. Refer to the Figures below for better understanding.

Location	
Define By	Geometry Selection
Geometry	1 Face
Coordinate System	Global Coordinate System
X Coordinate	-15. mm
Y Coordinate	27.071 mm
Z Coordinate	-828.36 mm
Definition	
ID (Beta)	27
Type	Rotating Force
Calculated From	Unbalanced Mass
☐ Mass	5.9e-002 kg
☐ Rotating Radius	1. mm
Unbalanced Force	5.9e-002 kg-mm
☐ Phase Angle	0. °
Hit Point Selection	Mesh Node
Suppressed	No

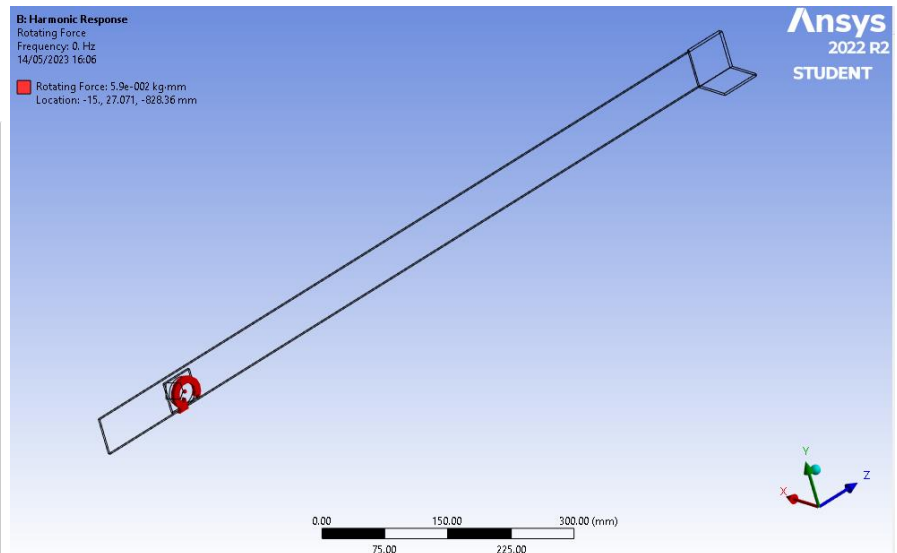


Figure 19. Location of rotating force

Total deformation of 0,27 mm of displacement was found during the Finite element method analysis. Refer to Figure. 20

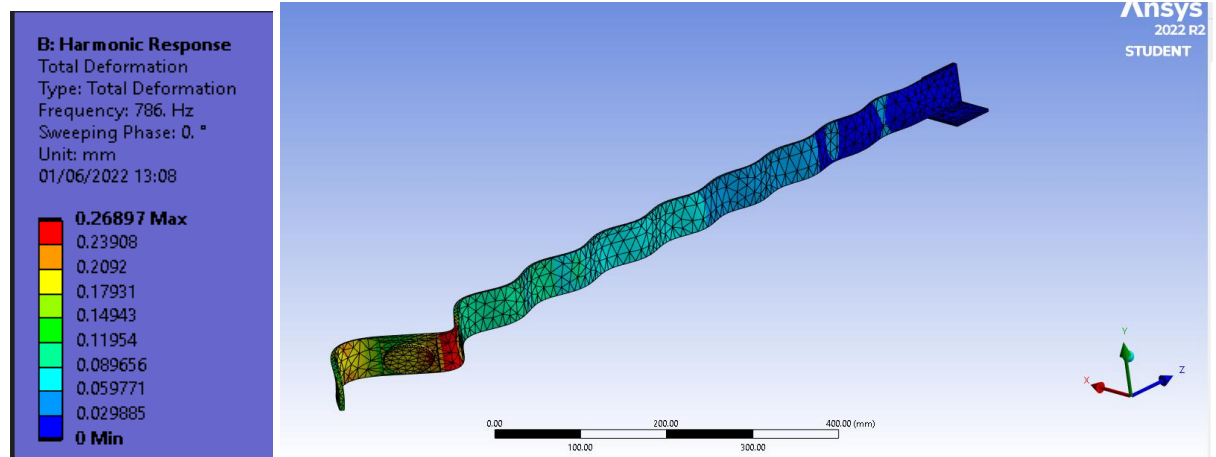


Figure 20. Results of Harmonic Analysis

Frequency response analysed during this condition is shown in figure 22. The maximum amplitude occurred was at the frequency of 628.8 Hz which is quite effective in such condition and also it seems to have similar values as we reviewed during the literature reviews.

Set	Frequency [Hz]	Object Name	Frequency Response	Phase Response
1.	78.6			
2.	157.2			
3.	235.8			
4.	314.4			
5.	393.			
6.	471.6			
7.	550.2			
8.	628.8			
9.	707.4			
10.	786.			
Results				
Maximum Amplitude		4.1899e-002 mm		
Frequency		628.8 Hz		
Phase Angle		-1.9178 °	179.19 °	
Real		4.1876e-002 mm	-3.9859e-002 mm	
Imaginary		-1.4022e-003 mm	5.6204e-004 mm	
Amplitude			3.9863e-002 mm	
Reported Frequency			786. Hz	

Figure 21. Results and set of frequencies occurred during analysis

Frequency Response

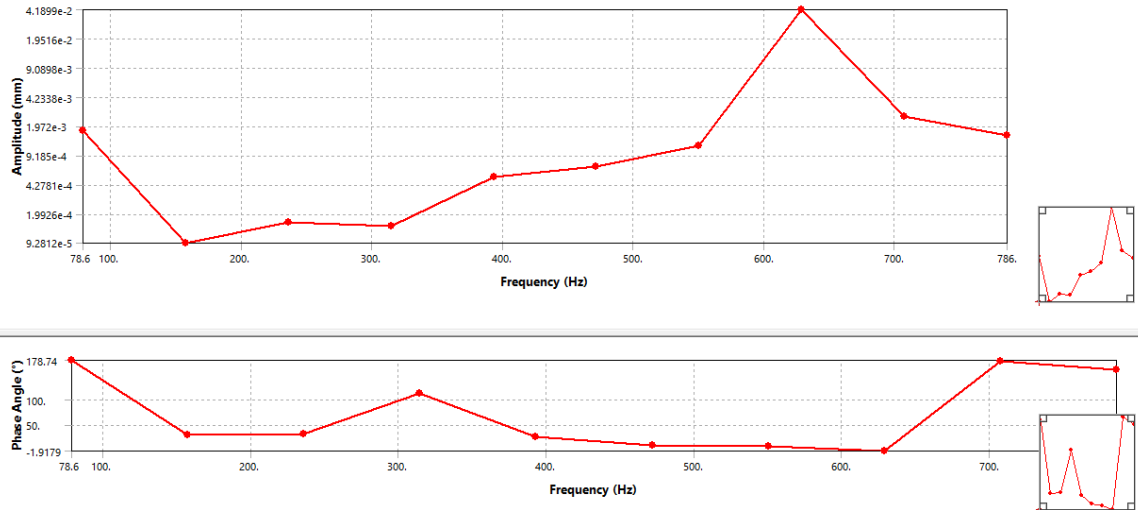


Figure 22. Frequency Response

Analysis of Reaction wheel attached to cantilever beam at an angle:

Same conditions were applied while performing Harmonic response for this condition. The Rotating force or measured imbalance of Reaction wheel i.e. 59,8 g-mm is used for the calculation. Frequency is calculated by the software automatically. Refer to the Figures below for better understanding.

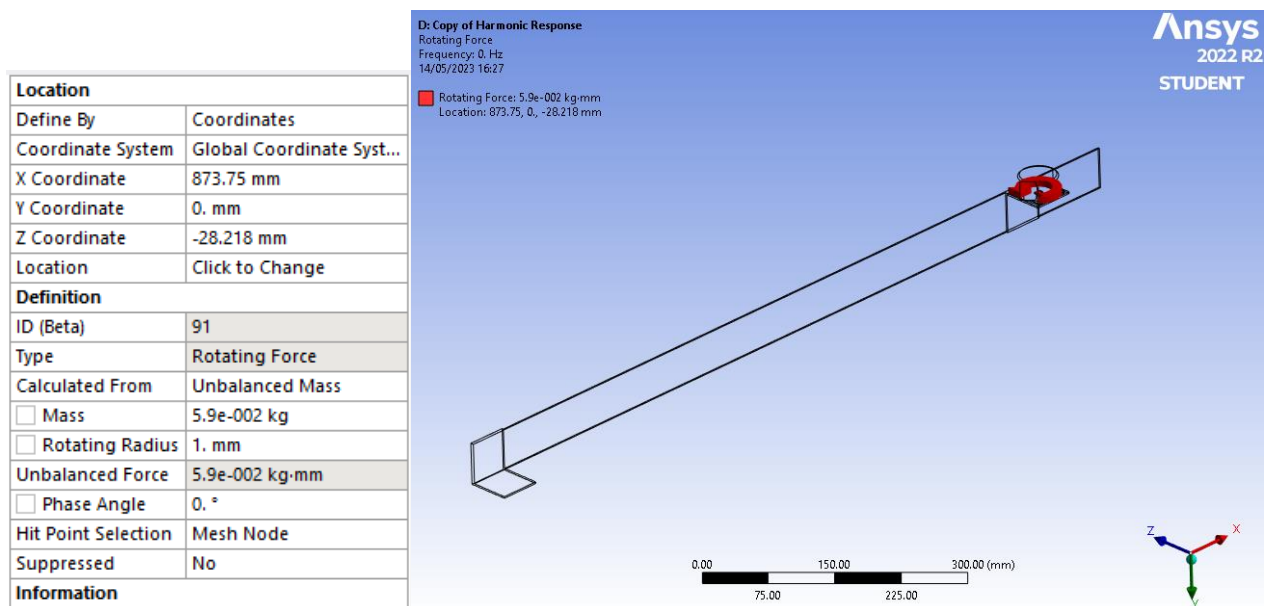


Figure 23. Location of rotating force

Total deformation of 0,97 mm of displacement was found during the Finite element method analysis, when reaction wheel was attached with an angle. Refer to Figure. 24

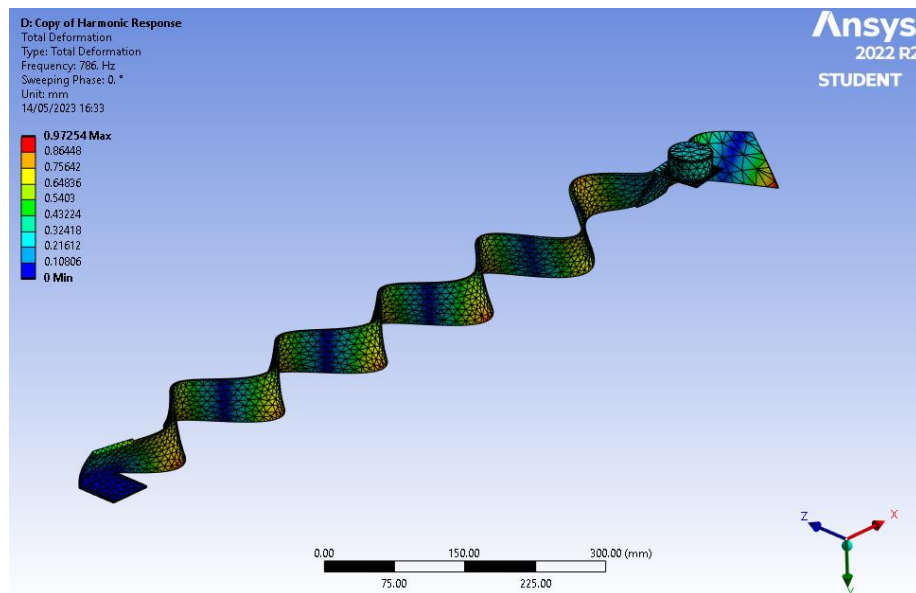


Figure 24. Results of Harmonic Analysis

Frequency response analysed during this condition is shown in figure 26. The maximum amplitude occurred was at the frequency of 314.4 Hz which used half of frequency to generate highest amplitude comparing to the condition 1.

Set	Frequency [Hz]	Object Name	Frequency Response	Phase Response
1.	78.6	Results		
2.	157.2	Maximum Amplitude	0.15425 mm	
3.	235.8	Frequency	314.4 Hz	
4.	314.4	Phase Angle	14.757 °	33.121 °
5.	393.	Real	0.14916 mm	0.16045 mm
6.	471.6	Imaginary	3.9291e-002 mm	0.10468 mm
7.	550.2	Amplitude		0.19158 mm
8.	628.8	Reported Frequency		786. Hz
9.	707.4			
10.	786.			

Figure 25. Results and set of frequencies occurred during analysis

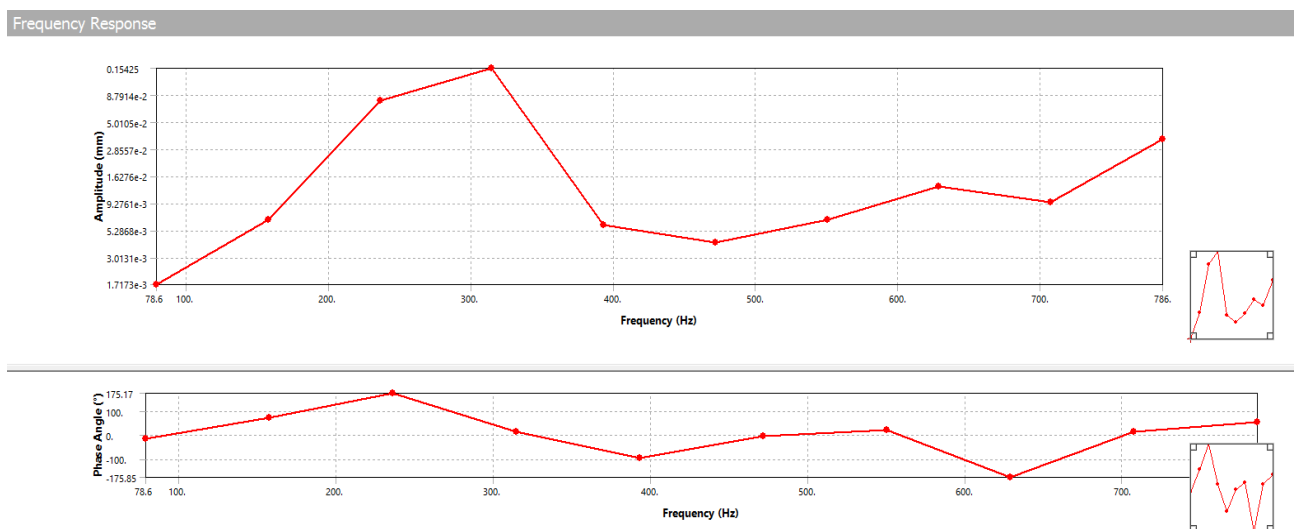


Figure 26. Frequency Response

3. EXPERIMENTAL ANALYSIS

Analysing the vibration and oscillation properties of a reaction wheel system is necessary for the experimental examination of reaction wheel jitter. Installing the reaction wheel, providing controlled inputs, and measuring the vibrations that result are the usual steps in an experiment. To record the motion and measure the jitter, a variety of sensors are employed, such as strain gauges and accelerometers. The gathered data is then examined to determine the jitter's underlying causes, assess how they affect system performance, and create mitigating measures to lessen or completely get rid of unpleasant vibrations. The behaviour of reaction wheels in applications like spaceship attitude control or precision instruments can be better understood and optimized by engineers with the aid of experiments

3.1. Vibration Analysis of Reaction Wheel

The vibration analysis of a reaction wheel attached to a beam can be conducted in two scenarios: when the reaction wheel is directly attached to the beam and when it is attached at an angle.

In the case of the reaction wheel directly attached to the beam (refer to figure 27), the vibration analysis involves studying the dynamic response of the beam due to the rotational motion of the reaction wheel. The interaction between the rotating wheel and the beam can lead to vibrations and oscillations in the system. The natural frequencies, mode shapes, and damping characteristics of the combined system need to be determined to assess the potential for resonance or instability.

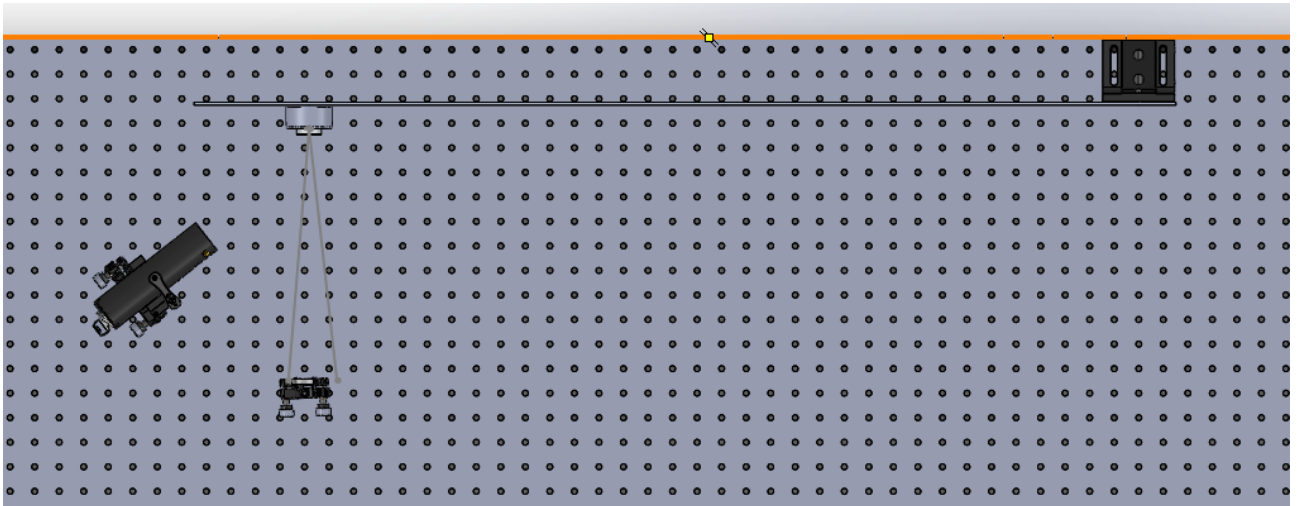


Figure 27. Solidworks model to show reaction wheel attached to beam directly

When the reaction wheel is attached to the cantilever beam at an angle (refer to figure 28), additional complexities arise due to the introduction of a moment about the beam's axis. This angular momentum affects the beam's bending behaviour, potentially resulting in different vibration modes and frequencies compared to the direct attachment scenario. The angle of attachment introduces coupling effects between the rotational motion of the wheel and the beam's bending and torsional modes, necessitating a more comprehensive analysis to understand the system dynamics.

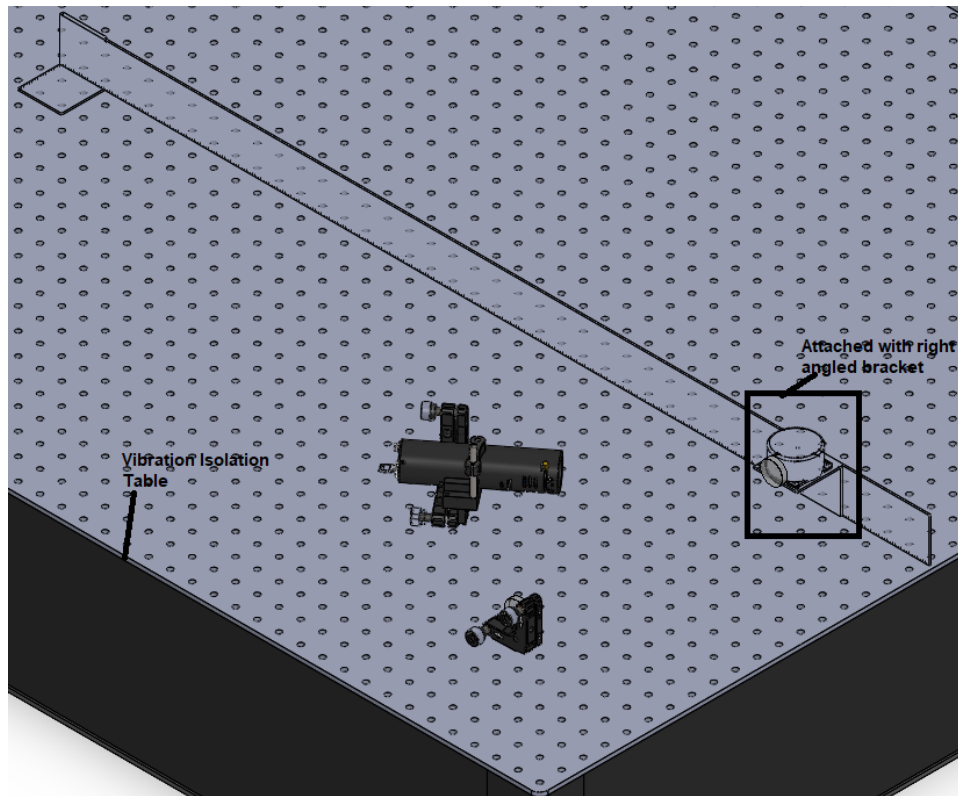


Figure 28. Reaction wheel attached to beam with an angle using right angled bracket.

Both scenarios require careful experimental analysis, which may involve modal testing techniques, accelerometers, strain gauges, and other sensors to measure vibrations and oscillations. The collected data is then analysed to determine the natural frequencies, mode shapes, and dynamic characteristics of the system. This analysis helps in optimizing the design, ensuring stability, and minimizing unwanted vibrations and oscillations in the reaction wheel and beam system.

3.2. Instrumentation used in this experiment

Several equipment and sensors are frequently employed to measure and collect pertinent data during the experimental examination of vibration in a reaction wheel attached to a beam. The following are some instruments that are frequently used in this experiment:

- **Rectangular beam:** Structural steel beam of thickness of 2 mm, breadth 50mm and length 1000mm was used during the experiment.
- **Vibration isolation table:** P/N M-RS2000-36-18 is used during this experiment which has high performance pneumatic vibration isolators. Graph below showing Horizontal/vertical frequency vs Transmissibility.

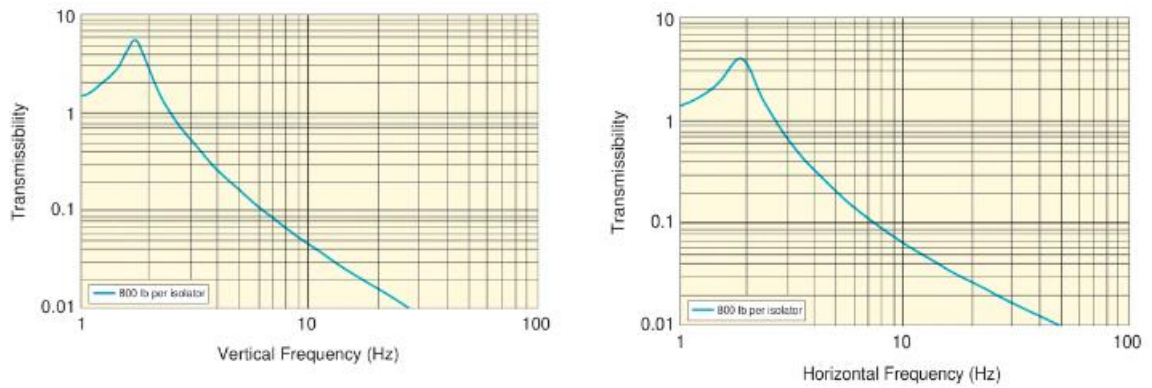


Figure 29. Graph showing frequency response

- **Vernier Calliper:** A *Vernier calliper* is a tool to precisely and reliably determine an object's dimensions. It comprises a fixed Vernier scale and a significant scale. Depending on whether the calliper is measuring in millimetres or inches, the major scale is marked with graduations in those units. This Vernier scale has finer divisions and glides along the main scale.
- **Multicomp pro P/N MP710086**, which is a single channel linear bench top DC power supply.



Figure 30. Power supply (Source: multicomp-pro-mp710086-dc-power-supply-review)

During experimental analysis we have used **10V** (Volts) current to supply power to reaction wheel.

- **Laser:** The P/N **CPS635R**'s output is circular using a set of specially designed openings. With shapes designed to trim the beam being sent out, these openings produce minimal noise in the immediate area and reflect less light to the diode output than regular openings. A result of this design is the formation of Airy diffraction caused by cutting the beam's edges.



Figure 31. Laser used during the experiment (Thorlabs, n.d.)

- **Camera:** We have used camera **P/N BFS-U3-31S4M-C** USB 3.1, Monochrome Camera.



Figure 32. FLIR Blackfly S USB 3 Camera(Edmund, n.d.)

Specifications of Camera:

Video Output:	USB 3.1 Gen 1	Type:	Monochrome Camera
Manufacturer:	FLIR	Mounting Threads:	¼-20 with Tripod Mount Adapter
Image Buffer:	240MB	Camera Family:	Blackfly® S
Model Number:	BFS-U3-31S4M-C	Imaging Sensor:	Sony IMX265
Type of Sensor:	Progressive Scan CMOS	Type of Shutter:	Global
Camera Sensor Format:	1/1.8"	Resolution (MegaPixels):	3.10
Pixels (H x V):	2,048 x 1,536	Pixel Size, H x V (µm):	3.45 x 3.45
Sensing Area, H x V (mm):	7.07 x 5.3	Frame Rate (fps):	55.00
Pixel Depth:	12 bit	Exposure Time:	11µs – 30s
Weight (g):	36	Mount:	C-Mount
Housing:	Full	Interface Port Orientation:	Back Panel
Dynamic Range (dB):	71.83	Operating Temperature (°C):	0 to +50
Storage Temperature (°C):	-30 to +60	GPIO Connector Type:	6-pin Hirose (HR10)
Machine Vision Standard:	USB3 Vision v1.0		

This camera has a sensor format of 1/1.8", with the resolution of 3.10 megapixels. Pixel Size of

each image is $3.45\text{ }\mu\text{m}$, This has a frame rate of 55fps, which uses C-mount sensor Sony IMX265. The objective is of FUJINON P/N **HF35XA-5M** which produces a Field of view of 54.4 degrees. Focal length of this objective is about 35mm.



Figure 33. Objective used for Monochrome camera

3.3. Software's used during this experiment:

- a) **YAT** - (Yet Another Terminal) is a software that allows users to interact with computer systems using a command-line interface (CLI). It is a terminal emulator that provides a graphical user interface (GUI) for interacting with the command line on Windows and Linux operating systems. YAT supports multiple tabs, different terminal types, and has a built-in terminal history. It also has features such as configurable key bindings and terminal options, as well as support for various text encodings. YAT can be used for tasks such as remotely accessing other computers, running commands, and managing files on a computer.
- b) **SpinView** - SpinView is a software application that is used to control, configure and monitor the operation of the Thorlabs' motorized rotation stages and controllers. It provides a simple graphical user interface for communicating with the motor controller. This allows the user to move the stage to a specific position, change the velocity of the motor, and set the acceleration of the motor. The software also provides real-time feedback on the position and status of the stage. SpinView can also be used to create and execute sequences of moves, and it can be used to generate simple scripts for automating repetitive tasks. This software was used to gain images of the distraction of the laser beams.
- c) **Python** - A popular approach for quickly computing the discrete Fourier transform (DFT) and its inverse in Python is the Fast Fourier Transform (FFT). Python's NumPy package offers a practical method for doing FFT calculations.

3.4. Experiment procedure:

Experimental set up is performed as shown in the solidworks model. Refer to Figure 34. for the setup input.

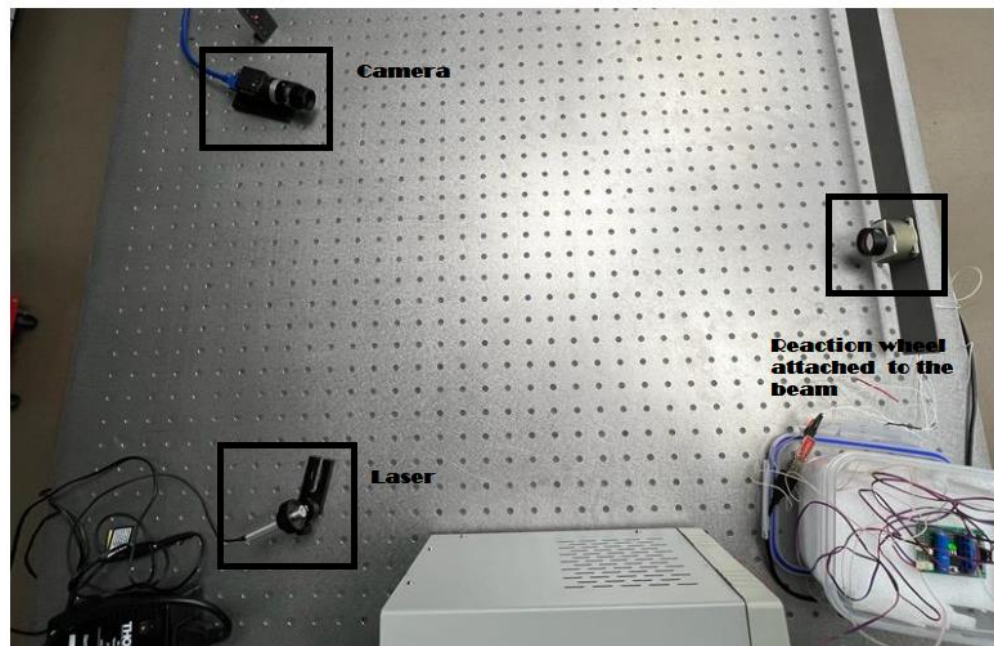


Figure 34. Experimental setup

After fixing cantilever beam attached with reaction wheel, perform trail fit to locate exact location to identify the laser beam pointing. During this experiment we have used Laser beam was attached to the vibration isolation table with a distance of 60 cm also with an angle of 40 degrees.

Reaction wheel is attached with simple mirror to achieve proper reflection of laser beam.



Figure 35. Mirror attached to the reaction wheel.

Monochrome camera and Reaction wheel will be connected to Laptop which uses required software which is YAT and Spin View, to extract data required.

Laser object is directly connected to power supply as it doesn't require any special conditions.

Camera is placed within the field of view and it is attached with the Filters to avoid exposure of unwanted rays. We used **Filter NE30A** and **NE20A** with **OD = 3.0** and **2.0** respectively.



Figure 36. Camera attached with the filters.

A laser beam was pointed towards the centre of the mirror attached to the reaction wheel (Ref. Figure 37).

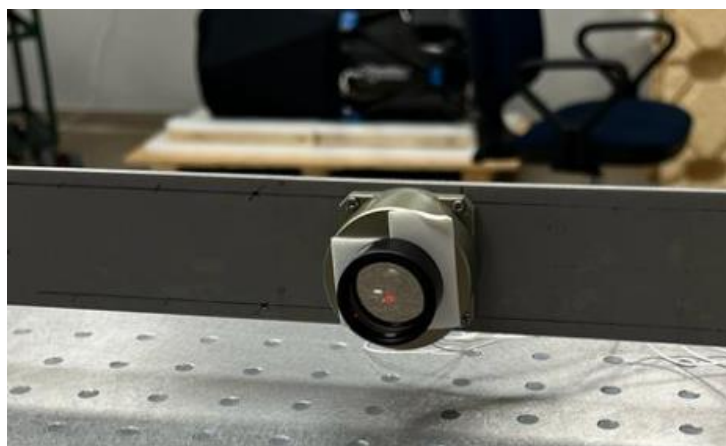


Figure 37. Laser beam pointing towards the centre.

After setting up the experimental setup, need to focus on setting up the software where we have to focus on the region of interest. Camera will be able to detect wider area rather just focusing on the laser beam so we create region of interest to collect data.

Spin View software is capable of extracting data which is given as region of interest. Also it is important to setup to receive proper data by changing Acquisition mode, exposure time etc. In this analysis we have set pixel of each image to 296 X 344, with a frame rate of 55,69 per image. Refer to Figure 38 which explains the region of interest.

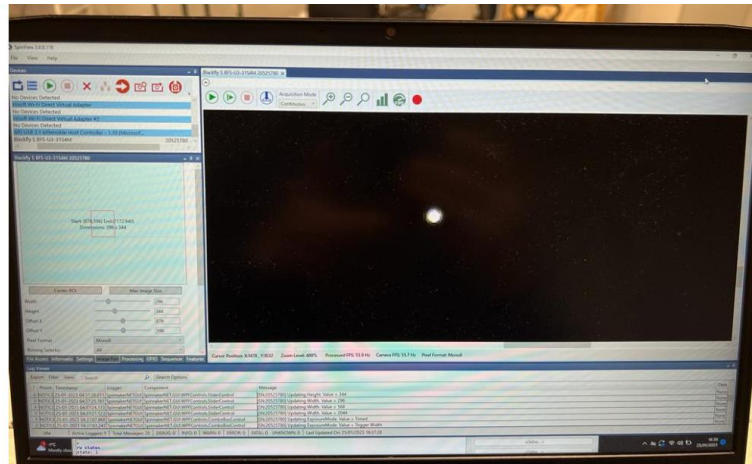


Figure 38. Image showing the region of interest.

Using YAT software we can adjust the rpm of Reaction wheel. Measurement performed at different Reaction wheel speeds in the range (0-6500 rpm) (see figure 39). During the measurements it was found that maximum rotational speed of Flywheel is not 6500 RPM, but 6486 RPM.

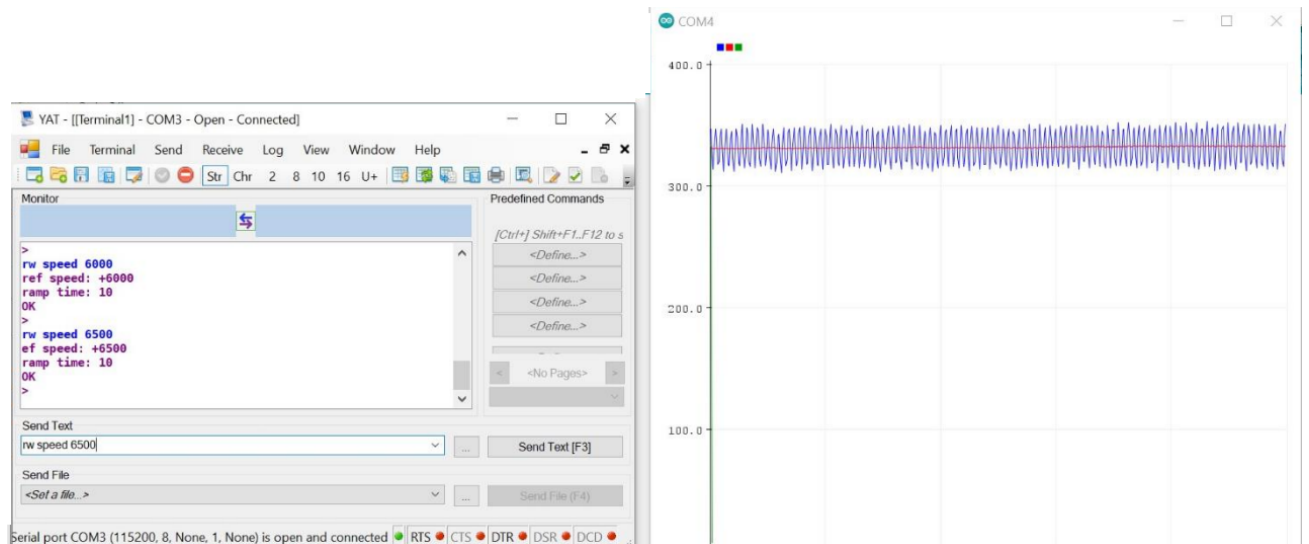


Figure 39. Reaction wheel terminal control and signal from serial plotter

Obtained data are in the form of JPG format, so extracted needs to be analysed using Python to extract Frequency and time domain. See figure 40 for the image detected by monochrome camera.

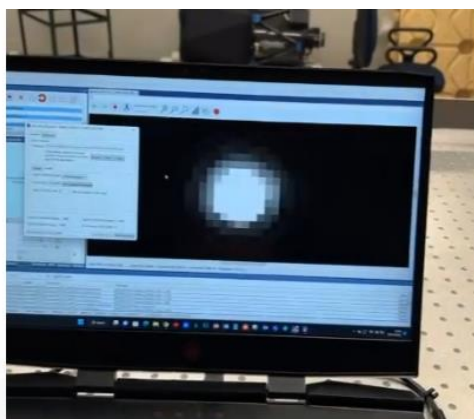


Figure 40. Image showing the laser beam reflected from the mirror.

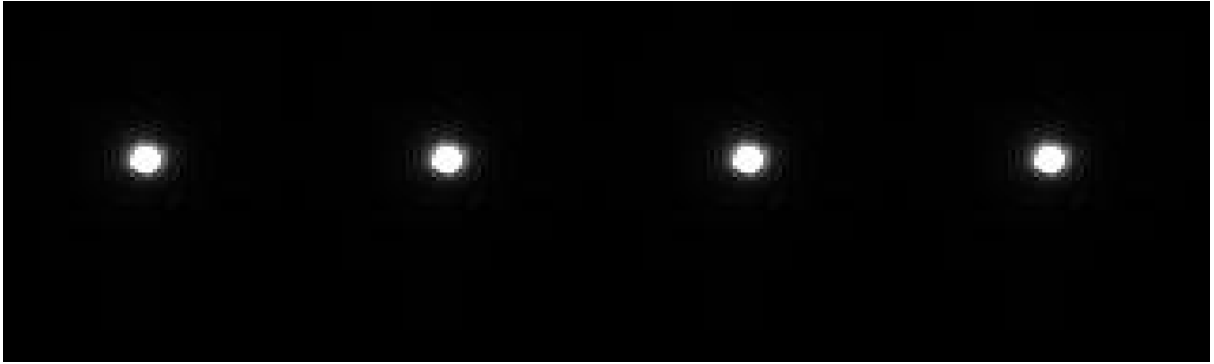


Figure 41. Obtained images from the experiment

Total of 6000 images were collected after the experiment was performed. All these images were merged by using Python software to obtain and to track centroid of the laser beam. Also displacement from the initial pointing to the final pointing of the beam was calculated. The green spot showing the result of obtained centroid of the laser beam (see figure 42).

The result obtained for total displacement was **2.0 pixel**, which is **0,53mm**.

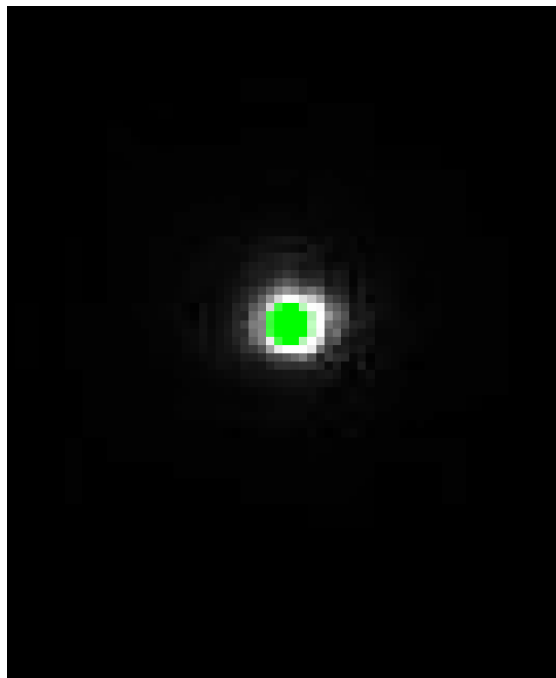


Figure 42. Merged obtained images using Python.

Maximum and minimum frequencies obtained during FFT analysis shows the frequency which is obtained during different RPM's (see Figure 43)

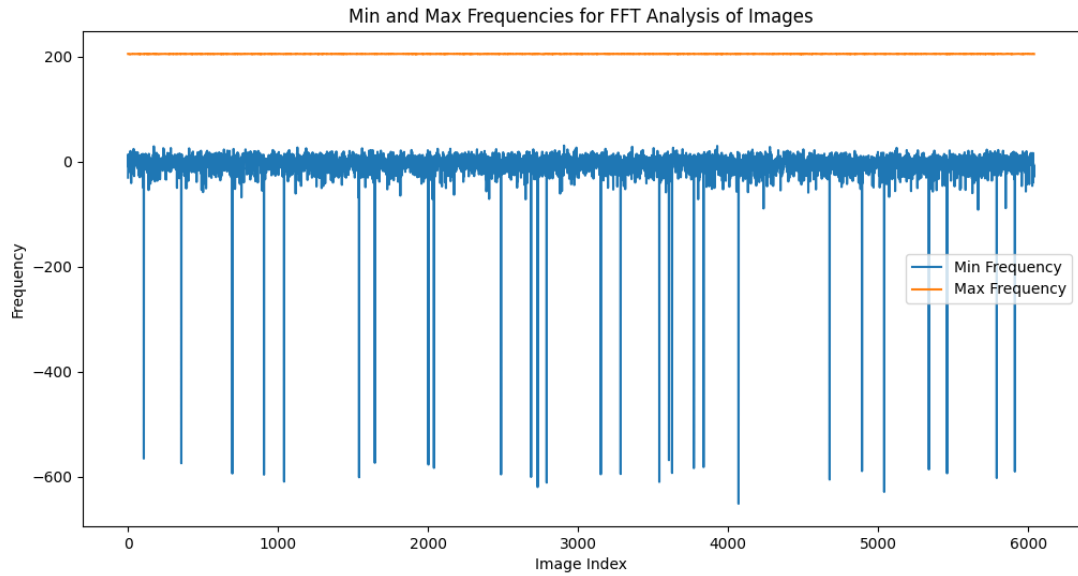


Figure 43. Maximum and minimum frequencies obtained

Measurements were performed several times at different and same time intervals, also it was repeated several times at different days to compare results. For all signals FFT algorithm was adapted and displacement at beam resonant conditions was calculated via Python software, results presented below.

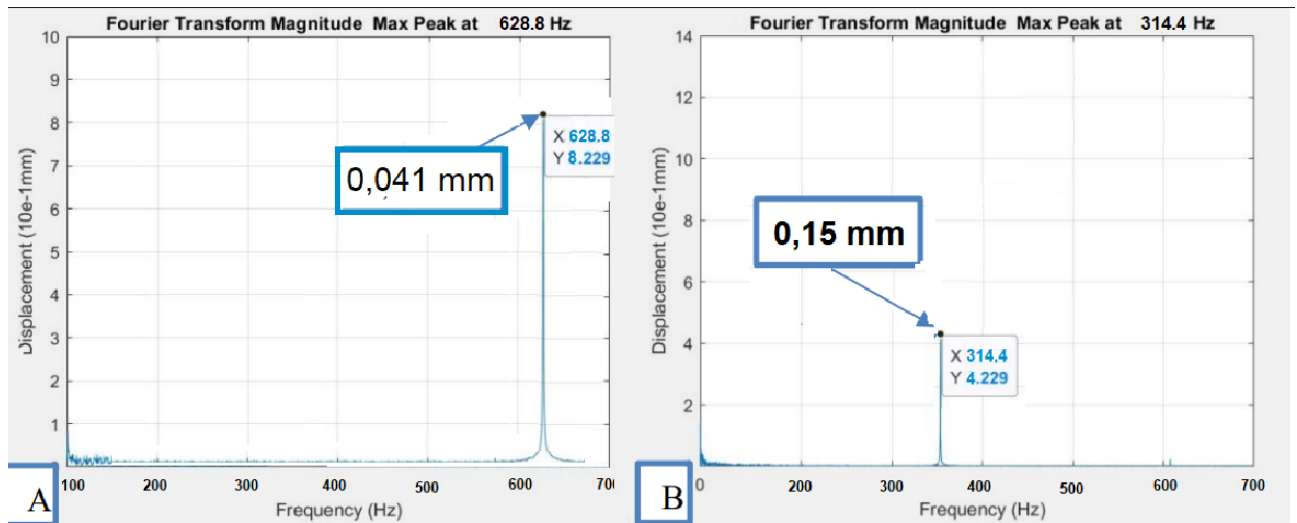


Figure 44. FFT applied to measurements. A - RW directly attached; B- RW attached to beam at an angle.

After performing FFT analysis maximum displacement of 0,15mm occurred at 314.4 Hz frequency.

3.5. Angular Displacement

Mapping the linear displacement of a beam spot on the detector to angular deformations of the mirror on a rotating wheel (RW) can be achieved using a mathematical relationship between the two variables.

Define the variables:

- Linear displacement of the beam spot on the detector: Let us denote this as "D" (in length units, such as meters).
- Angular deformation of the mirror on the rotating wheel: Let us denote this as " θ " (in units of radians).

We need to consider the system's geometry to establish a relationship between D and θ . The critical parameter is the distance between the rotating wheel and the detector, which we will denote as "L" (in units of length, such as meters). Additionally, we will assume that the beam spot remains on the detector plane, simplifying the problem.

When the mirror on the rotating wheel undergoes an angular deformation θ , it changes the path length of the light beam from the source to the detector. This path length change causes the beam spot to move linearly on the detector plane by a distance of D.

By considering the basic trigonometry involved, we can establish the following relationship:

$$D = L \times \theta \text{ in 'm'} \quad (3.1)$$

This equation represents the linear displacement (D) in meter of the beam spot on the detector as a function of the angular deformation (θ) in degree of the mirror on the rotating wheel.

By taking maximum displacement occurred 0,15mm as linear displacement (D), and Distance between the rotating wheel and the detector , L is 60 cm as mentioned before.

Therefore angular deformation θ , 0.00025 radians, which is equal to 0.014 degrees.

3.6. Results

Detailed Reaction Wheel finite element and experimental analysis were performed to measure pointing errors due displacement caused by jitter/vibration of Reaction Wheel. Results presented in table below:

Condition 1 refers to Reaction wheel directly attached to the cantilever beam

Condition 2 refers to Reaction wheel attached to the beam with an angle bracket.

Table 5. Results

Displacement amplitude (FEM method) Condition 1	0.0318 mm at 618.8 Hz
Displacement amplitude (experimental analysis) Condition 1	0.041 mm at 618.8 Hz
Displacement amplitude (FEM method) Condition 2	0,15425 mm at 314.4 Hz
Displacement amplitude (experimental analysis) Condition 2	0,15 mm at 314.4 Hz
Angular deformation (considered for maximum displacement)	0.014 degrees

After analysing the data of FEM method vs Experimental, we noticed that the results obtained are similar, so we can consider that analysis are performed correctly. However. The difference between obtained results may occur due to low accuracy of experimental set up, external noises or principle of calculation methods.

4. ANALYSIS OF RESULTS

Pointing is essential for optical communication between satellites. One of the main causes of internal vibrations are reaction wheels. In this thesis, the reaction wheel of a tiny satellite was examined using FEM and experimental techniques, as well as its impact on the stability of the 6U CubeSat.

4.1. Analysis and development of an experimental methodology

Analysing and developing an experimental methodology to directly measure pointing errors caused by reaction wheel jitter on a CubeSat satellite involves several steps. Here is an outline of the process:

- a) **Problem Description:** Clearly define the objective of the experiment, which is to measure the pointing errors resulting from reaction wheel jitter on a CubeSat satellite. Specify the specific parameters and metrics to be measured, such as angular error or variation from the desired pointing direction.
- b) **Experimental Setup:** Design and set up the experimental conditions that closely corresponds the operational conditions of the CubeSat satellite. Consider factors such as vibration isolation, environmental control, and appropriate mounting setups for the CubeSat and measurement devices.
- c) **Measurement Devices:** Select and incorporate appropriate measurement devices to capture the pointing errors caused by the reaction wheel jitter. This may involve using sensors like accelerometers, gyroscopes, or optic devices capable of measuring angular deviations.
- d) **Jitter Simulation:** Simulate reaction wheel jitter using a controlled mechanism or hardware-in-the-loop setup. This can involve introducing intentional disruptions to the reaction wheels or using simulated jitter profiles.
- e) **Data Acquisition:** Develop a data acquisition system to capture the measurements from the sensors during the experiment. This may include setting up data logging tools, specifying synchronization between measurement devices, and ensuring the sampling rate is adequate.
- f) **Test Execution:** Conduct the experiment by activating the reaction wheel jitter simulation and collecting data over a defined duration. Ensure that the experiment is replicated multiple times to gather statistically significant data and account for any variability.
- g) **Data Analysis:** Investigate the collected data to quantify the pointing errors caused by the reaction wheel jitter. This can involve processing the raw sensor data, applying appropriate signal processing techniques, and figuring relevant metrics such as root mean square (RMS) error or spectral analysis of the pointing errors.
- h) **Validation and Verification:** Validate the experimental results by comparing them with theoretical models or known performance specifications of the CubeSat design. This step helps ensure the precision and reliability of the experimental methodology.

- i) **Documentation and Reporting:** Document the experimental methodology, including the setup details, measurement procedures, data analysis procedures, and results. Prepare a comprehensive report summarizing the conclusions, observations, and any recommendations for mitigating the pointing errors caused by reaction wheel jitter.
- j) **Iterative Improvement:** If necessary, refine the experimental methodology based on the initial results or feedback, and repeat the experiment to further validate the conclusions or explore different scenarios.

By following these steps, you can systematically investigate the pointing errors caused by reaction wheel jitter on a CubeSat satellite and develop an experimental methodology to directly measure and characterize those errors.

CONCLUSIONS

1. When a spacecraft needs to be moved, or its attitude stabilized, reaction wheels are used as the primary actuators. However, a revolving device called the reaction wheel causes spacecraft disturbances. The unbalance of the spinning mass and the ball bearing mechanism are the main causes of disturbance from the reaction wheels. A spacecraft's pointing stability is decreased by the disturbances, even though they are often minor and referred to as micro-disturbances.

2. The work presented in this thesis was about the analysis and development of experimental methodology to directly measure pointing errors caused by reaction wheel jitter, given by supervisor.

3. Reaction wheel pointing errors calculated both by Finite element and Experimental methods. Experimental method was produced as new, low budget method. Measurements resembles that such an experiment method may be appropriate.

4. Measurement of Laser beam pointing errors was performed successfully, laser beam pointing was captured and FFT analysis was performed.

5. Displacement caused due to Reaction wheel imbalance was simulated with Ansys software.

RECOMMENDATIONS

In the mentioned thesis, a cantilever beam was utilized as a means to mount the reaction wheel and conduct the experimental analysis. However, the suggestion for future experiments is to enhance the setup by using a CubeSat and designing a dedicated test bench to achieve more accurate and precise experimental results. This concept is depicted in Figure 45, which showcases the proposed scenario.

By transitioning from a cantilever beam setup to a dedicated test bench using a CubeSat, several advantages can be gained:

1. **Realistic Representation:** A CubeSat is a standardized satellite platform that closely resembles the size, weight, and structural aspects of actual CubeSat missions. By using a CubeSat in the experimental setup, the test conditions better mimic real-world techniques, leading to more accurate results.

2. **System Integration:** A dedicated test bench allows for proper integration of the reaction wheel with other elements typically found in CubeSat missions. This enables a more comprehensive analysis of the interaction between the reaction wheel and the overall satellite design.

3. **Vibration Isolation:** While in operation, CubeSats frequently endure vibrations and disturbances that can impair the response wheel's functionality. These impacts can be lessened and a more stable testing environment can be created by designing a test bench with vibration isolation systems.

4. **Sensor Integration:** A CubeSat-based testing configuration has the option of directly integrating different sensors and measurement tools into the satellite's framework. This makes it easier to measure aiming errors and other appropriate factors directly, leading to more accurate and precise experimental results.

5. **Better Control of Environmental Aspects:** A separate test bench enables better control over environmental aspects like temperature, stress, and magnetic fields. This control makes sure that the studies are carried out in a controlled environment, producing results that are more predictable and reproducible.

Overall, transitioning from a cantilever beam setup to a dedicated test bench using a CubeSat in future experiments offers improved realism, system integration, vibration isolation, sensor integration, and experimental control. These enhancements are expected to yield much more precise and

reliable experimental results, enabling a deeper understanding of the effects of reaction wheel jitter on CubeSat pointing accuracy.

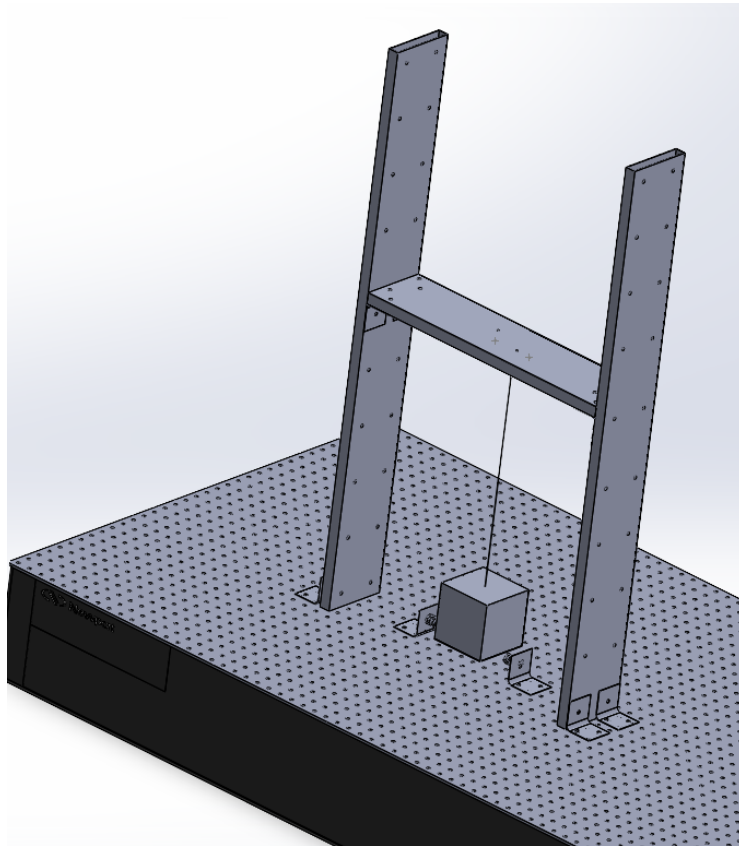


Figure 45. Future recommendation design

REFERENCES

- Ampatzoglou, A., & Kostopoulos, V. (2018). Design, Analysis, Optimization, Manufacturing, and Testing of a 2U Cubesat. *International Journal of Aerospace Engineering*, 2018, 9724263. <https://doi.org/10.1155/2018/9724263>
- Andrews, L. C., Phillips, R. L., & Society of Photo-optical Instrumentation Engineers. (2005). *Laser beam propagation through random media*. 782.
- Chen, S. B., Xuan, M., Zhang, L., Gu, S., Gong, X. X., & Sun, H. Y. (2019a). Simulating and testing microvibrations on an optical satellite using acceleration sensor-based jitter measurements. *Sensors (Switzerland)*, 19(8). <https://doi.org/10.3390/s19081797>
- Chen, S. B., Xuan, M., Zhang, L., Gu, S., Gong, X. X., & Sun, H. Y. (2019b). Simulating and Testing Microvibrations on an Optical Satellite Using Acceleration Sensor-Based Jitter Measurements. *Sensors (Basel, Switzerland)*, 19(8). <https://doi.org/10.3390/S19081797>
- de Lellis, S., Stabile, A., Aglietti, G. S., & Richardson, G. (2019). A semiempirical methodology to characterise a family of microvibration sources. *Journal of Sound and Vibration*, 448, 1–18. <https://doi.org/10.1016/j.jsv.2019.02.005>
- Duijsens, A. :, Promoteur, S., & Bruls, O. (n.d.). *Master thesis : Cubesat deployment*. <http://matheo.ulg.ac.be>
- Edmund. (n.d.). <https://www.edmundoptics.eu/p/bfs-u3-31s4m-c-usb3-blackflyreg-s-monochrome-camera/41348/>.
- Fast Steering Mirrors (FSM) | Controller Performance Tuning | Piezo Tip/Tilt | Manufacturer. (n.d.). Retrieved June 21, 2022, from <https://www.pi-usa.us/en/tech-blog/design-performance-and-tuning-of-fast-steering-mirrors-based-on-piezo-drives-and-flexure-guides/>
- Fast-Steering-Mirrors-Improve-Active-Beam-Stabilization. (n.d.).
- GOCE platform micro-vibration verification by test and analysis - NASA/ADS. (n.d.). Retrieved June 20, 2022, from <https://ui.adsabs.harvard.edu/abs/2005ESASP.581E..40W/abstract>
- Hemmati, H. (2009). *Near-earth laser communications*.
- Inamori, T., Wang, J., Saisutjarit, P., & Nakasuka, S. (2013a). Jitter reduction of a reaction wheel by management of angular momentum using magnetic torquers in nano- and micro-satellites. *Advances in Space Research*, 52(1), 222–231. <https://doi.org/10.1016/J.ASR.2013.02.014>
- Inamori, T., Wang, J., Saisutjarit, P., & Nakasuka, S. (2013b). Jitter reduction of a reaction wheel by management of angular momentum using magnetic torquers in nano- and micro-satellites. *Advances in Space Research*, 52(1), 222–231. <https://doi.org/10.1016/j.asr.2013.02.014>
- Israr, A. (2014a). Vibration and modal analysis of low earth orbit satellite. *Shock and Vibration*, 2014. <https://doi.org/10.1155/2014/740102>
- Israr, A. (2014b). Vibration and modal analysis of low earth orbit satellite. *Shock and Vibration*, 2014. <https://doi.org/10.1155/2014/740102>

- Kaymak, Y., Rojas-Cessa, R., Feng, J., Ansari, N., Zhou, M., & Zhang, T. (2018). A Survey on Acquisition, Tracking, and Pointing Mechanisms for Mobile Free-Space Optical Communications. *IEEE Communications Surveys and Tutorials*, 20(2), 1104–1123. <https://doi.org/10.1109/COMST.2018.2804323>
- Kennedy Space Center, J. F., McKissock, D., Loyselle, P., & Vogel, E. (2007). UL Standard for Safety for Lithium Batteries, UL 1642. UL Standard. 4th ed. Northbrook, IL, Underwriters Laboratories. In *Range Safety User Requirements Manual* (Vol. 3).
- Kim, D. K. (2014). Micro-vibration model and parameter estimation method of a reaction wheel assembly. *Journal of Sound and Vibration*, 333(18), 4214–4231. <https://doi.org/10.1016/J.JSV.2014.04.032>
- Kim, D. K., Oh, S. H., & Yong, K. L. (2010). Numerical study on a reaction wheel and wheel-disturbance modeling. *17th International Congress on Sound and Vibration 2010, ICSV 2010*, 5, 3384–3390. <https://doi.org/10.5139/jksas.2010.38.7.702>
- Laurynas Šišovas MT. (2021). *Investigation of mechanical disturbances in a Cubesat caused by the ReactionWheel*.
- Mankour, A., Smahat, A., Guy, R., Wang, R., & Khatir, M. (2021). Experimental investigation of microvibrations induced by reaction wheels on earth observation satellite. *Advances in Space Research*, 68(11), 4484–4495. <https://doi.org/10.1016/J.ASR.2021.08.035>
- Pointing, Acquisition, and Tracking Systems for Free-Space Optical Communication Links*. (n.d.). Retrieved June 21, 2022, from <https://drum.lib.umd.edu/handle/1903/6734>
- Sonawane, A. R., & Talmale, P. S. (2017). Modal Analysis of Single Rectangular Cantilever Plate by Mathematically, FEA and Experimental. *International Research Journal of Engineering and Technology*. www.irjet.net
- Song, T., Wang, Q., Wu, M. W., Ohtsuki, T., Gurusamy, M., & Kam, P. Y. (2017). Impact of Pointing Errors on the Error Performance of Intersatellite Laser Communications. *Journal of Lightwave Technology*, 35(14), 3082–3091. <https://doi.org/10.1109/JLT.2017.2705132>
- TANIWAKI, S., & OHKAMI, Y. (2003). Experimental and Numerical Analysis of Reaction Wheel Disturbances. *JSME International Journal Series C Mechanical Systems, Machine Elements and Manufacturing*, 46(2), 519–526. <https://doi.org/10.1299/jsmec.46.519>
- Thorlabs. (n.d.). <https://www.thorlabs.com/thorproduct.cfm?partnumber=CPS635R>.
- time domain vs frequency domain | difference between time domain and frequency domain*. (n.d.). Retrieved June 22, 2022, from <https://www.rfwireless-world.com/Terminology/time-domain-vs-frequency-domain.html>
- Valentin, P. (2017). *Robust Microvibration Control and Worst-case Analysis for High Pointing Stability Space Missions*.
- Walter Frei. (2016). <https://www.comsol.com/blogs/part-1-modeling-the-harmonic-excitations-of-linear-systems/>. Modeling the Harmonic Excitations of Linear Systems.
- Wang, Y., & Jing, X. (2019). Nonlinear stiffness and dynamical response characteristics of an asymmetric X-shaped structure. *Mechanical Systems and Signal Processing*, 125, 142–169. <https://doi.org/10.1016/J.YMSSP.2018.03.045>

- Weipeng, L., Hai, H., Xubin, Z., Xintao, Z., & Yang, B. (2014). Design and experiments of an active isolator for satellite micro-vibration. *Chinese Journal of Aeronautics*, 27(6), 1461–1468. <https://doi.org/10.1016/j.cja.2014.10.012>
- Zhang, Z., Aglietti, G. S., & Ren, W. (2013). Coupled microvibration analysis of a reaction wheel assembly including gyroscopic effects in its accelerance. *Journal of Sound and Vibration*, 332(22), 5748–5765. <https://doi.org/10.1016/j.jsv.2013.06.011>
- Zhang, Z., Aglietti, G. S., Ren, W., & Addari, D. (2014). Microvibration analysis of a cantilever configured reaction wheel assembly. *Advances in Aircraft and Spacecraft Science*, 1(4), 379–398. <https://doi.org/10.12989/AAS.2014.1.4.379>
- Zhang, Z., Aglietti, G. S., & Zhou, W. (2012). Microvibrations Induced by a Cantilevered Wheel Assembly with a Soft-Suspension System. *Https://Doi.Org/10.2514/1.J050791*, 49(5), 1067–1079. <https://doi.org/10.2514/1.J050791>
- Zhou, W., Li, D., Luo, Q., & Liu, K. (2012). Analysis and Testing of Microvibrations Produced by Momentum Wheel Assemblies. *Chinese Journal of Aeronautics*, 25(4), 640–649. [https://doi.org/10.1016/S1000-9361\(11\)60430-5](https://doi.org/10.1016/S1000-9361(11)60430-5)

THE ANNEXES

(If necessary)