



Transport and Telecommunication, 2022, volume 23, no. 3, 207–219
Transport and Telecommunication Institute, Lomonosova 1, Riga, LV-1019, Latvia
DOI 10.2478/tjt-2022-0017

THE CONCEPT OF DEPENDENCY GAME TREE GRAPHS AS A BLACK BOX IN THE ANALYSIS OF AUTOMATIC TRANSMISSIONS

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The methodology of graph theory has long had applications in mechanics. Graphs allow the simultaneous consideration and synthesis of the structure of a real system and the idealized structure of an equivalent model of a mechanical system. In particular, there are many applications of graph theory in the analysis of automatic transmissions. This paper presents continuing research on applications of game tree graphs in the analysis of automatic transmissions. Black box ideas for dependency graphs are presented. This makes it possible to automatically generate simplified models from given physical models of an automatic transmission and to determine the optimal number of teeth. The developed methodology allows also to perform real-time simulations. In a further stage, the methods of decision logic trees can be applied to analyze the functional diagrams of selected gears.

Keywords: Computational model, mechanical engineering, optimization, graph theory, automatic gearboxes, black box

1. Introduction

Mechanical gears are one of the most important inventions in the world of mechanics and transportations. They make it possible to freely change the speed and rotational force transmitted from the feeding device to the peeling device. The history of the use of gears is closely related to the invention of the wheel. Mechanical transmissions are usually wheeled mechanisms designed to transmit power from the shaft of an engine performing a rotational movement to the drive member of a working machine, an executive mechanism or directly to the working member. It is by far the most common drive mechanism used in electrical machinery and equipment. Gears are used in dozens of industries: from automotive to agricultures, electronics etc. In most cases, gears are made of black steel or stainless steel, but modern variants created from plastics are becoming more and more popular; however, they do not always perform well under heavy loads (Wang *et al.*, 2018). In such situations, graphite-doped gears are increasingly proposed. One of the most popular gears are planetary gears. Gears are also used, among others, in traditional clocks, turbines, household appliances (mixer, blender), as well as in mining, paper machines and other equipment equipped with power transmission systems. This widespread use is a consequence of the key advantages to be gained from cyclic maintenance (Feng *et al.*, 2012). They owe their popularity to their small dimensions, simple design and manufacturing technology, and low production costs.

Automatic transmissions are transmissions that automatically change gears depending on the load of the vehicle engine. An automatic transmission allows smooth driving – optimizing the gear ratio in relation to the engine speed. The classic automatic transmission is not an ordinary transmission with automatic control. It differs in design and transmission method – a manual transmission is equipped with a conventional clutch, while automatic transmissions use for example a torque converter. Automatic gearboxes based on ordinary gearboxes and clutches with added control are mainly used in trucks, etc. (Tseng *et al.*, 2015; Yu *et al.*, 2017).

The design of planetary gears can be aided by various methods of decision support schemes, such as the use of decision trees (Yu *et al.*, 2013 and Deptuła *et al.*, 2011) or graphs (Deptuła *et al.*, 2017; Chen, 2013; Pennestri'e *et al.*, 2013). The objectives of modelling gears with graphs have been varied- including

dynamic analysis, kinematic analysis (Uematsu, 1997) synthesis, structure analysis (Huang *et al.*, 1996), enumeration (Tsai, 2001), gear train optimization, and automatic design based on so-called graph grammars. Publications on the problem of solving gear design tasks using artificial intelligence methods for gears and other engineering objects can be found, for example, in Deptula, 2020. An extensive literature review is described, among others, by S. Zawiślak in work Tseng and Yu (2015). In particular, the author described a method of generating a system of kinematics equations for Hsu graphs modelling an automatic transmission, and also indicated many other areas of research in the field of graph applications in mechanical engineering. J. Wojnarowski *et al.*, 2006 in their work presented a review of works related to the problem of modelling gearboxes by means of universal models of graph theory. In addition, graph methods in the analysis of various types of gears were dealt with by many authors abroad. Nelson *et al.* (2005) and Pennestri *et al.* (2003) presented kinematic, power flow, and efficiency analysis of epicycles gear sets.

The advantage of modelling gears with graphs is that graph models are solved algorithmically. Many of the steps are automated, which speeds up conceptual gearbox design activities. The present work is a continuation of co-authored research on the game tree graphs method. Currently, the authors have proposed a black-box approach to game tree graphs. Thus, it is possible to replace the complex numerical sub models of the physical model of an automatic transmission with a black box model.

2. Game tree structures

The directed dependency graph (Deptula *et al.*, 2011; Deptula, 2020) defines analytical expressions representing this graph, and thus constituting its analytical model. There are studies in the literature describing the use of dependency graphs and parametric structures in the study of dynamic properties of machine systems. A graph is defined by an ordered pair of sets. The first contains the graph's vertices, and the second contains the edges of the graph, i.e. an ordered pair of vertices. Figure 1 shows an example of the directed dependency graph G (playing parametrically).

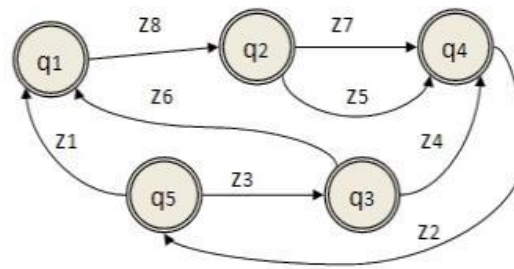


Figure 1. Example dependency graph

A graph is called an ordered pair $G = (V, E)$, in which V is a finite set of elements called the vertices of the graph, and E is a set of $(v_i, v_j) (v_i, v_j \in V)$ pairs called the edges of the graph. To fully specify the graph, it must be also specify the relationship P formed by individual elements of the set of vertices $V(G)$ and edges $E(G)$. Then the graph can be called ordered three:

$$G = (V, E, P), \quad (1)$$

where: V – set of graph vertices, E – set of edges (graph branches), P – three – member relationship $G = (V, E, P)$ which meets the following conditions: There is such a pair of vertices $(x, y \in V)$ for each branch e , such that $\langle x, e, y \rangle \in P$. If for branch e exist $\langle x, e, y \rangle \in P$ and $\langle w, e, z \rangle \in P$ either $x = w$ and $y = z$ or $x = y = z$. A dependency graph is an ordered pair $G = (X, R)$, in which X is a finite set of elements called vertices of a graph, and R is a set of pairs $(x_i, x_j) (x_i, x_j \in X)$ called the edges of the graph. In the case of parametric graphs, the notation introduced by Deptula, 2020 defines the signs: $G = (Q, Z)$, where Z is a set of pairs $(z_i, z_j) (z_i, z_j \in Z)$. The oriented dependency (game) graph is shown in Figure 1 is composed of a set of vertices Q :

$$Q = \{q_1, q_2, q_3, q_4, q_5\} \quad (2)$$

and of a set of edges Z , that is an ordered pair of vertices:

$$Z = \{z_1, z_2, z_3, z_4, z_5, z_6, z_7, z_8\}. \quad (3)$$

The algorithm for the analysis, structuring and distribution of the dependency graph is presented, among others in the research by Deptula, 2020. For example, for the start vertices q_3 , q_4 and q_5 (Figure 1) the analytical expressions are obtained: Gq_3^{++} , Gq_4^{++} and Gq_5^{++} :

$$\begin{cases} G_{q_3}^{++} = ({}^0q_3({}^1z_4q_4({}^2z_2q_5({}^3z_1q_1({}^4z_8q_2({}^5z_5q_4^1, z_7q_4^2)^5)^4, z_3q_3^1)^3)^2, \\ z_6q_1({}^2z_8q_2({}^3z_5q_4^3, z_7q_4^4)^3)^2)^1)^0 \\ G_{q_4}^{++} = ({}^0q_4({}^1z_2q_5({}^2z_1q_1({}^3z_8q_2({}^4z_5q_4^1, z_7q_4^2)^4)^3, z_3q_3({}^3z_4q_4^3, z_6q_1^1)^3)^2)^1)^0 \\ G_{q_5}^{++} = ({}^0q_5({}^1z_1q_1({}^2z_8q_2({}^3z_5q_4({}^4z_2q_5^1)^4, z_7q_4({}^4z_2q_5^2)^4)^3)^2, z_3q_3({}^2z_4q_4({}^3z_2q_5^3)^3, \\ z_6q_1({}^3z_8q_2({}^4z_5q_4({}^5z_2q_5^5)^5, z_7q_4({}^5z_2q_5^5)^4)^3)^2)^1)^0 \end{cases} \quad (4)$$

3. Analysis of automatic transmission considering game tree graphs in the black box view

The element that controls the rotation of the engine is the gearbox. A given gear determines the torque of the engine and thus the speed at which the car moves. The transmission also allows to change the direction of travel. After the engine, this is the device that significantly influences the optimization of the functioning of the car. The requirements for gearboxes are primarily:

- guaranteeing driving comfort,
- smooth, jerk- free and quiet engine management;
- minimum space requirement;
- fuel economy (achieved, for example, by an economical transmission strategy);
- good driving dynamics – appropriate gearshift times adapted to the prevailing driving conditions (recognition of the driver's preferences, acceleration, engine braking e.g. in the mountains, shift inhibition in fast cornering).

In the Figure 2 shows an example of how an automatic transmission works.

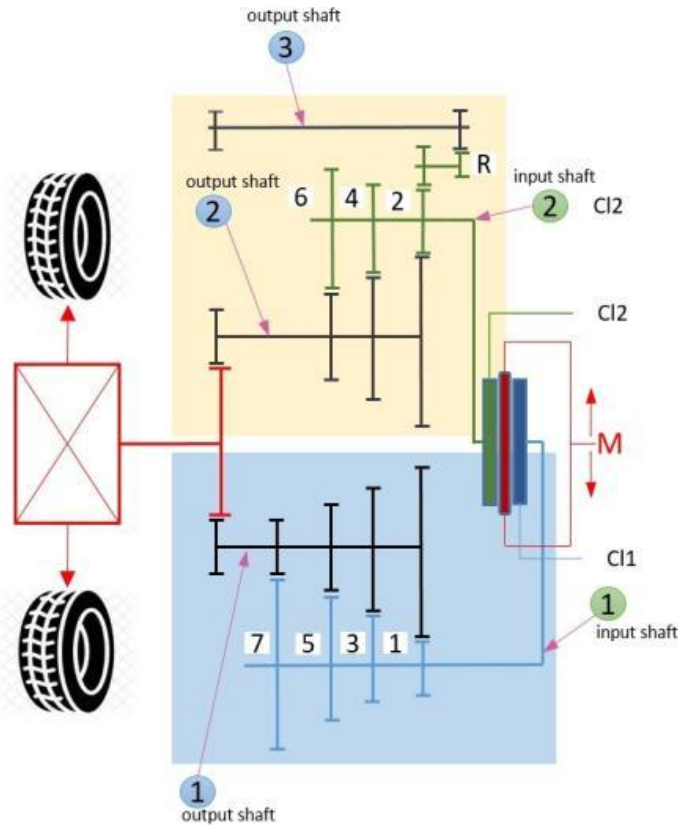


Figure 2. Double-clutch automatic transmission operating diagram

The modern world strives to automate as much as possible of the activities associated with daily life. The introduction of electronic transmission control systems has made it possible to automate the regulation of engine operation. The control device is a printed circuit board with electronic components. This computer analyses the signals coming from the sensors. The transmission of data from external devices (these are sensors and actuators) is carried out through a wiring harness and a multi-pin connector. These signals can be:

- analogy – battery voltage, transmission oil temperature;
- digital – take only two values: 0 and 1 (binary system). These are e.g. switch position signals (on/off), speed sensor signals;
- impulse – e.g. rotational speed, information about the reference level. They are converted into digital form after the elimination of noise.

3.1. The idea of using a game tree structures as a black box

For methodological support, a numerical derivation scheme is developed as a simplified automatic transmission model based on a given physical model. A general black box model structure (in discrete time) based on a local model is proposed. The game tree graphs entering the black box structure allow to determine the optimal values of the number of teeth and to determine the optimal gear ratios. Figure 3 shows a schematic of the gearbox under analysis along with the conventional model.

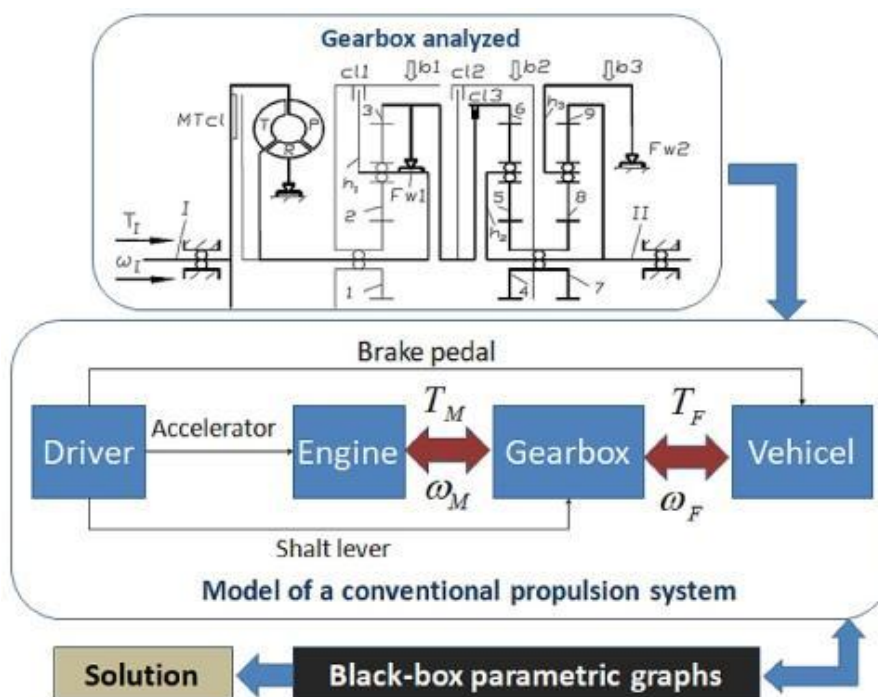


Figure 3. The analysed transmission together with model of the conventional drive system

The main task of an automatic transmission is to adjust the ratio between speeds or torques according to the requirements of the vehicle. The control unit determines when to shift gears and performs the shifting operations. The mechanics of the transmission consist of planetary stages connected to each other by clutches and brakes. The clutches and brakes used are sliding elements that are opened or closed by hydraulic control. Thus, the planetary stages can be opened and closed by means of hydraulic control in various switching operations by Isermann, 2010.

The physical model equations are numerically complex. Coupling the hydraulic control to the transmission mechanics results in a rigid differential. A system of differential equations with high natural frequencies is then obtained. The method proposed by the authors replaces the numerically complex partial models, consisting of planetary stages, switching elements and hydraulic control, with discrete-time. Figure 4 shows a simplified model derivation scheme using the black box concept.

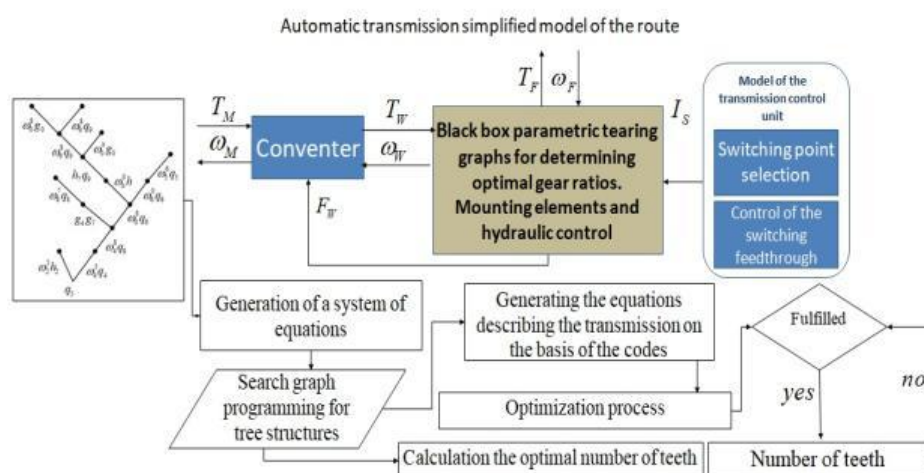


Figure 4. Simplified model derivation scheme with the idea of using a black box

4. Analysis of an example automatic transmission

In the process of determining kinematic structures using graph theory, an important and complicated problem is the identification of graph isomorphism's. There are many approaches to detecting isomorphic graphs, and these approaches are largely algorithmic. The main objective of kinematic analysis of an automatic transmission is to determine the kinematic ratio and possibly detect redundant gears.

- I. In the first stage, the graphs determine the optimal number of teeth.
- II. The next task is to optimize the parameters of the static function approximator model to approximate the given behavior.

The parametrically playing out structures for a game graphs with an approximator can be further divided into models with and without output feedback. The advantage of the model with output feedback is that the system dynamics can be approximated by only a few time delay operators, and thus the number of model parameters remains limited. The disadvantage of this solution is that the created models may become unstable in some simulations. In order to be able to exploit the advantages of stability of models without output feedback and to reduce the number of model parameters at the same time, orthonormal basis function filters (OBF filters) are used instead of simple time delay operators, which make it possible to incorporate knowledge about the dynamics of the technical process (Hartmann, 2014). The object of analysis is a complex four- speed automatic transmission with overdrive type A4LD (Deptula, 2020). Individual gears are implemented using brakes, clutches and backstops (free wheels) is shown in Table 1 according to Deptula, 2020 and Zawislak, 2010. The diagram of the analysed gearbox is shown on Figure 3.

Table 1. Functional matrix of the A4 ALD gearbox

Position	DE (1 2 3 4)	1 st gear	2 nd gear	Reverse
<i>cl1</i>	()	X	X	X
<i>b1</i>	(X)	-	-	-
<i>Fw1</i>	(X X X)	X	X	X
<i>cl2</i>	(X X)	-	-	X
<i>cl3</i>	(X X X X)	X	X	-
<i>b2</i>	(X)	X	-	-
<i>b3</i>	(X X X X)	-	X	X
<i>Fw2</i>	(X)	-	-	-

4.1. Generating graph and game tree structures- finding the optimum number of teeth

Power is transmitted through the torque converter, *h1* yoke, free wheel *Fw1*, *Cl3* and *Cl2* clutches, gears 6, 5, 4 and yoke *h2* and in parallel through wheels 7, 8, 9 on output shaft II (*Fw2* backstop

activated) – Figure 3. Following the rules of building according to the contour graph methods, the dependency graph shown in Figure 5 is built for the transmission.

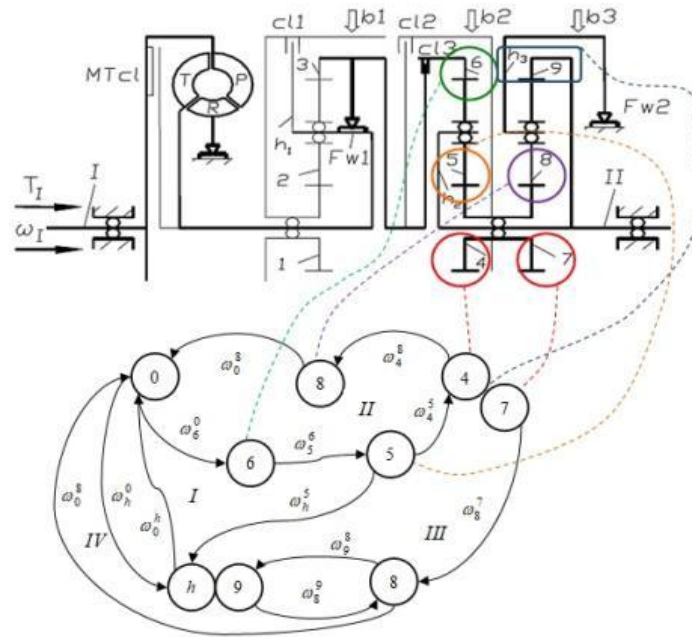


Figure 5. Signal dependency graph for DE1

For the above Figure 5, observing the appropriate algorithm, it is possible to generate a set of parametric trees:

$$DE1 = \{G_{(q_0^{++})}, G_{(q_4^{++})}, G_{(q_5^{++})}, G_{(q_6^{++})}, G_{(q_8^{++})}, G_{(q_9^{++})}\}. \quad (5)$$

Tree structures are shown in Figure 6.

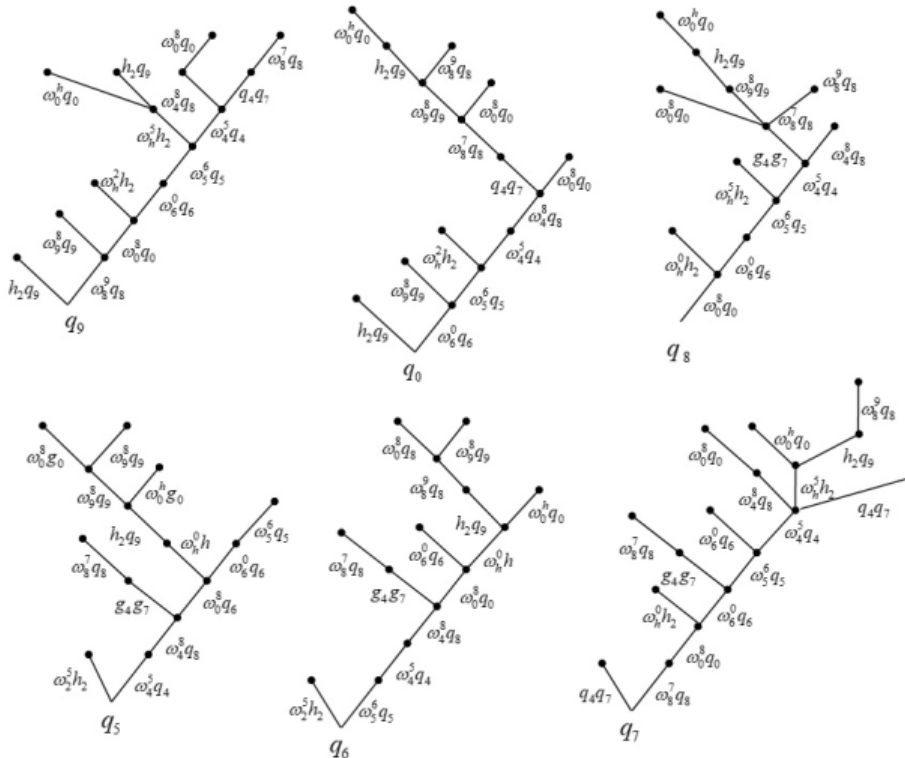


Figure 6. Game-tree structures for the signal dependency graph for DE1 (showed in the Figure 5)

Each of the structures has an appropriate analytical formula (G_i^+ and G_i^{++} , where i denotes the vertex from which the decomposition of the graph was made), that clearly defines the way of transition from a dependency graph to a tree structure. The structures in Figure 6 are defined by the Equations (6–11):

$$G_{Z0}^{++} = ({}^0g_0({}^1\omega_0g_6({}^2\omega_5g_5({}^3\omega_4g_4({}^4\omega_8g_8({}^5\omega_0g_0)^5, g_4g_7({}^5\omega_8g_{8b}({}^6\omega_9g_9({}^7\omega_8g_8, g_9h_2({}^8\omega_0g_0)^8)^7, \omega_0g_0)^6)^5)^4, \omega_5h_2)^3)^2, \omega_h^0h)^1)^0 \quad (6)$$

$$G_{Z9}^{++} = ({}^0g_9({}^1\omega_9g_8({}^2\omega_0g_0({}^3\omega_6g_6({}^4\omega_5g_5({}^5\omega_4g_4({}^6g_4g_7({}^7\omega_8g_{8b})^7, \omega_4g_8({}^7\omega_0g_0)^7)^6, \omega_h^5h_2({}^6h_2g_9, \omega_0g_0)^6)^5)^4, \omega_h^0h_2)^3, \omega_9g_9)^2, h_2g_9)^1)^0 \quad (7)$$

$$G_{Z8}^{++} = ({}^0g_8({}^1\omega_0g_0({}^2\omega_6g_6({}^3\omega_5g_5({}^4\omega_4g_4({}^5\omega_8g_8, g_4g_7({}^6\omega_8g_{8b}({}^7\omega_0g_0, \omega_9g_9({}^8h_2g_9({}^9\omega_0g_0)^8, \omega_8g_{8b})^7)^6)^5, \omega_h^5h_2)^4)^3, \omega_h^0h_2)^2)^1)^0)^2 \quad (8)$$

$$G_{Z7}^{++} = ({}^0g_7({}^1\omega_8g_8({}^2\omega_0g_0({}^3\omega_6g_6({}^4\omega_5g_5({}^5\omega_4g_4({}^6g_4g_7, \omega_4g_{8b}({}^7\omega_0g_0)^7)^6, \omega_h^5h_2({}^6\omega_0g_0, h_2g_9({}^7\omega_8g_{8b})^7)^6)^5)^4, \omega_h^0h_2)^3)^2, g_4g_7)^1)^0 \quad (9)$$

$$G_{Z6}^{++} = ({}^0g_6({}^1\omega_5g_5({}^2\omega_4g_4({}^3\omega_8g_8({}^4\omega_0g_0({}^5\omega_6g_6, \omega_h^0h({}^6h_2g_9({}^7\omega_8g_{8b}({}^8\omega_0g_0, \omega_9g_9)^8)^7, \omega_0g_0)^6)^5)^4, g_4g_7({}^4\omega_8g_{8b})^4)^3, \omega_h^5h_2)^2)^1 \quad (10)$$

$$G_{Z5}^{++} = ({}^0g_5({}^1\omega_4g_4({}^2\omega_8g_8({}^3\omega_0g_0({}^4\omega_6g_6({}^5\omega_5g_5)^5, \omega_h^0h({}^5h_2g_9({}^6\omega_8g_{8b}({}^7\omega_0g_0, \omega_9g_9)^7)^6, \omega_0g_0)^5)^4)^3, g_4g_7({}^3\omega_8g_{8b})^3)^2, \omega_h^5h_2)^1)^0 \quad (11)$$

Structures can be searched in a computer program:

```

/ Returns the value of z and if the target is reachable z
// src within the bounds of max_depth
bool IDDFS(src, target, max_depth)
    for limit from 0 to max_depth //take into account the initial and design
    conditions
        if DLS(src, target, limit) == true
            return true
        return false
bool DLS(src, target, limit)
    if (src == target)
        return true;
    // If reached the maximum depth,
    // stop recursing.
    if (limit <= 0)
        return false;
    foreach adjacent i of src
        if DLS(i, target, limit-1)
            return true

```

An important point is that the lowest-level nodes are visited multiple times through return cycles. The last level (or maximum depth) is visited once, the next-to-last level is visited twice, and so on. Two cases may occur:

- When the graph has no cycle; this case is simple. We can use DFS multiple times with different height limits. Most frequently at the initial levels of the structure.
- When the graph has cycles. Most frequently at the end nodes of the structure.

The algorithm operates parametrically. The initial number of teeth is input:

The input attributes are t numbers of teeth $z_1, z_2, z_3, z_4, z_5, z_6, z_7, \dots, z_k$. The values $h_1, h_1, \dots, h_m$ are respectively hypotheses represented by the constraints. These are the output values (out) that determine the tooth values. For searching, ranges of values for the number of teeth searched are assumed: $z_1, z_2, z_3, z_4, z_5, z_6, z_7, z_8, z_9: z_1 \in (0, 1, 2, \dots, 100), z_2 \in (0, 1, 2, \dots, 100), \dots, z_9 \in (0, 1, 2, \dots, 100)$.

Figures 7–9 show selected steps in determining the optimal number of teeth by game tree structures.

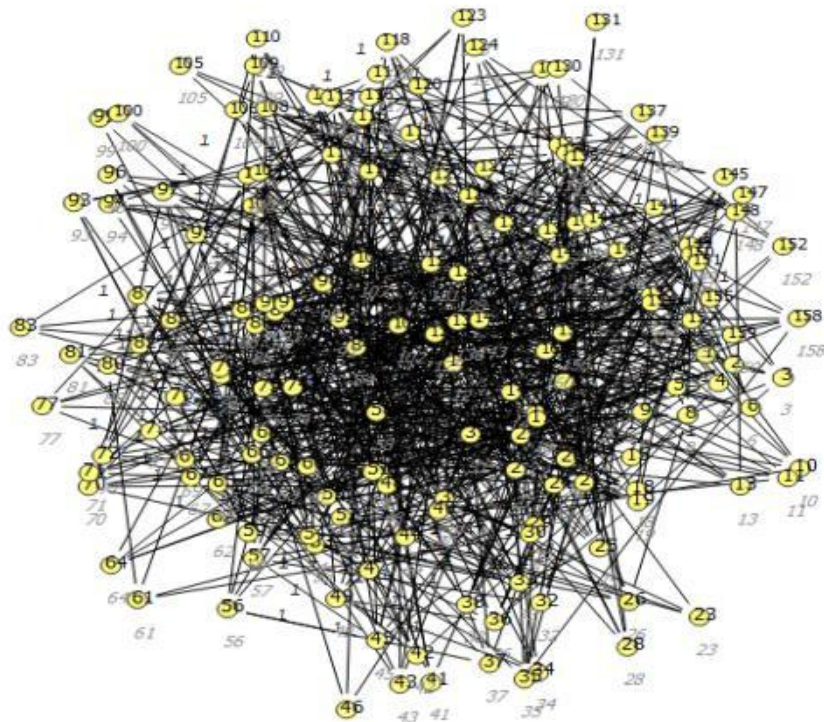


Figure 7. First stage of finding the optimal number of teeth – unordered search

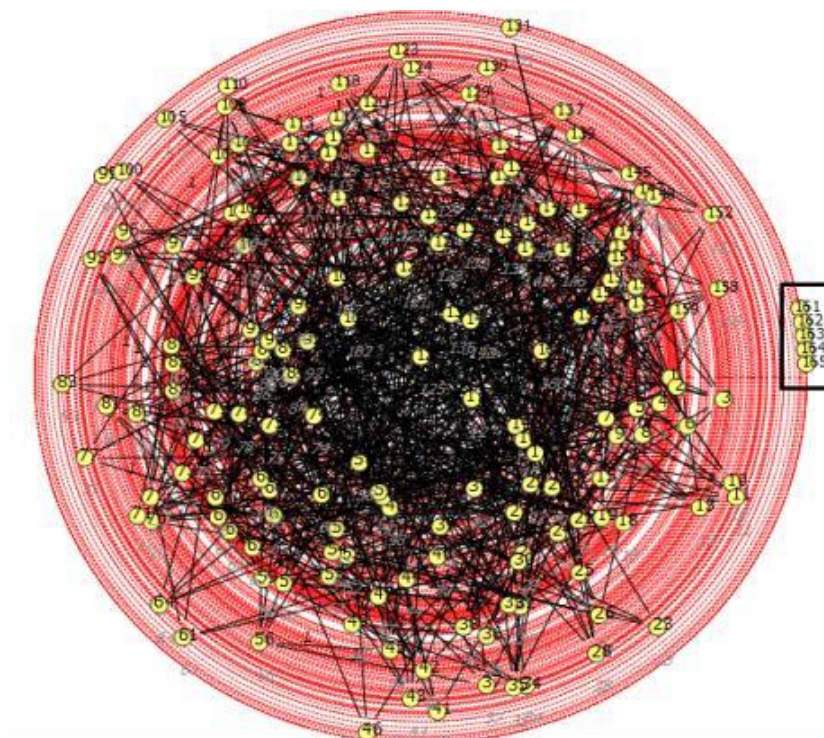


Figure 8. The next optimization stage of finding the optimal number of teeth – formulating the objective function

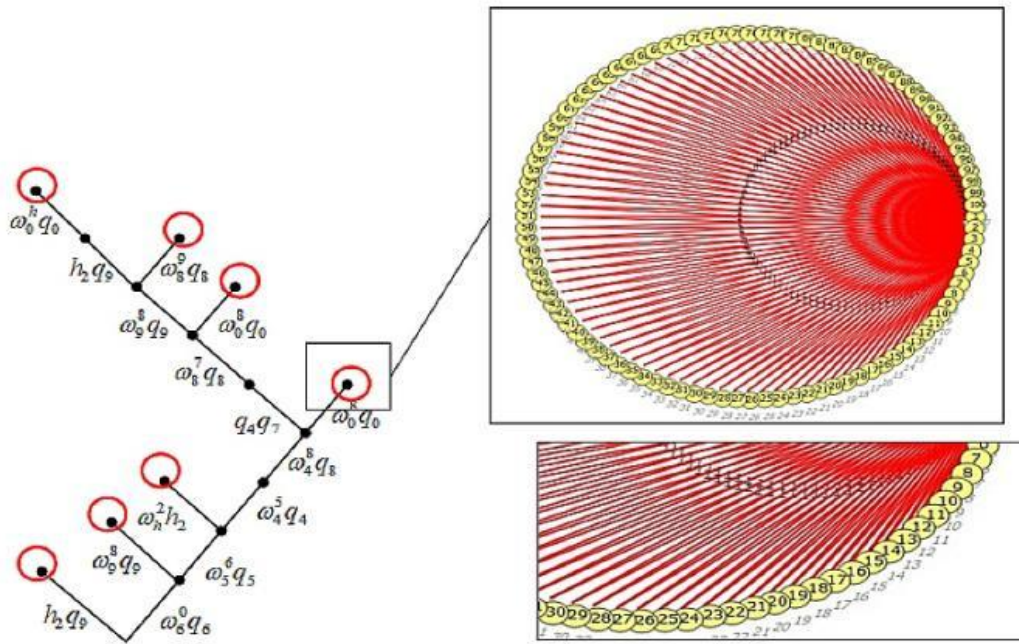


Figure 9. Ordered finding of the optimal number of teeth by parametric structure q_0

The optimal numbers of teeth calculated by the parametric structures are: $z_1 = z_4 = z_7 = 18$, $z_2 = z_5 = z_8 = 27$, modulus value $m = 2$. The next step is to optimize the parameters of the static function approximator model in order to approximate the given behaviour.

Laguerre filters are suitable for modelling strongly damped second-order processes and are defined by pole and row:

$$L_l(z, p) = \left(\frac{\sqrt{1-p^2}}{z-p} \right) \left(\frac{1-pz}{z-p} \right)^{l-1}, \text{ where } 0 \leq p < 1. \quad (12)$$

Depending on the choice of order and pole of the Laguerre filter, the dynamics of the engineering process can be approximated with different accuracy. The way clustering works can be explained as follows: The centre created so far is calculated. If it is less than the predefined threshold value, the data point is assigned to the centre to which it belongs. Once the Gaussian functions are created, they are normalized to obtain validity functions. In addition, the Gaussian standard deviations can be globally modified with a scaling parameter, so that transitions between the membership functions can be kept smooth.

4.2. Numerical simplification of automatic transmission models

For methodological support, a numerical derivation scheme is developed.

4.3. The idea of using a game tree structures as a black box

The idea of local model networks is to model a given globally nonlinear behavior with locally valid models. For this purpose, a validity function exists for each local model, which takes values between 0 and 1.

The possible input output variables for the dynamic local model network are given by: $\hat{u} = [I_s, T_w, \omega_F]$ and $\hat{y} = [T_F, \omega_w]$. Depending on the design of the hydraulic control and the number of switching elements, the number of control signals that are used as input variables changes. In addition, torque and output speed are used as physical input variables. Figure 10 shows the dynamic structure of a black box model with local model networks for a game- tree graph.

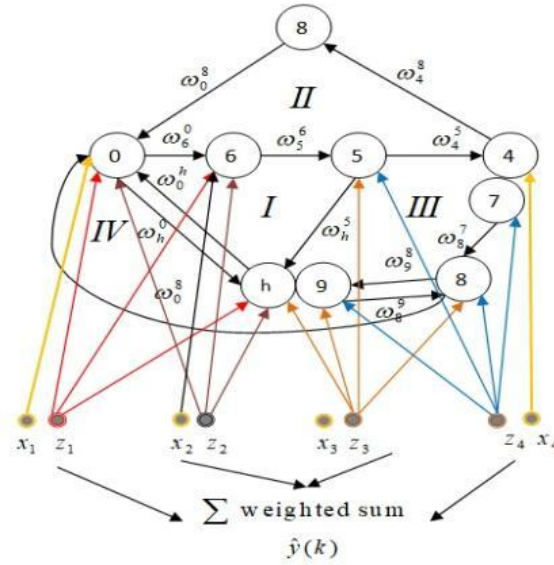


Figure 10. Dynamic black box model structure with local model networks for game graph

For the graph model a system of contour equations of velocity $\omega_{i,i-1}$, peripheral velocities $\omega_{i,i-1} \times r_{Ai}$, angular accelerations $\varepsilon_{i,i-1}$, and tangential $\varepsilon_{i,i-1} \times r_{Ai}$ and centripetal accelerations $\omega_i^2 \cdot r_{Ai,Ai+1}$ was generated systems of equations (13 – 16):

$$\left\{ \begin{array}{l} \sum_{(i)} \omega_{i,i-1} = 0 \\ \omega_{6,0} + \omega_{5,6} + \omega_{h2,5} + \omega_{0,h2} = 0 \\ \omega_{6,0} + \omega_{5,6} + \omega_{4,5} + \omega_{8,7} + \omega_{0,8} = 0 \\ \omega_{4,5} + \omega_{8,7} + \omega_{9,8} + \omega_{5,h2} = 0 \\ \omega_{6,0} + \omega_{5,6} + \omega_{h2,5} + \omega_{8,9} + \omega_{0,8} = 0 \\ \sum_{(i)} r_{Ai} \times \omega_{i,i-1} = 0 \end{array} \right. \quad (13)$$

$$\left\{ \begin{array}{l} \sum_{(i)} r_{Ai} \times \omega_{i,i-1} = 0 \\ r_6 \times \omega_{5,6} + (r_4 + r_5) \times \omega_{h2,5} = 0 \\ r_6 \times \omega_{5,6} + r_4 \times \omega_{4,5} + r_7 \times \omega_{8,7} + r_9 \times \omega_{9,8} = 0 \\ r_4 \times \omega_{4,5} + r_7 \times \omega_{8,7} + r_9 \times \omega_{9,8} + (r_4 + r_5) \times \omega_{5,h2} = 0 \\ r_6 \times \omega_{5,6} + (r_4 + r_5) \times \omega_{h2,5} + r_9 \times \omega_{8,9} = 0 \end{array} \right. \quad (14)$$

$$\left\{ \begin{array}{l} \sum_{(i)} \varepsilon_{i,i-1} = 0 \\ \varepsilon_{6,0} + \varepsilon_{5,6} + \varepsilon_{h2,5} + \varepsilon_{0,h2} = 0 \\ \varepsilon_{6,0} + \varepsilon_{5,6} + \varepsilon_{4,5} + \varepsilon_{8,7} + \varepsilon_{0,8} = 0 \\ \varepsilon_{4,5} + \varepsilon_{8,7} + \varepsilon_{9,8} + \varepsilon_{5,h2} = 0 \\ \varepsilon_{6,0} + \varepsilon_{5,6} + \varepsilon_{h2,5} + \varepsilon_{8,9} + \varepsilon_{0,8} = 0 \end{array} \right. \quad (15)$$

$$\left\{ \begin{array}{l} \sum_{(i)} r_{Ai} \times \varepsilon_{i,i-1} - \omega_i^2 \cdot r_{Ai,Ai+1} = 0 \\ r_6 \times \varepsilon_{5,6} + (r_4 + r_5) \times \varepsilon_{h2,5} = 0 \\ r_6 \times \varepsilon_{5,6} + r_4 \times \varepsilon_{4,5} + r_7 \times \varepsilon_{8,7} + r_9 \times \varepsilon_{9,8} = 0 \\ r_4 \times \varepsilon_{4,5} + r_7 \times \varepsilon_{8,7} + r_9 \times \varepsilon_{9,8} + (r_4 + r_5) \times \varepsilon_{5,h2} = 0 \\ i_{6,9} = \omega_{6,0} / \omega_{9,0} = \\ r_6 \times \varepsilon_{5,6} + (r_4 + r_5) \times \varepsilon_{h2,5} + r_9 \times \varepsilon_{8,9} = 0 \end{array} \right. \quad (16)$$

Where, in the equations, the designations mean successively: $\omega_{i,i-1}$ – relative angular velocity vector of the element i relative to the previous element $i-1$, $\omega_{i,0}$ – vector of the absolute angular velocity vector of the element (relative to the fixed base), $r_{Ai} = r_{OAi}$ – the radius of the vector of point A_i (point A on element i), $r_{Ai,Ai+1} = r_{Ai+1} - r_{Ai}$, $\varepsilon_{i,j-1}$ – vector of relative angular acceleration of the element relative to the previous element $i-1$.

The described numerical simplification procedure was applied to numerically simplify the model of a complex four-speed automatic transmission. The required algorithms were implemented in Matlab. In addition, local model networks and simulation data were created and taught from the development of the transmission control unit functions, in which a wide range of different switching. To apply the black-box idea for parametric graphs in the analysis of planetary gears, the following steps, which are shown in the diagram in Figure 11, must be followed.

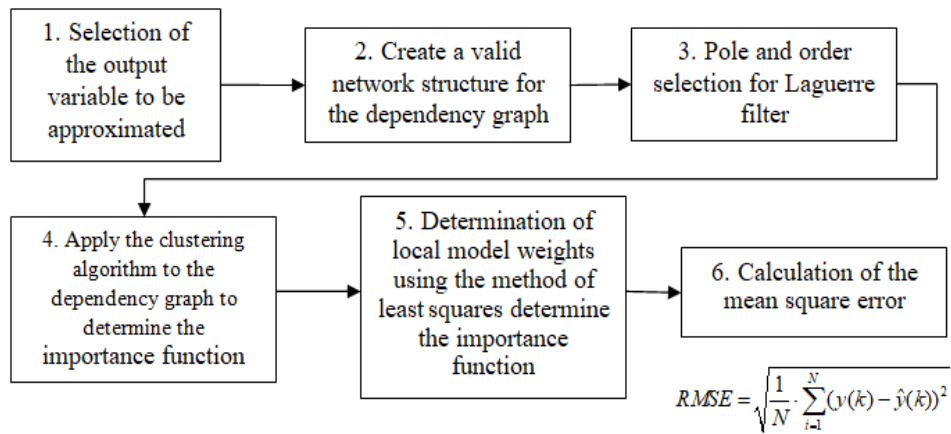


Figure 11. Steps to be taken for numerical modelling of automatic transmissions using dynamic local model networks

The sampling time of the simulation data was 0.01 seconds, the simulation time started at zero seconds and ended at 2500 seconds. The first 80% of this data was used as learning files (.lrm) while the last 20% was used as testing files. The poles of the two Laguerre filters were set to values between 0.75 and 0.90, corresponding to fading times between 0.15 and 0.25 seconds.

Depending on the order of the Laguerre filter in the operation of the automatic transmission, a very large number of shift elements can be quickly obtained. Figure 12 shows an example of the torque output (learning file). Figure 13 shows an example of the output torque waveform (test file).

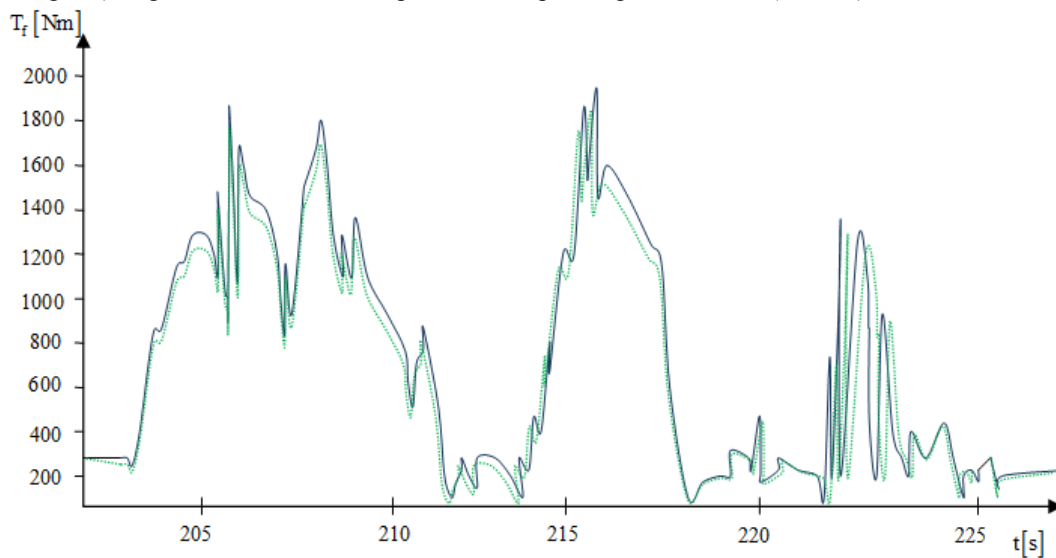


Figure 12. Example of output torque waveform (learning), without local filter —, reference

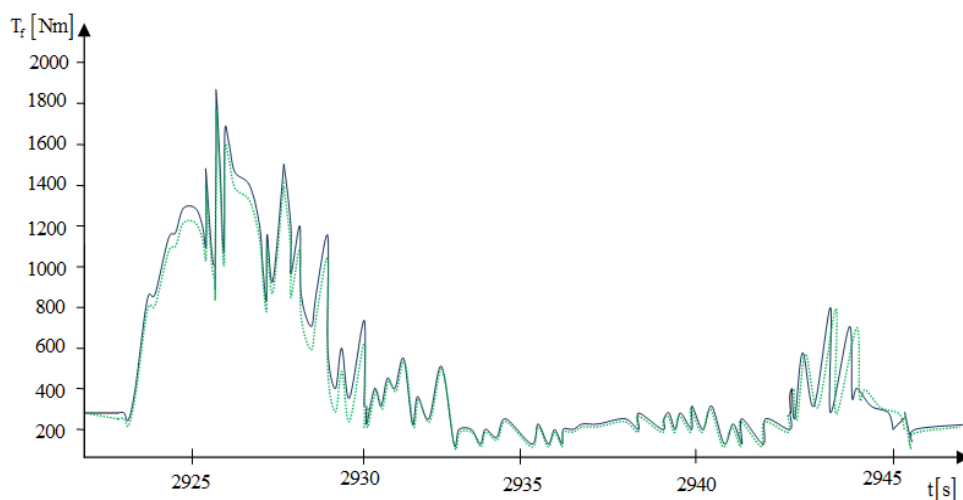


Figure 13. Example of output torque waveform (testing), without local filter —, reference

5. Conclusions

This paper presents the concept of black box in the analysis of automatic transmissions, where the role of the black box is played by game tree graphs. The mathematical model of automatic transmissions (or planetary gearboxes) can be written analytically as a system of algebraic-differential-integral equations or graphically as a dependence graph in which the links (relations) are any arithmetically determined input variables and the vertices are time-dependent output quantities. If one were to change the numerical values of the input variables in the mathematical model, one would obtain changes in the values of the output variables. To obtain a different planned behavior of the system (element), one can often do many ways of changing the numerical values of the input variables: which to change? how to change – increase, no change, decrease? in what order? etc.. Such conjecturing is akin to subjectively (according to a given designer) changing numerical values in a mathematical model. It means that another designer, according to his own experience, may subjectively do quite differently redesigning of the system (element) for identical new working conditions as the previous designer. Of course, objective matters are not solved even by CAD using Matlab, and therefore "structuring the designer's thoughts" must be done so that the sequence of changes and controls follows the properties of the mathematical and physical model. In such a situation, the designer's subjective behavior in the design process is practically limited to only the initial vertex of the dependency graph decomposition. Therefore, ultimately, different tree structures with cycles and different tree structures playing out parametrically for different initial vertices of the decomposition can be obtained as corresponding sets of design guidelines. This approach is different from the existing literature on parametric-playing automata and their applications related to control systems, operating systems, natural language level knowledge representation, sentence semantics recognition, natural language processing, cybernetic system behaviour programming, etc. The paper presents a scheme for the automatic determination of numerically simplified automatic transmissions as well as the possibility of determining the optimum number of teeth. The idea builds on existing studies in the literature on the Black Box Model for systematic numerical simplification of physical automatic transmission models. However, the authors additionally made changes and generalizations to describe the idea for a complex four-speed automatic transmission. The paper describes the concept and applicability of the presented methodology. Further work should focus on extending the research to real operating parameters of different types of gearboxes. Additionally, an integrated decision-making system should be built in order to optimize the operation of gearboxes.

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