Improved algorithm for a system with different stiffness for positive and negative displacements

K. Ragulskis[1], B. Spruogis[2], A. Matuliauskas[3], V. Mištinas[4], L. Ragulskis[5]
[1]Kaunas University of Technology, K. Donelaičio str. 73, LT-44249, Kaunas, Lithuania
[2, 3, 4]Department of Mobile Machinery and Railway Transport, Faculty of Transport Engineering, Vilnius Gediminas Technical University, Plytinės str. 27, LT-10105, Vilnius, Lithuania
[5]Department of Systems Analysis, Faculty of Informatics, Vytautas Magnus University, Vileikos str. 8, LT-44404, Kaunas, Lithuania
[1]Corresponding author
E-mail: [1]*kazimieras3@hotmail.com*, [2]*bronislovas.spruogis@gmail.com*, [3]*arvydas.matuliauskas@vgtu.lt*, [4]*vygantas.mistinas@gmail.com*, [5]*l.ragulskis@if.vdu.lt*

Received 6 May 2022; received in revised form 14 June 2022; accepted 24 June 2022
DOI https://doi.org/10.21595/mme.2022.22748

Copyright © 2022 K. Ragulskis, et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Abstract. In the process of investigation of dynamics of elements of robots, systems with different values of stiffness for positive and negative displacements play an important role. This change of the value of stiffness has an effect to the results of numerical calculation of dynamics of the system. For more precise investigation of dynamics a special numerical procedure is proposed in the paper. Numerical results for two values of time steps: large and medium ones, are presented for two cases: without application of the proposed procedure and with application of it. Comparison of the presented results indicates advantages of the proposed numerical procedure.

Keywords: different values of stiffness, harmonic excitation, transient processes.

1. Introduction

In the process of investigation of dynamics of elements of robots, systems with different values of stiffness for positive and negative displacements play an important role. This change of the value of stiffness has an effect to the results of numerical calculation of dynamics of the system.

For more precise investigation of dynamics a special numerical procedure is proposed in the paper. Numerical results for two values of time steps: large and medium ones, are presented for two cases: without application of the proposed procedure and with application of it.

Comparison of the presented results indicates advantages of the proposed numerical procedure.

Resonances in nonlinear systems are described in [1]. Systems with impacts are investigated in [2]. Stabilisation of dynamical systems is presented in [3]. Impacts in vibrating systems are investigated in [4]. Periodic orbits are analysed in [5]. Vibro-impact energy sink is investigated in [6]. Particle impact with a wall is presented in [7]. Frequencies of a multibody system are analysed in [8]. Pendulum and its dynamics are investigated in [9]. Piecewise linearity is analysed in [10]. Resonant zones are investigated in [11]. Sommerfeld effect is presented in [12]. Isolated resonances are investigated in [13].

First the model of the system with a single degree of freedom and harmonic excitation is described. Then the proposed procedure for improved calculation of dynamics is presented. Graphical results without application of the proposed procedure and with it are presented and mutually compared.

2. Model of the system with different values of stiffness for positive and negative displacements

The investigated system has a single degree of freedom and is described by the following differential equation:

$$\ddot{x} + 2h\dot{x} + p^2 x = -f \sin\omega t, \tag{1}$$

where x is the displacement of the system, h is the coefficient of viscous damping, p^2 is the nonlinear coefficient of stiffness, f is the amplitude of harmonic excitation, ω is the frequency of harmonic excitation, t is the time and the upper dot denotes differentiation with respect to it.

It is assumed that the nonlinear stiffness has the following form:

$$p^2 = \begin{cases} p_2^2, & x > 0, \\ p_1^2, & \text{elsewhere}, \end{cases} \tag{2}$$

where p_1 and p_2 are constant values.

Further the force of stiffness is denoted as:

$$P = p^2 x. \tag{3}$$

3. Improved procedure for calculation of dynamics of the system with different values of stiffness for positive and negative displacements

Further the following notation is used: T denotes the time step, the subscript 0 denotes the value at the beginning of a time step and the subscript T denotes the value at the end of a time step.

It is assumed that:

$$p^2 = \begin{cases} p_2^2, & (x > 0) \vee \big((x = 0) \& (\dot{x} > 0)\big), \\ p_1^2, & \text{elsewhere}. \end{cases} \tag{4}$$

If the following conditions are satisfied:

$$x_T > 0, \tag{5}$$

and:

$$x_0 < 0, \tag{6}$$

or if the following conditions are satisfied:

$$x_T < 0, \tag{7}$$

and:

$$x_0 > 0, \tag{8}$$

then the reduced time step is used in the process of calculations:

$$T_r = -\frac{x_0}{x_T - x_0} T. \tag{9}$$

Then it is assumed that:

$$x_{T_r} = 0, \tag{10}$$

$$\dot{x}_{T_r} = \dot{x}_0 + (\dot{x}_T - \dot{x}_0) \frac{T_r}{T}, \tag{11}$$

$$\ddot{x}_{T_r} = \ddot{x}_0 + (\ddot{x}_T - \ddot{x}_0) \frac{T_r}{T}. \tag{12}$$

4. Investigation of dynamics of the system with different values of stiffness for positive and negative displacements

The following parameters of the investigated system with different values of stiffness for positive and negative displacements were assumed:

$$\omega = 1, \quad f = 1, \quad h = 0.1, \quad p_1 = 1, \quad p_2 = 2. \tag{13}$$

Zero initial conditions were assumed:

$$x(0) = 0, \quad \dot{x}(0) = 0. \tag{14}$$

Results for two values of a time step are presented: large time step:

$$T = \frac{2\pi}{\omega 40}, \tag{15}$$

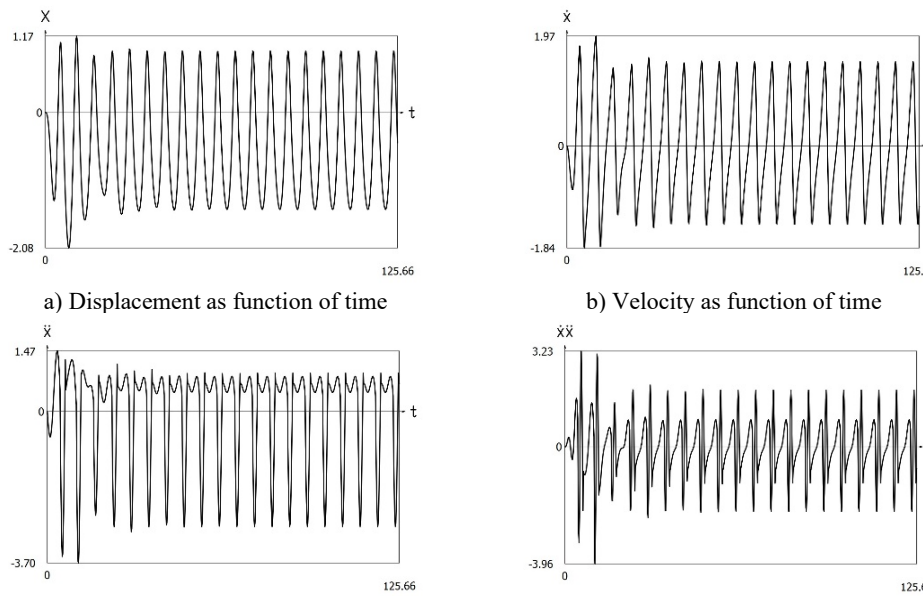
and medium time step:

$$T = \frac{2\pi}{\omega 80}. \tag{16}$$

4.1. Results of investigation of the system with different values of stiffness for positive and negative displacements for large time step

4.1.1. Conventional procedure for calculation of dynamics

Displacement, velocity, acceleration, and velocity multiplied by acceleration as functions of time are presented in Fig. 1.



a) Displacement as function of time

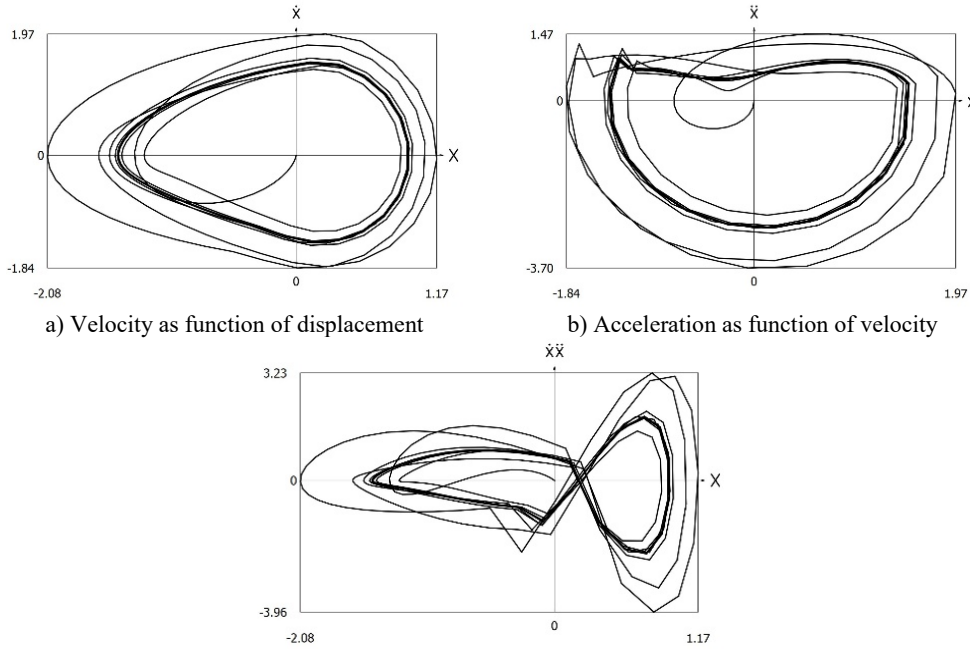
b) Velocity as function of time

c) Acceleration as function of time

d) Velocity multiplied by acceleration as function of time

Fig. 1. Dynamics of the system with different values of stiffness for positive and negative displacements when $\omega = 1$, $f = 1$, $h = 0.1$, $p_1 = 1$, $p_2 = 2$

Phase trajectories of motion of the investigated system are presented in Fig. 2.
Force of stiffness as function of displacement is shown in Fig. 3.



a) Velocity as function of displacement

b) Acceleration as function of velocity

c) Velocity multiplied by acceleration as function of displacement

Fig. 2. Phase trajectories of the system with different values of stiffness for positive and negative displacements when $\omega = 1$, $f = 1$, $h = 0.1$, $p_1 = 1$, $p_2 = 2$

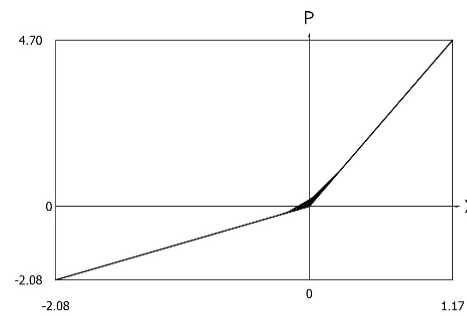


Fig. 3. Force of stiffness as function of displacement of the system with different values of stiffness for positive and negative displacements when $\omega = 1$, $f = 1$, $h = 0.1$, $p_1 = 1$, $p_2 = 2$

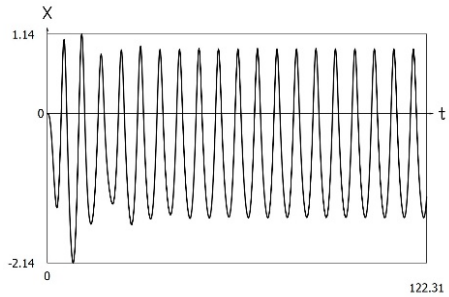
4.1.2. Improved procedure for calculation of dynamics

Displacement, velocity, acceleration, and velocity multiplied by acceleration as functions of time are presented in Fig. 4.

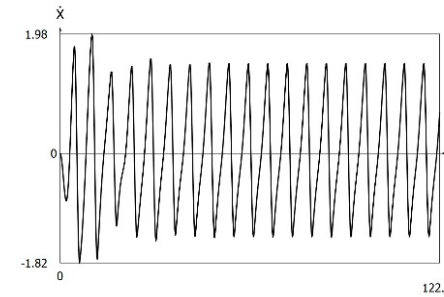
Phase trajectories of motion of the investigated system are presented in Fig. 5.

Force of stiffness as function of displacement is shown in Fig. 6.

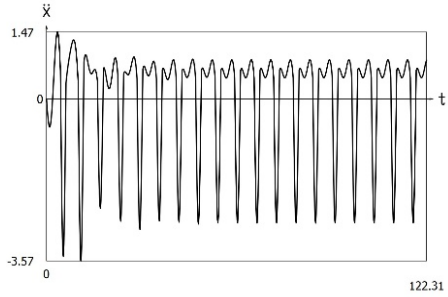
From the presented results it is seen that the improved procedure for calculation of dynamics has advantages, which are seen from the comparison of corresponding graphical representations. This is especially evident from the comparison of corresponding drawings involving accelerations as well as from the comparison of representations of force of stiffness as function of displacement in the vicinity of the origin of the coordinate system.



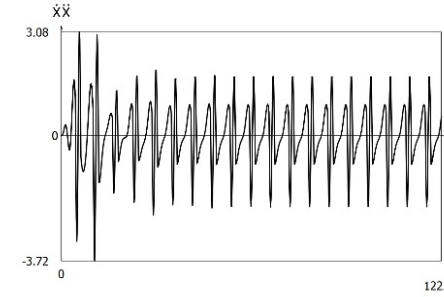
a) Displacement as function of time



b) Velocity as function of time

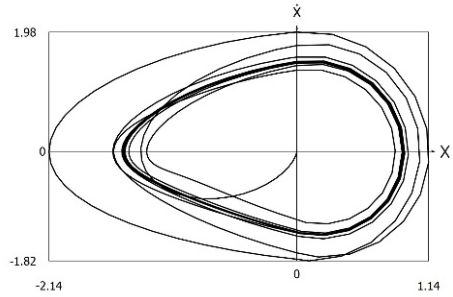


c) Acceleration as function of time

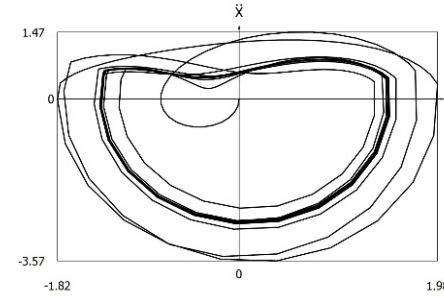


d) Velocity multiplied by acceleration as function of time

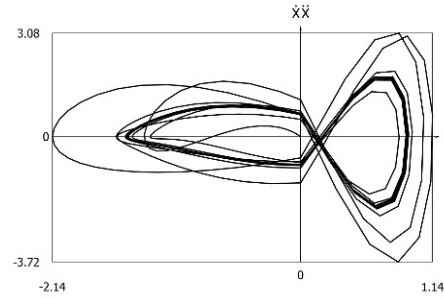
Fig. 4. Dynamics of the system with different values of stiffness for positive and negative displacements when $\omega = 1$, $f = 1$, $h = 0.1$, $p_1 = 1$, $p_2 = 2$



a) Velocity as function of displacement



b) Acceleration as function of velocity



c) Velocity multiplied by acceleration as function of displacement

Fig. 5. Phase trajectories of the system with different values of stiffness for positive and negative displacements when $\omega = 1$, $f = 1$, $h = 0.1$, $p_1 = 1$, $p_2 = 2$

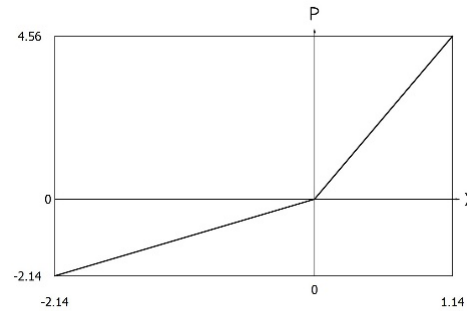
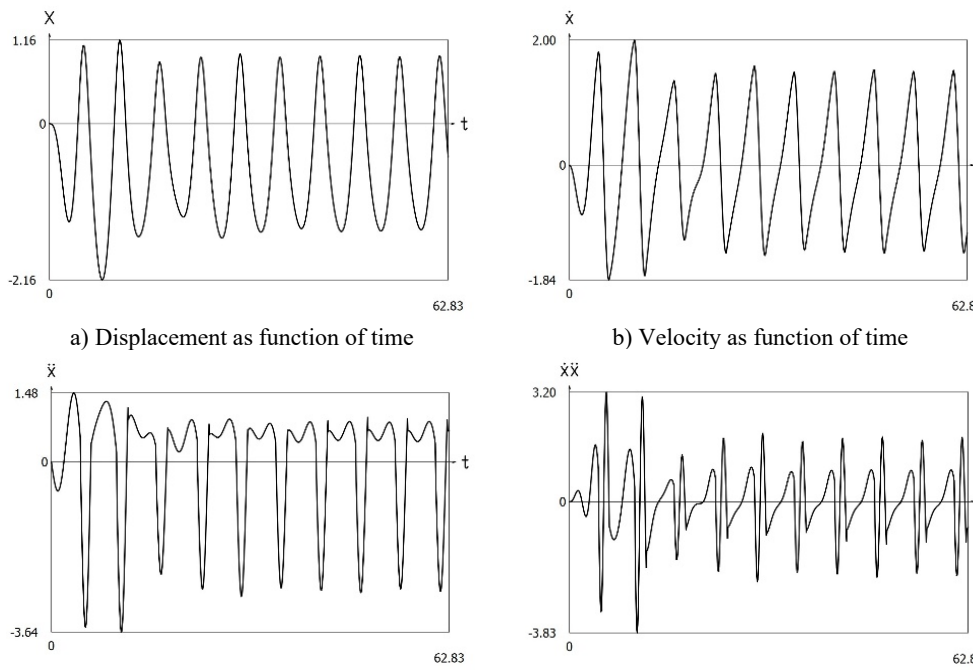


Fig. 6. Force of stiffness as function of displacement of the system with different values of stiffness for positive and negative displacements when $\omega = 1$, $f = 1$, $h = 0.1$, $p_1 = 1$, $p_2 = 2$

4.2. Results of investigation of the system with different values of stiffness for positive and negative displacements for medium time step

4.2.1. Conventional procedure for calculation of dynamics

Displacement, velocity, acceleration, and velocity multiplied by acceleration as functions of time are presented in Fig. 7.



a) Displacement as function of time

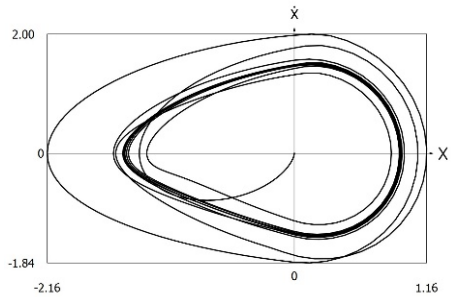
b) Velocity as function of time

c) Acceleration as function of time

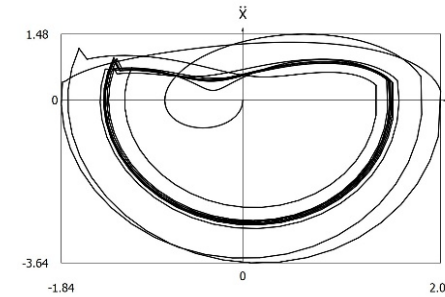
d) Velocity multiplied by acceleration as function of time

Fig. 7. Dynamics of the system with different values of stiffness for positive and negative displacements when $\omega = 1$, $f = 1$, $h = 0.1$, $p_1 = 1$, $p_2 = 2$

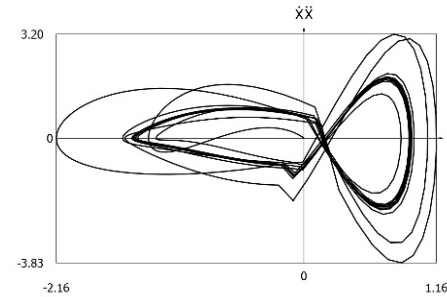
Phase trajectories of motion of the investigated system are presented in Fig. 8.
Force of stiffness as function of displacement is shown in Fig. 9.



a) Velocity as function of displacement



b) Acceleration as function of velocity



c) Velocity multiplied by acceleration as function of displacement

Fig. 8. Phase trajectories of the system with different values of stiffness for positive and negative displacements when $\omega = 1$, $f = 1$, $h = 0.1$, $p_1 = 1$, $p_2 = 2$

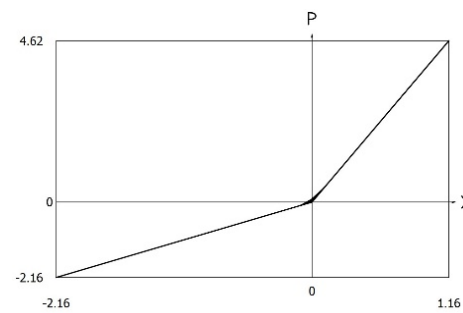


Fig. 9. Force of stiffness as function of displacement of the system with different values of stiffness for positive and negative displacements when $\omega = 1$, $f = 1$, $h = 0.1$, $p_1 = 1$, $p_2 = 2$

4.2.2. Improved procedure for calculation of dynamics

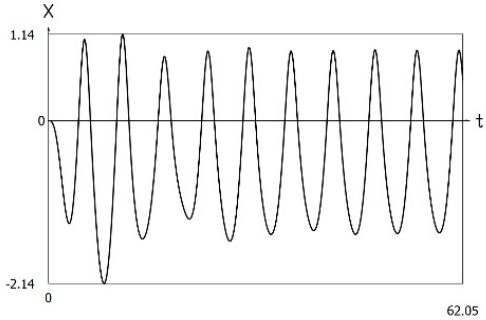
Displacement, velocity, acceleration, and velocity multiplied by acceleration as functions of time are presented in Fig. 10.

Phase trajectories of motion of the investigated system are presented in Fig. 11.

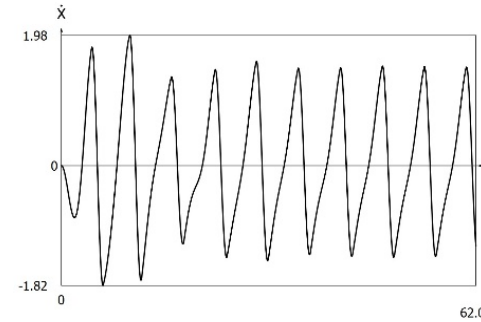
Force of stiffness as function of displacement is shown in Fig. 12.

From the presented results it is seen that the improved procedure for calculation of dynamics has advantages, which are seen from the comparison of corresponding graphical representations. This is especially evident from the comparison of corresponding drawings involving accelerations as well as from the comparison of representations of force of stiffness as function of displacement in the vicinity of the origin of the coordinate system.

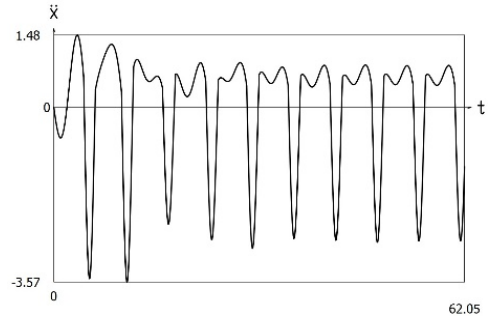
The presented graphical representations show the advantages of the improved procedure for calculation of dynamics of the system with different values of stiffness for positive and negative displacements.



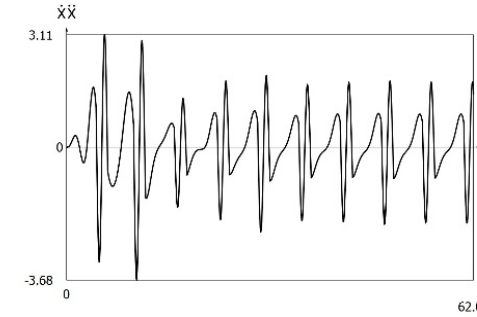
a) Displacement as function of time



b) Velocity as function of time

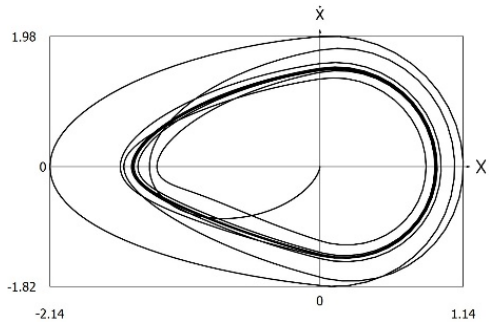


c) Acceleration as function of time

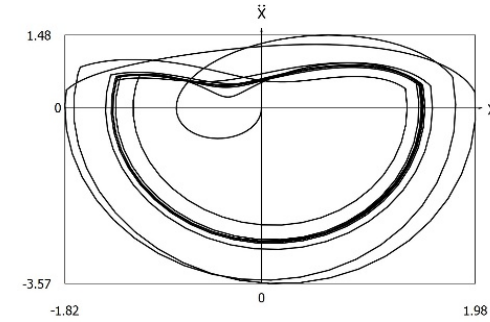


d) Velocity multiplied by acceleration as function of time

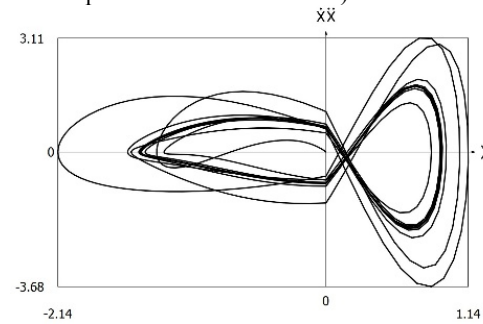
Fig. 10. Dynamics of the system with different values of stiffness for positive and negative displacements when $\omega = 1$, $f = 1$, $h = 0.1$, $p_1 = 1$, $p_2 = 2$



a) Velocity as function of displacement



b) Acceleration as function of velocity



c) Velocity multiplied by acceleration as function of displacement

Fig. 11. Phase trajectories of the system with different values of stiffness for positive and negative displacements when $\omega = 1$, $f = 1$, $h = 0.1$, $p_1 = 1$, $p_2 = 2$

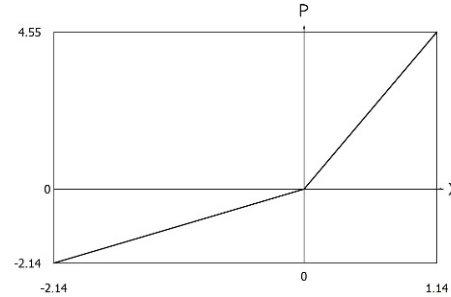


Fig. 12. Force of stiffness as function of displacement of the system with different values of stiffness for positive and negative displacements when $\omega = 1$, $f = 1$, $h = 0.1$, $p_1 = 1$, $p_2 = 2$

5. Conclusions

Systems with different values of stiffness for positive and negative displacements are important in engineering applications. This change of the value of stiffness has an effect to the results of numerical calculation of dynamics of the system. For more precise investigation of dynamics a special numerical procedure is proposed in the paper. Numerical results for two values of time steps are presented without application of the proposed procedure and with application of it.

Displacement, velocity, acceleration, and velocity multiplied by acceleration as functions of time are investigated. Also phase trajectories of motion of the investigated system are presented. Force of stiffness as function of displacement is calculated and graphically represented.

From the presented results it is seen that the improved procedure for calculation of dynamics has advantages, which are seen from the comparison of corresponding graphical representations. This is especially evident from the comparison of corresponding drawings involving accelerations as well as from the comparison of representations of force of stiffness as function of displacement in the vicinity of the origin of the coordinate system. The presented graphical representations show the advantages of the improved procedure for calculation of dynamics of the system with different values of stiffness for positive and negative displacements.

The results of the performed investigation are applied in the process of design of elements of robots.

References

[1] W. Wedig, "New resonances and velocity jumps in nonlinear road-vehicle dynamics," *Procedia IUTAM*, Vol. 19, pp. 209–218, 2016.

[2] T. Li, E. Gourc, S. Seguy, and A. Berlioz, "Dynamics of two vibro-impact nonlinear energy sinks in parallel under periodic and transient excitations," *International Journal of Non-Linear Mechanics*, Vol. 90, pp. 100–110, Apr. 2017, https://doi.org/10.1016/j.ijnonlinmec.2017.01.010

[3] V. A. Zaitsev, "Global asymptotic stabilization of periodic nonlinear systems with stable free dynamics," *Systems and Control Letters*, Vol. 91, pp. 7–13, May 2016, https://doi.org/10.1016/j.sysconle.2016.01.004

[4] H. Dankowicz and E. Fotsch, "On the analysis of chatter in mechanical systems with impacts," *Procedia IUTAM*, Vol. 20, pp. 18–25, 2017, https://doi.org/10.1016/j.piutam.2017.03.004

[5] S. Spedicato and G. Notarstefano, "An optimal control approach to the design of periodic orbits for mechanical systems with impacts," *Nonlinear Analysis: Hybrid Systems*, Vol. 23, pp. 111–121, 2017, https://doi.org/10.48550/arxiv.1702.04544

[6] W. Li, N. E. Wierschem, X. Li, and T. Yang, "On the energy transfer mechanism of the single-sided vibro-impact nonlinear energy sink," *Journal of Sound and Vibration*, Vol. 437, pp. 166–179, Dec. 2018, https://doi.org/10.1016/j.jsv.2018.08.057

[7] J. S. Marshall, "Modeling and sensitivity analysis of particle impact with a wall with integrated damping mechanisms," *Powder Technology*, Vol. 339, pp. 17–24, Nov. 2018, https://doi.org/10.1016/j.powtec.2018.07.097

[8] E. Salahshoor, S. Ebrahimi, and Y. Zhang, "Frequency analysis of a typical planar flexible multibody system with joint clearances," *Mechanism and Machine Theory*, Vol. 126, pp. 429–456, Aug. 2018, https://doi.org/10.1016/j.mechmachtheory.2018.04.027

[9] U. Starossek, "Forced response of low-frequency pendulum mechanism," *Mechanism and Machine Theory*, Vol. 99, pp. 207–216, May 2016, https://doi.org/10.1016/j.mechmachtheory.2016.01.004

[10] S. Wang, L. Hua, C. Yang, Y.O. Zhang, and X. Tan, "Nonlinear vibrations of a piecewise-linear quarter-car truck model by incremental harmonic balance method," *Nonlinear Dynamics*, Vol. 92, No. 4, pp. 1719–1732, Jun. 2018, https://doi.org/10.1007/s11071-018-4157-6

[11] P. Alevras, S. Theodossiades, and H. Rahnejat, "On the dynamics of a nonlinear energy harvester with multiple resonant zones," *Nonlinear Dynamics*, Vol. 92, No. 3, pp. 1271–1286, May 2018, https://doi.org/10.1007/s11071-018-4124-2

[12] A. Sinha, S. K. Bharti, A. K. Samantaray, G. Chakraborty, and R. Bhattacharyya, "Sommerfeld effect in an oscillator with a reciprocating mass," *Nonlinear Dynamics*, Vol. 93, No. 3, pp. 1719–1739, Aug. 2018, https://doi.org/10.1007/s11071-018-4287-x

[13] G. Habib, G. I. Cirillo, and G. Kerschen, "Isolated resonances and nonlinear damping," *Nonlinear Dynamics*, Vol. 93, No. 3, pp. 979–994, Aug. 2018, https://doi.org/10.1007/s11071-018-4240-z

Kazimieras Ragulskis is a member of Academies of Sciences of the USSR (later of the Russian Academy of Sciences) and Lithuania. His research area is precise vibromechanics and vibroengineering, the basis of it are vibrations and waves in nonlinear dynamical systems. Kazimieras Ragulskis responsibilities in this paper: conceptualization, formal analysis, investigation, methodology, supervision, validation, visualization, writing – original draft preparation, writing – review and editing.

Bronislovas Spruogis is a Professor of the Department of Mobile Machinery and Railway Transport of the Faculty of Transport Engineering of Vilnius Gediminas Technical University. His main research interests are dynamics and design of mechanical and hydraulic systems. Bronislovas Spruogis responsibilities in this paper: formal analysis, investigation, methodology, supervision, validation, visualization, writing – original draft preparation, writing – review and editing.

Arvydas Matuliauskas is a Lecturer of the Department of Mobile Machinery and Railway Transport of the Faculty of Transport Engineering of Vilnius Gediminas Technical University. His main research interests are dynamics and design of robots. Arvydas Matuliauskas responsibilities in this paper: formal analysis, investigation, methodology, validation, visualization, writing – original draft preparation, writing – review and editing.

Vygantas Mištinas is a Lecturer of the Department of Mobile Machinery and Railway Transport of the Faculty of Transport Engineering of Vilnius Gediminas Technical University. His main research interests are dynamics and design of robots. Vygantas Mištinas responsibilities in this paper: formal analysis, investigation, methodology, validation, visualization, writing – original draft preparation, writing – review and editing.

Liutauras Ragulskis is working as a research associate at Vytautas Magnus University. His research interests are numerical calculations of vibrating systems. Liutauras Ragulskis responsibilities in this paper: visualization.